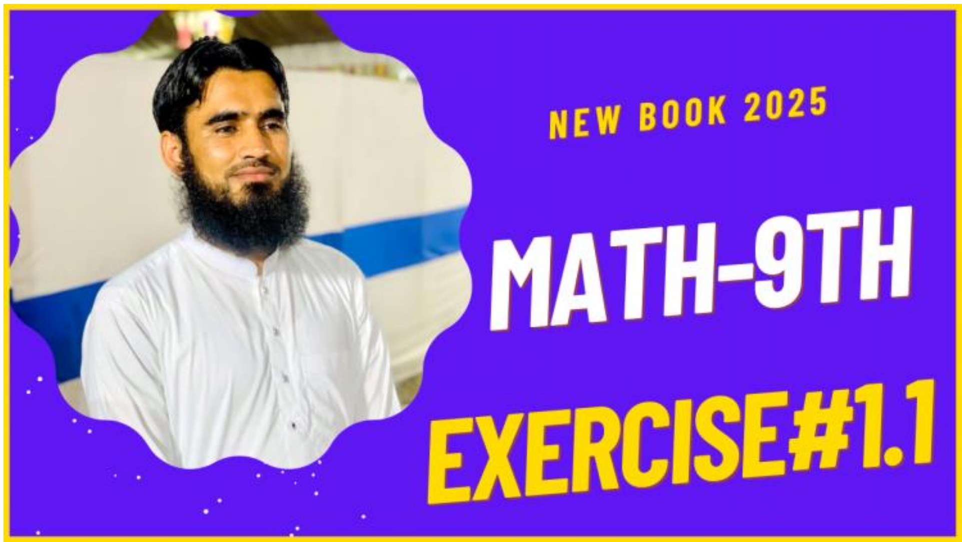
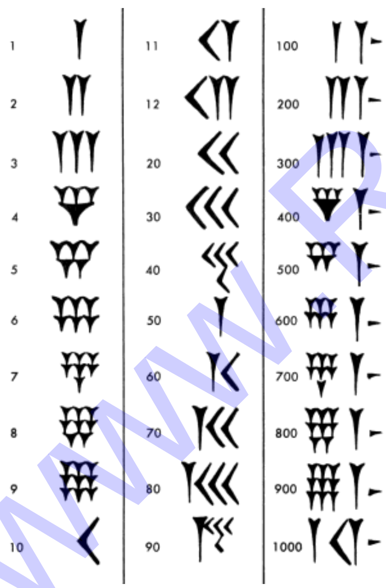


For video lecture please visit https://www.youtube.com/@MathwayAcademy

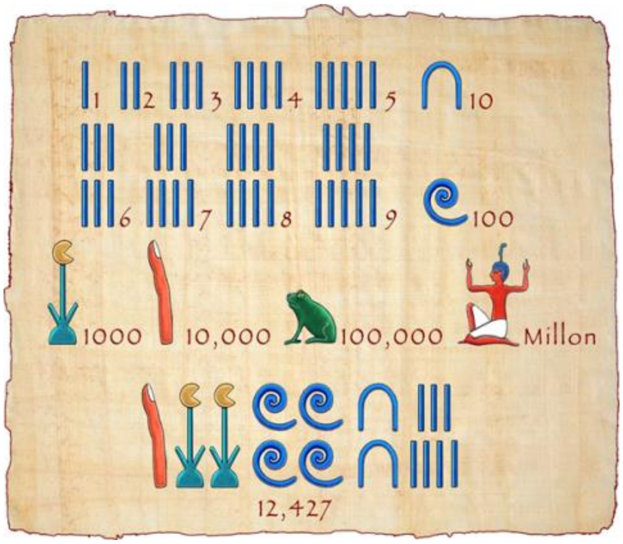
HISTORY OF NUMBERS:
Thousands of years from ancient civilization to modern Arabic system

Sumerians (4500-1900 BCE)

Sexagesimal (base 60) system for counting
Used cone , bead, large cone, large perforated cone, sphere and perforated sphere, corresponding to 1, 10. 60, 600.



Egyptians (3000 - 2000 BCE)
Base 10 (decimal system) for counting



Romans (500BCE-500CE)

Romans numerals use 7 letters to represent different numbers.

$I = 1,\ V = 5,\ X = 10,\ L = 50,\ \ C = 100,\ D = 500, M = 1000$

1	I	11	XI	50	L
2	II	12	XII	100	C
3	III	13	XIII	500	D
4	IV	14	XIV	1000	M
5	V	15	XV		
6	VI	16	XVI		
7	VII	17	XVII		
8	VIII	18	XVIII		
9	IX	19	XIX		
10	X	20	XX		

INDIANS (500 - 1200 CE)

- Concept of zero(0)
- Contribution to decimal (base 10) system

Number system used today was invented by Indians still called **Indo-Arabic**.
Indian invented and Arab merchants took to the Western world.

ARABS (800 - 1500 CE)

- Introduced Arabic numerals (0 - 9) to Europe.
- Muhammad ibn MUSA Al-Khwarizmi introduced algebra in 9th century
- Successors Al-Karaji expanded his work, contributing to advancements in various mathematical domains.

Modern era (1700 - present):

Modern number systems e.g. binary system(base -2) and hexadecimal system (base -16)

Denary/Decimal	Binary	Hexadecimal
Base 10 Number System	Base 2 Number System	Base 16 Number System
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Natural numbers:

- Natural numbers counts from 1 to infinity.
- They are the positive counting numbers.
- Represented by **'N'**.
- Generally use for counting.

$N = \{1,2,3,4,5,6,7,\ldots\}$

Whole numbers:

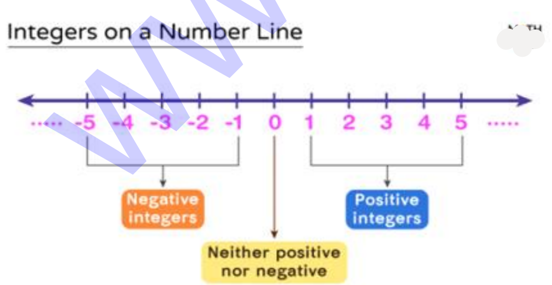
- count from zero to infinity.
- Do not include fractions or decimals.
- Represented by 'W'.

$W = \{0,1,2,3,4,5,\ldots\}$

Integers:

- Set of numbers including all the positive counting numbers, zero as well as all negative counting numbers which count from negative infinity to positive infinity.
- The set doesn't include fractions and decimals.
- Represented by 'Z'.

$Z = \{\ldots -5. -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

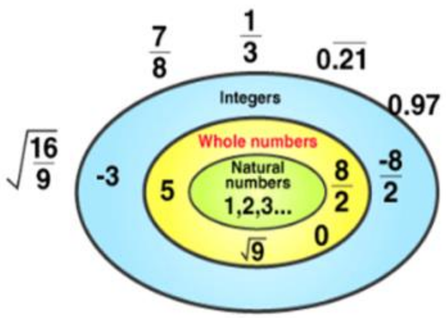


Rational number:

⇒ Any number which can be represented in the form of p/q where q ≠ 0.

⇒ Any fraction, where the denominator and numerator are integers and the denominator is not equal to zero.

⇒ When the rational number (i.e., Fraction) is divided,
the result will be in decimal form,
which may be either terminating decimal or
the repeating decimal.

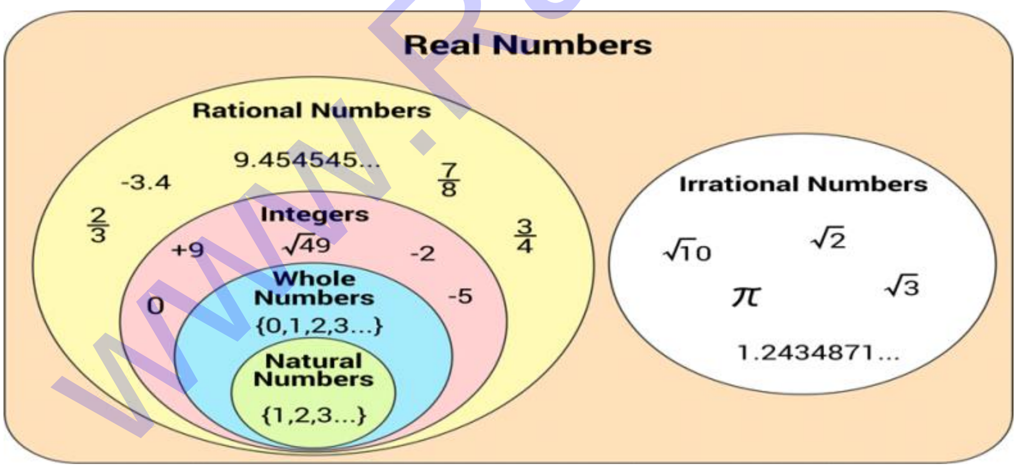


Irrational number:

- A real number that cannot be expressed as a ratio of integers;
- for example, $\sqrt{2}$ is an irrational number.
- We cannot express any irrational number in the form of a ratio, such as p/q, where p and q are integers, q≠0.
- The decimal expansion of an **irrational number is *neither terminating nor recurring***

Real numbers:

- Numbers that include both rational and irrational numbers.
- Rational numbers such as integers (-2, 0, 1),
- fractions(1/2, 2.5)
- irrational numbers such as $\sqrt{3}$, π(22/7), etc., are all real numbers.



PROPERTIES OF REAL NUMBERS

Name of property	$\forall$ **a, b, c** $\in$ **R**	
Operation	ADDITION	MULTIPLICATION

$2+3=5 \in R$

$1 \in R$

Name of property	$\forall\, a, b, c \in R$	
Operation	ADDITION	MULTIPLICATION
Closure	$a + b \in R$	$ab \in R$
Commutative	$a + b = b + a$	$ab = ba$
Associative	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$
Identity	$a + 0 = a = 0 + a$	$a \times 1 = a = 1 \times a$
Inverse	$a + (-a) = -a + a = 0$	$a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$

Distributive property	$a(b + c) = ab + ac$
	$(a + b)c = ac + bc$

Properties of Equality

Reflexive	$\forall\, a \in R,$	$a = a$
Symmetric	$\forall\, a, b \in R,$	$a = b \Rightarrow b = a$
Transitive	$\forall\, a, b, c \in R$	$a = b \wedge b = c \Rightarrow a = c$
Additive	$\forall\, a, b, c \in R$	$a = b \Rightarrow a + c = b + c$
Multiplicative	$\forall\, a, b, c \in R$	$a = b \Rightarrow ac = bc$
Cancellation (+)	$\forall\, a, b, c \in R$	$a + c = b + c \Rightarrow a = b$
Cancellation (×)	$\forall\, a, b, c \in R \wedge c \neq 0,$	$ac = bc \Rightarrow a = c$

Order properties

Trichotomy property	$\forall\, a, b, c \in R$ $a = b$ or $a > b$ or $a < b$
Transitive	$\forall\, a, b, c \in R$ • $a > b \wedge b > c \Rightarrow a > c$ • $a < b \wedge b < c \Rightarrow a < c$

$2 + 3 = 5 \in R$

$2 \times 3 = 6 \in R$

$2, 3, 4$

$(2+3)+4 = 2+(3+4)$

$5+4 = 2+7$

$9 = 9$

$a + 0 = a$

$2 = 2$

$2 = \frac{4}{2} \Rightarrow \frac{4}{2} = 2$

$2 = \frac{4}{2},\ \frac{4}{2} = \frac{8}{4}$

$2 = \frac{8}{4}$

P/q, Terminating, Repeating

EXERCISE 1.1

1. **Identify each of the following as a rational or irrational number:**

(i) 2.353535 (ii) $0.\overline{6}$ (iii) 2.236067… (iv) $\sqrt{7}$

$0.6666....$

(i) Rational

(ii) Rational

(iii) Irrational

(iv) Irrational

$\frac{22}{7}$ Q

(v) e (vi) π (vii) $5+\sqrt{11}$ (viii) $\sqrt{3}+\sqrt{13}$

(v) Irrational

(vi) Irrational

(vii) Irrational

(vi) Irrational

(vii) Irrational

(viii) Irrational

(ix) $\frac{15}{4}$ (x) $(2-\sqrt{2})(2+\sqrt{2})$

ix) Rational

$(a-b)(a+b) = a^2 - b^2$

$2^2 - (\sqrt{2})^2$

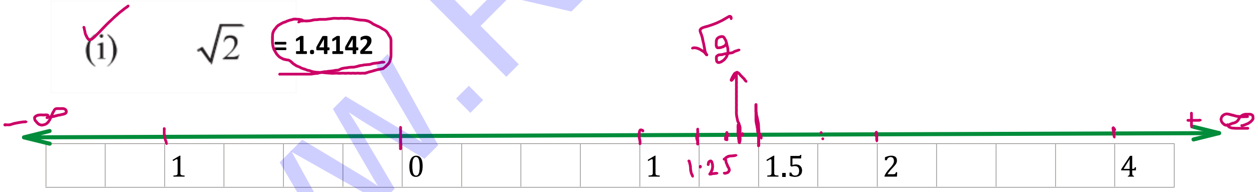
$= 4 - 2$

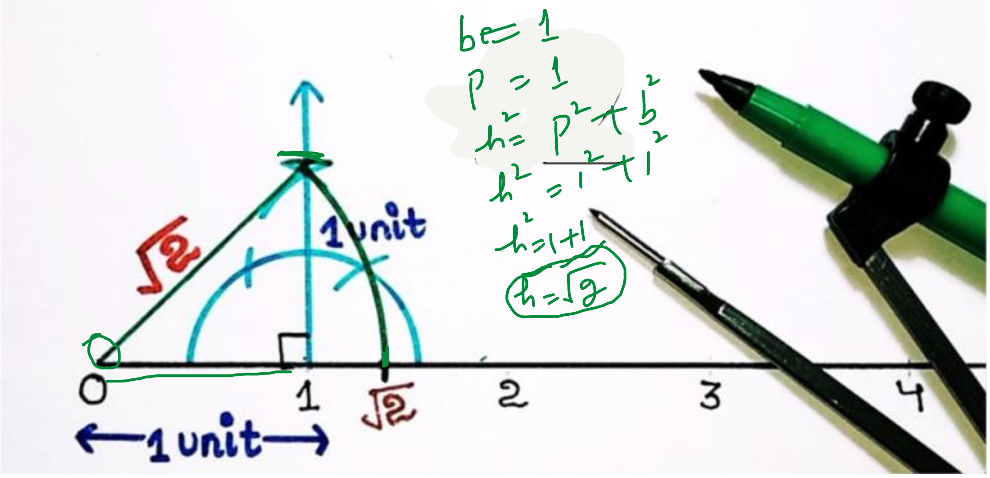
$= 2$

x) Rational

2. Represent the following numbers on number line:

(i) $\sqrt{2}$ = 1.4142

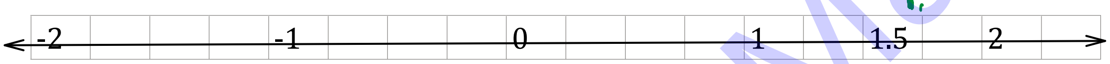




$b = 1$
$p = 1$
$h^2 = p^2 + b^2$
$h^2 = 1^2 + 1^2$
$h^2 = 1 + 1$
$h = \sqrt{2}$

(ii) $\sqrt{3}$ **=1.7320**

$\sqrt{3}$



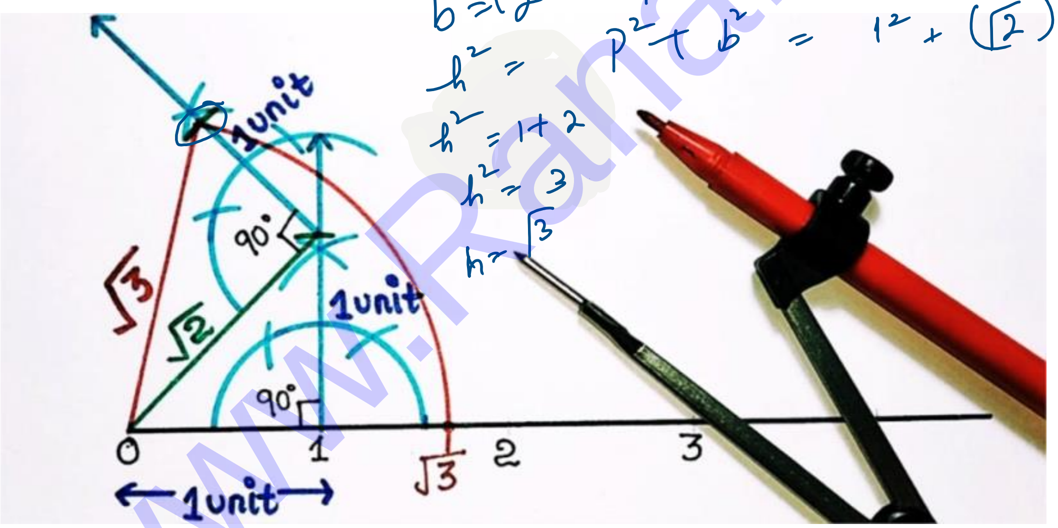
$b = \sqrt{2}$ $p = 1$ $h = ?$

$h^2 = p^2 + b^2 = 1^2 + (\sqrt{2})^2$

$h^2 = 1 + 2$

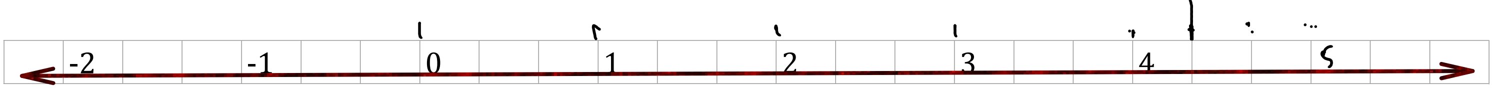
$h^2 = 3$

$h = \sqrt{3}$

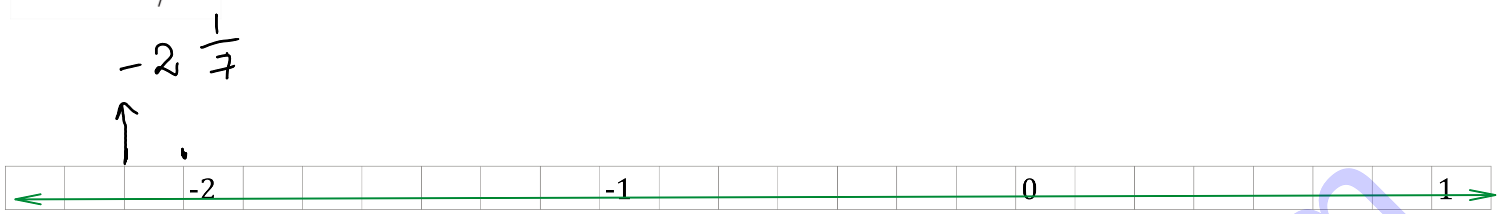


(iii) $4\frac{1}{3}$

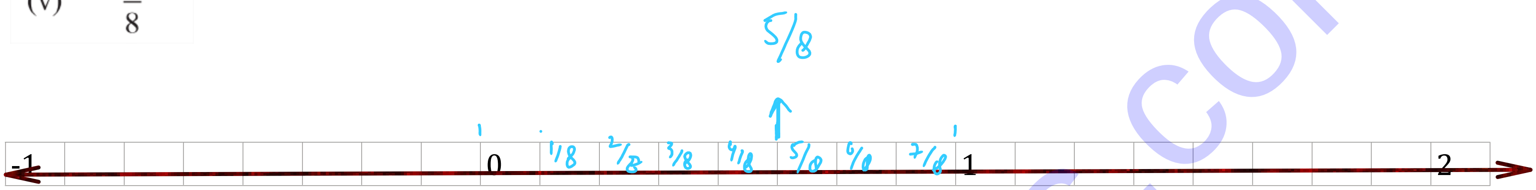
$4\frac{1}{3}$



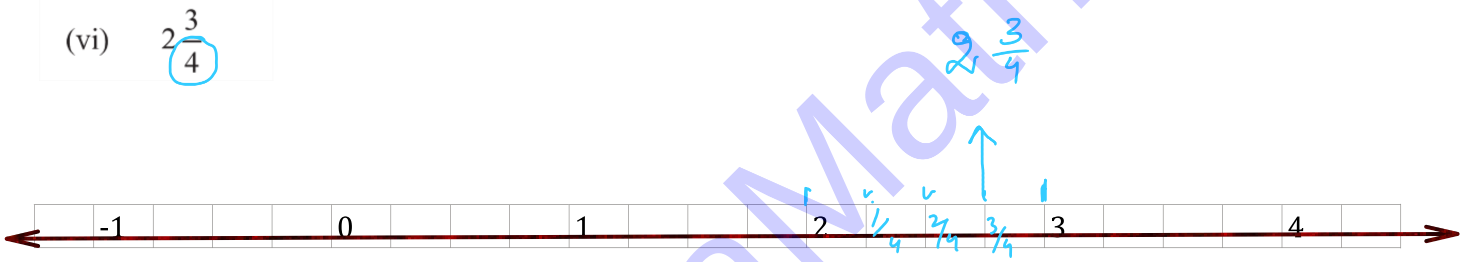
(iv) $-2\frac{1}{7}$



(v) $\frac{5}{8}$



(vi) $2\frac{3}{4}$



3. Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i) $0.\overline{4}$

Let $x = 0.\overline{4}$

$x = 0.444\ldots \quad \text{—} ①$

multiplying 10 b.s

$10x = 10(0.444\ldots)$

$10x = 4.444\ldots$

$10x = 4 + 0.444\ldots$

Using ①

$10x = 4 + x$

$10x - x = 4$

$9x = 4$

$$\boxed{x = \frac{4}{9}}$$

$10x = 4 + x$

$10x - x = 4$

(ii) $0.\overline{37}$

Let $x = 0.\overline{37}$

$x = 0.373737\cdots$ —— (1)

multiplying by 100

$100x = 100(0.373737\cdots)$

$100x = 37.373737\cdots$

$100x = 37 + 0.373737\cdots$

$100x = 37 + x$

$100x - x = 37$

$99x = 37$

$\boxed{x = \frac{37}{99}}$

(iii) $0.\overline{21}$

Let $x = 0.\overline{21}$

$x = 0.212121\cdots$

multiplying 100 b.s

$100x = 100(0.212121\cdots)$

$100x = 21.212121\cdots$

$100x = 21 + 0.212121\cdots$

using (1)

$100x = 21 + x$

$100x - x = 21$

$99x = 21$

$\boxed{x = \frac{21}{99}}$

4. Name the property used in the following:

(i) $(a + 4) + b = a + (4 + b)$

Associative property w.r.t '+'

(ii) $\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$

Commutative Property w.r. '+'

(iii) $x - x = 0$ $\quad x + (-x) = 0$

Additive Inverse

(iv) $a(b + c) = ab + ac$

Left Distributive property

(v) $16 + 0 = 16$

Additive Identity

(vi) $100 \times 1 = 100$

multiplicative Identity

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$

Associative property w.r.t '×'

(viii) $ab = ba$

Commutative property w.r. '×'

5. Name the property used in the following:

(i) $-3 < -1 \Rightarrow 0 < 2$ $\quad -3 + 3 < -1 + 3$

Additive property

(ii) If $a < b$ then $\frac{1}{a} > \frac{1}{b}$

$\frac{a}{ab} < b \times \frac{1}{ab}$

$\frac{1}{b} < \frac{1}{a}$

multiplicative property.

(iii) If $a < b$ then $a + c < b + c$

Additive property

(iv) If $ac < bc$ and $c > 0$ then $a < b$

Cancellation property

(v) If $ac < bc$ and $c < 0$ then $a > b$

Cancellation property

(v) If $ac < bc$ and $c < 0$ then $a > b$

Cancellation property

(vi) Either $a > b$ or $a = b$ or $a < b$

Trichotomy Property

6. Insert two rational numbers between:

(i) $\frac{1}{3}$ and $\frac{1}{4}$

1st No. $= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{4}\right] = \frac{1}{2}\left[\frac{4+3}{12}\right]$

$= \frac{1}{2}\left[\frac{7}{12}\right] = \frac{7}{24}$

2nd No. $= \frac{1}{2}\left[\frac{1}{3} + \frac{7}{24}\right] = \frac{1}{2}\left[\frac{8+7}{24}\right]$

$= \frac{15}{48}$

$\left\{\frac{7}{24}, \quad \frac{15}{48}\right\}$

(ii) 3 and 4

1st no. $= \frac{1}{2}[3+4] = \frac{7}{2}$

(ii) 3 and 4

1st no. $= \frac{1}{2}[3+4] = \frac{7}{2}$

2nd no. $= \frac{1}{2}\left[3 + \frac{7}{2}\right]$

$= \frac{1}{2}\left[\frac{6+7}{2}\right]$

$= \frac{1}{2}\left[\frac{13}{2}\right] = \frac{13}{4}$ Ans

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

1st no. $= \frac{1}{2}\left[\frac{3}{5} + \frac{4}{5}\right]$

$= \frac{1}{2}\left[\frac{3+4}{5}\right] = \frac{1}{2}\left[\frac{7}{5}\right]$

$= \frac{7}{10}$

2nd no. $= \frac{1}{2}\left[\frac{7}{5} + \frac{4}{5}\right] = \frac{1}{2}\left[\frac{7+4}{5}\right]$

$= \frac{11}{10}$