N11 EX 1.2

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FOR VIDEO LECTURE VISIT
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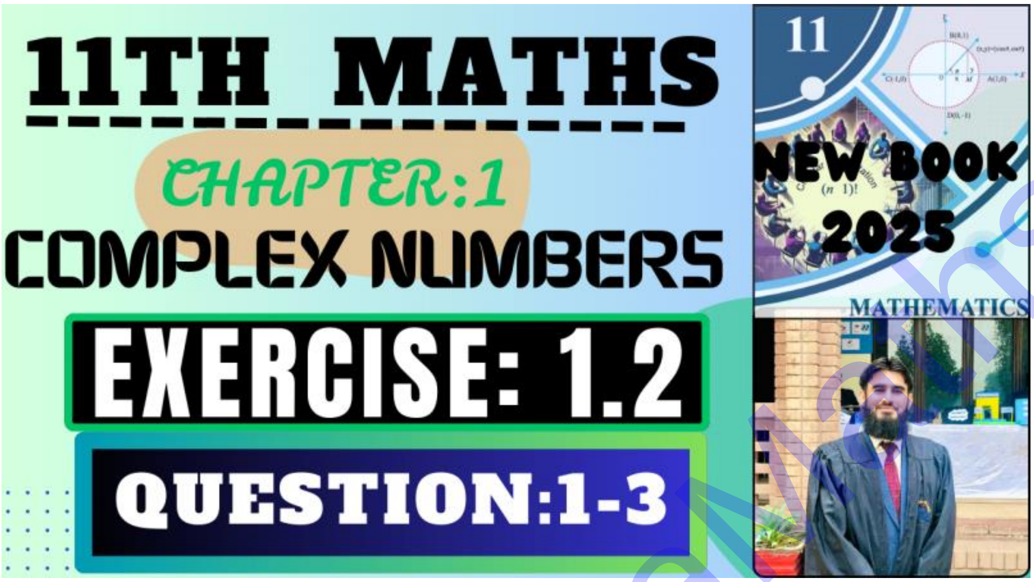


Equality of two complex numbers

The two complex numbers $z_1 = a + bi$ and $z_2 = c + di$ are said to be equal

iff their real and imaginary parts are equal

i.e., $a + bi = c + di \Leftrightarrow a = c$ and $b = d$.

Square root of a complex number

The square root of a complex number is another complex number that,

when squared give the original complex number.

$z = x + yi \qquad \sqrt{z} = \sqrt{x+yi}$

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

Exercise 1.2

1. **Find the values of x and y in each of the following:**

(i) $x + iy + 2 - 3i = i(5 - i)(3 + 4i)$

$$x+iy+2-3i = i(15+20i-3i-4i^2)$$

$$x+iy+2-3i = i(15+17i+4)$$

$$x+iy+2-3i = i(19+17i) \qquad \because i^2=-1$$

$$x+iy+2-3i = 19i+17i^2$$

$$x+iy+2-3i = 19i-17$$

$$x+iy = 19i-17-2+3i$$

$$x+iy = -19+22i$$

By comparing real and Imaginary Part

$$\boxed{x=-19} \qquad y=22$$

(ii) $(x + iy)(1 - i) = (2 - 3i)(-5 + 5i)\left(-\frac{i3}{5}\right)$

$$(x+iy)(1-i) = [2(-5+5i)-3i(-5+5i)]\left(-\frac{i3}{5}\right)$$

$$(x+iy)(1-i) = [-10+10i+15i-15i^2]\left(-\frac{i3}{5}\right)$$

$$(x+iy)(1-i) = [-10+25i-15(-1)]\left(-\frac{i3}{5}\right)$$

$$(x+iy)(1-i) = [-10+25i+15]\left(-\frac{i3}{5}\right)$$

$$= [5+25i]\left(-\frac{i3}{5}\right)$$

$$(x+iy)(1-i) = \cancel{5}\left(-\frac{i3}{\cancel{5}}\right) + \overset{5}{\cancel{25}}i\left(-\frac{i3}{\cancel{5}}\right)$$

$$(x+iy)(1-i) = -3i+5i(-i3)$$

$$(x+iy)(1-i) = -3i-15i^2 \qquad \because i^2=-1$$

$$(x+iy)(1-i) = -3i-15(-1)$$

$$(x+iy)(1-i) = -3i - 15(-1)$$

$$(x+iy)(1-i) = -3i + 15$$

$$(x+iy)(1-i) = 15 - 3i$$

$$x+iy = \frac{15-3i}{1-i}$$

$$x+iy = \frac{15-3i}{1-i} \times \frac{1+i}{1+i}$$

$$x+iy = \frac{15(1+i) - 3i(1+i)}{1^2 - i^2}$$

$$x+iy = \frac{15 + 15i - 3i - 3i^2}{1 - i^2}$$

$\because i^2 = -1$

$$x+iy = \frac{15 + 12i - 3(-1)}{1-(-1)}$$

$$x+iy = \frac{15 + 12i + 3}{1+1}$$

$$x+iy = \frac{18+12i}{2}$$

$$x+iy = \frac{18}{2} + \frac{12i}{2}$$

$$x+iy = 9 + 6i$$

By Comparing real and Imaginary Part

$x = 9$ $\quad$ $y = 6$

(iii) $\frac{x}{2+i}+\frac{y}{3-i}=4+5i$

$$\frac{x}{2+i}+\frac{y}{3-i}=4+5i$$

$$\frac{x(3-i)+y(2+i)}{(2+i)(3-i)}=4+5i$$

$$\frac{3x-xi+2y+yi}{2(3-i)+i(3-i)}=4+5i$$

$$\frac{3x-xi+2y+yi}{6-2i+3i-i^2}=4+5i \qquad \because i^2=-1$$

$$\frac{3x+2y+yi-xi}{6+i-(-1)}=4+5i$$

$$\frac{3x+2y+(y-x)i}{6+i+1}=4+5i$$

$$\frac{3x+2y+(y-x)i}{7+i}=4+5i$$

$$3x+2y+(y-x)i=(4+5i)(7+i)$$

$$3x+2y+(y-x)i=28+4i+35i+5i^2$$

$$3x+2y+(y-x)i=28+39i-5$$

$$3x+2y+(y-x)i=23+39i$$

Comparing b.s

$$3x+2y=23 \quad \text{—(1)} \qquad y-x=39$$

$$y=39+x$$

$y = 39 + x$

Put in ①

$3x+2(39+x) = 23$

$3x + 78 + 2x = 23$

$5x = 23 - 78$

$5x = -55$

$x = -\cancel{\frac{55}{5}}$

$x = -11$

$y = 39 + x$

Put $x = -11$

$y = 39 - 11$

$y = 28$

2. If $z_1 = -13 + 24i$ and $z_2 = x + yi$, find the values of x and y such $z_1 - z_2 = -27 + 15i$

$z_1 = -13 + 24i$

$z_2 = x + yi$

$z_1 - z_2 = -27 + 15i$

$-13 + 24i - (x + yi) = -27 + 15i$

$-13 + 24i + 27 - 15i = x + yi$

$-13 + 27 + 24i - 15i = x + yi$

$14 + 9i = x + yi$

Comparing real and Imaginary Parts

$14 = x$

$9 = y$

$x^2 + (iy)^2 + 2(x)(yi) = 25 + 60i$

$\ldots = 25 + 60$

$x^2 + (iy)^2 + 2(x)(yi) = 25 + 60$

$\boxed{x^2 - y^2} + 2xyi = 25 + 60$

3. Find the value of x and y if :

(i) $(x + iy)^2 = 25 + 60i$

$x^2 - y^2 = 25 \quad \text{--- (1)}$

$2xy = 60$

$xy = \frac{60}{2}$

$xy = 30$

$y = \frac{30}{x}$

put in (1)

$(x+iy)^2 = 25 + 60i$

Taking square root b:s

$\sqrt{(x+iy)^2} = \pm\sqrt{25 + 60i}$

$z = x + iy = \pm\sqrt{25 + 60i}$

$|z| = \sqrt{x^2 + y^2} = \sqrt{25^2 + 60^2}$

$= \sqrt{625 + 3600} = \sqrt{4225}$

$|z| = 65$

By Using square root formula

$\sqrt{z} = \pm\left(\sqrt{\frac{|z| + x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z| - x}{2}}\right)$

$= \pm\left(\sqrt{\frac{65 + 25}{2}} + \frac{i(60)}{|60|}\sqrt{\frac{65 - 25}{2}}\right)$

$= \pm\left(\sqrt{\frac{90}{2}} + \frac{i(\cancel{60})}{\cancel{60}}\sqrt{\frac{40}{2}}\right)$

$= \pm\left(\sqrt{45} + i\sqrt{20}\right)$

$\sqrt{z} = \pm\left(\sqrt{9 \times 5} + i\sqrt{4 \times 5}\right)$

$= \quad \; i\,2\sqrt{5})$

$$x+iy = \pm(3\sqrt{5} + i2\sqrt{5})$$

$$\boxed{x = \pm 3\sqrt{5}} \quad \boxed{y = \pm 2\sqrt{5}}$$

(ii) $(x+iy)^2 = 64+48i$

$$(x+iy)^2 = 64+48i$$

Taking square root b.s

$$x+iy = \pm\sqrt{64+48i}$$

$$|z| = \sqrt{x^2+y^2} = \sqrt{64^2+48^2}$$

$$|z| = \sqrt{4096+2304}$$

$$|z| = \sqrt{6400} = 80$$

By using square root formula

$$x+yi = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{80+64}{2}} + \frac{i(48)}{|48|}\sqrt{\frac{80-64}{2}}\right)$$

$$x+iy = \pm\left(\sqrt{\frac{144}{2}} + \frac{i(\cancel{48})}{\cancel{48}}\sqrt{\frac{16}{2}}\right)$$

$$x+iy = \pm\left(\sqrt{72} + i\sqrt{8}\right)$$

$$x+iy = \pm(\sqrt{72} + i\sqrt{8})$$

$$x+iy = \pm(\sqrt{36\times 2} + i\sqrt{4\times 2})$$

$$x+iy = \pm(6\sqrt{2} + i2\sqrt{2})$$

$$\boxed{\begin{aligned} x &= \pm 6\sqrt{2} \\ y &= \pm 2\sqrt{2} \end{aligned}}$$

(iii) $x + iy = \dfrac{-2-5i}{(1+3i)^3}$

$$x+iy = \frac{-2-5i}{(1+3i)^3}$$

$$\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$x+iy = \frac{-2-5i}{(1)^3 + (3i)^3 + 3(1)(3i)(1+3i)}$$

$$x+iy = \frac{-2-5i}{1 + 27i^3 + 9i(1+3i)}$$

$$x+iy = \frac{-2-5i}{1+27i^2\cdot i + 9i + 27i^2}$$

$$x+iy = \frac{-2-5i}{1+27(-1)\cdot i + 9i + 27(-1)}$$

$$x+iy = \frac{-2-5i}{1-27i+9i-27}$$

$$x+iy = \frac{-2-5i}{-26-18i}$$

$$x+iy = \frac{-2-5i}{-26-18i} \times \frac{-26+18i}{-26+18i}$$

$$x+iy = \frac{-2(-26+18i) -5i(-26+18i)}{(-26)^2 - (18i)^2}$$

$$\because a^2-b^2 = (a+b)(a-b)$$

$$x+iy = \frac{+52-36i+130i-90i^2}{676-324i^2}$$

$$\because i^2=-1$$

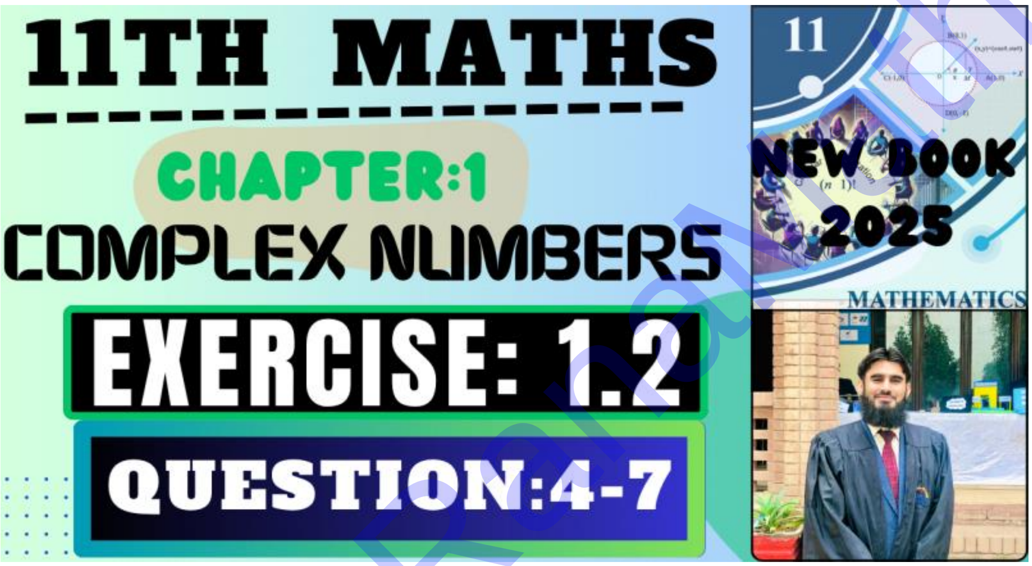
$$x+iy = \frac{52-94i+90}{676+324}$$

$$x+iy = \frac{142-94i}{1000}$$

$$x+iy = \frac{142}{1000} - \frac{94i}{1000}$$

$$x+iy = \frac{71}{500} - \frac{47i}{500}$$

$x = \frac{71}{500}$, $y = \frac{47}{500}$



Exercise 1.2

4. If $z_1 = 2 + 3i$ and $z_2 = 1 - \alpha$, find the value of α such that $Im(z_1 z_2) = 7$.

$z_1 = 2+3i \qquad z_2 = 1-\alpha$

$Z_1 = 2+3i \qquad Z_2 = 1-\alpha$

$Z_1Z_2 = (2+3i)(1-\alpha)$

$= 2(1-\alpha) + 3i(1-\alpha)$

$= 2 - 2\alpha + 3i - 3\alpha i$

$Z_1Z_2 = 2-2\alpha + (3-3\alpha)i$

By given Condition

$Im(Z_1Z_2) = 7$

$3-3\alpha = 7$

$3-7 = 3\alpha$

$-4 = 3\alpha$

$\boxed{\frac{-4}{3} = \alpha}$

$Z_1 = 2+3i \qquad Z_2 = 1-\alpha i$

$Z_1Z_2 = (2+3i)(1-\alpha i)$

$= 2(1-\alpha i) + 3i(1-\alpha i) \qquad \because i^2 = -1$

$= 2 - 2\alpha i + 3i - 3\alpha i^2$

$Z_1Z_2 = 2 - 2\alpha i + 3i + 3\alpha$

$= 2 + 3\alpha + (-2\alpha + 3)i$

$Im(Z_1Z_2) = 7$

$$Im(Z_1Z_2) = 7$$

$$-9\alpha + 3 = 7$$

$$-9\alpha = 7 - 3$$

$$-2\alpha = 4$$

$$\alpha = \frac{4}{-2}$$

$$\alpha = -2$$

5. If $z_1 = x + yi$ and $z_2 = a + bi$, find x, y, a and b such that $z_1 + z_2 = 10 + 4i$ and $z_1 - z_2 = 6 + 2i$

$Z_1 = x + yi \quad Z_2 = a + bi$

$Z_1 + Z_2 = 10 + 4i$

$x + yi + a + bi = 10 + 4i$

$(x + a) + (y + b)i = 10 + 4i$

$x + a = 10$ — (I) $\quad y + b = 4$ — (II)

$Z_1 - Z_2 = 6 + 2i$

$x + yi - (a + bi) = 6 + 2i$

$x + yi - a - bi = 6 + 2i$

$\ldots (y - b)i = 6 + 2i$

$x + yi - a - bi = 6 + 2i$

$x - a + (y - b)i = 6 + 2i$

$x - a = 6$ — (iii) $\quad y - b = 2$ — (iv)

Adding (i) & (iii) , (ii) & (iv)

$x + \cancel{a} = 10$

$x - \cancel{a} = 6$

$2x = 16$

$x = \frac{16}{2}$

$x = 8$

Put in (i)

$8 + a = 10$

$a = 10 - 8$

$a = 2$

$y + \cancel{b} = 4$

$y - \cancel{b} = 2$

$2y = 6$

$y = \frac{6}{2}$

$y = 3$

Put in (ii)

$3 + b = 4$

$b = 4 - 3$

$b = 1$

6. **Show that** $\forall\, z_1, z_2 \in \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

Let $z_1 = a + bi$ & $z_2 = c + di \in C$

then $\bar{z}_1 = \overline{a + bi} \qquad \bar{z}_2 = \overline{c + di}$

$\bar{z}_1 = a - bi \qquad \bar{z}_2 = c - di$

$R.H.S = \bar{z}\,\bar{z}$

$$R.H.S = \bar{z}_1 \bar{z}_2$$
$$= (a-bi)(c-di)$$
$$= a(c-di) - bi(c-di)$$
$$= ac - adi - bci + bdi^2$$
$$= ac - adi - bci + bd(-1)$$
$$\because i^2 = -1$$
$$= ac - adi - bci - bd$$
$$\bar{z}_1 \bar{z}_2 = (ac-bd) - (ad+bc)i$$

$$z_1 z_2 = (a+bi)(c+di)$$
$$= a(c+di) + bi(c+di)$$
$$= ac + adi + bci + bdi^2$$
$$= ac + adi + bci + bd(-1)$$
$$= ac + adi + bci - bd$$
$$z_1 z_2 = (ac-bd) + (ad+bc)i$$

$$L.H.S = \overline{z_1 z_2}$$
$$= (ac-bd) - (ad+bc)i$$
$$L.H.S = R.H.S$$

7. **Find the square root of the following complex numbers:**

(i) $-7-24i$

Let $z = -7 - 24i$

square root b.s

$$\sqrt{z} = \pm\sqrt{-7-24i}$$

$$|z| = \sqrt{x^2 + y^2} = \sqrt{(-7)^2 + (-24)^2}$$

$$|z| = \sqrt{49 + 576} = \sqrt{625} = 25$$

Square root $= \sqrt{z}$

$$= \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{25+(-7)}{2}} + \frac{i(-24)}{|-24|}\sqrt{\frac{25-(-7)}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{25-7}{2}} - \frac{i(24)}{24}\sqrt{\frac{25+7}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{18}{2}} - i\sqrt{\frac{32}{2}}\right)$$

$$= \pm\left(\sqrt{9} - i\sqrt{16}\right)$$

$$\sqrt{z} = \pm(3 - 4i)$$

$$\sqrt{z} = 3 - 4i$$

$$z = -3 + 4i$$

(ii) $8-6i$

Let $z = 8 - 6i$

Taking sq root b.s

Let $Z = 8 - 6i$

Taking sq root b.s

$$\sqrt{Z} = \pm\sqrt{8 - 6i}$$

$$|Z| = \sqrt{x^2 + y^2} = \sqrt{8^2 + (-6)^2}$$

$$|Z| = \sqrt{64 + 36} = \sqrt{100} = 10$$

Square Root $= \sqrt{Z}$

$$= \pm\left(\sqrt{\frac{|Z| + x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|Z| - x}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{10+8}{2}} + \frac{i(-6)}{|-6|}\sqrt{\frac{10-8}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{18}{2}} - \frac{i(6)}{6}\sqrt{\frac{2}{2}}\right)$$

$$= \pm\left(\sqrt{9} - i\sqrt{1}\right)$$

$$= \pm\left(3 - i(1)\right)$$

$$\boxed{\sqrt{Z} = \pm(3 - i)}$$

$$\sqrt{Z} = 3 - i$$

$$\sqrt{Z} = -3 + i$$

(iii) $-15 - 36i$

Let $Z = -15 - 36i$

Taking square root b.s

$$\sqrt{Z} = \pm\sqrt{-15 - 36i}$$

$\sqrt{z} = \pm\sqrt{-15-36i}$

$|z| = \sqrt{x^2+y^2} = \sqrt{(-15)^2+(-36)^2}$

$|z| = \sqrt{225+1296} = \sqrt{1521} = 39$

Square Root $= \sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

$= \pm\left(\sqrt{\frac{39+(-15)}{2}} + \frac{i(-36)}{|-36|}\sqrt{\frac{39-(-15)}{2}}\right)$

$= \pm\left(\sqrt{\frac{39-15}{2}} - \frac{i(36)}{36}\sqrt{\frac{39+15}{2}}\right)$

$= \pm\left(\sqrt{\frac{24}{2}} - i\sqrt{\frac{54}{2}}\right)$

$= \pm\left(\sqrt{12} - i\sqrt{27}\right)$

$= \pm\left(\sqrt{4\times 3} - i\sqrt{9\times 3}\right)$

$= \pm\left(\sqrt{4}\sqrt{3} - i\sqrt{9}\sqrt{3}\right)$

$\sqrt{z} = \pm\left(2\sqrt{3} - i\,3\sqrt{3}\right)$

(iv) $\mathbf{119 + 120}\boldsymbol{i}$

Let $z = 119 + 120i$

Taking square root b.s

$\sqrt{z} = \pm\sqrt{119+120i}$

$|z| = \sqrt{x^2+y^2} = \sqrt{(119)^2+(120)^2}$

$$|z| = \sqrt{x^2+y^2} = \sqrt{(119)^2+(120)^2}$$

$$|z| = \sqrt{14161+14400} = \sqrt{28561} = 169$$

$$\text{Square Root} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{169+119}{2}} + \frac{i(120)}{|120|}\sqrt{\frac{169-119}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{288}{2}} + \frac{i(120)}{120}\sqrt{\frac{50}{2}}\right)$$

$$= \pm\left(\sqrt{144} + i\sqrt{25}\right)$$

$$= \pm(12 + i(5))$$

$$\boxed{\sqrt{z} = \pm(12+5i)}$$

.....

8. Find the square root of $13 - 20\sqrt{3}i$ and represent them on an argand diagram.

8. **Find the square root of $13 - 20\sqrt{3}i$ and represent them on an argand diagram.**

Let $z = 13 - 20\sqrt{3}i$

Taking square root b.s

$$\sqrt{z} = \pm\sqrt{13 - 20\sqrt{3}}$$

$$|z| = \sqrt{x^2 + y^2} = \sqrt{13^2 + (20\sqrt{3})^2}$$

$$|z| = \sqrt{169 + 400(3)} = \sqrt{169 + 1200}$$

$$|z| = \sqrt{1369} = 37$$

$$\text{square Root} = \pm\left(\sqrt{\frac{|z| + x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z| - x}{2}}\right)$$

$$\sqrt{z} = \pm\left(\sqrt{\frac{37+13}{2}} + \frac{i(-20\sqrt{3})}{|-20\sqrt{3}|}\sqrt{\frac{37-13}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{50}{2}} - \frac{i(20\sqrt{3})}{20\sqrt{3}}\sqrt{\frac{24}{2}}\right)$$

$$= \pm\left(\sqrt{25} - i\sqrt{12}\right)$$

$$\sqrt{z} = \pm\left(5 - i\sqrt{4 \times 3}\right)$$

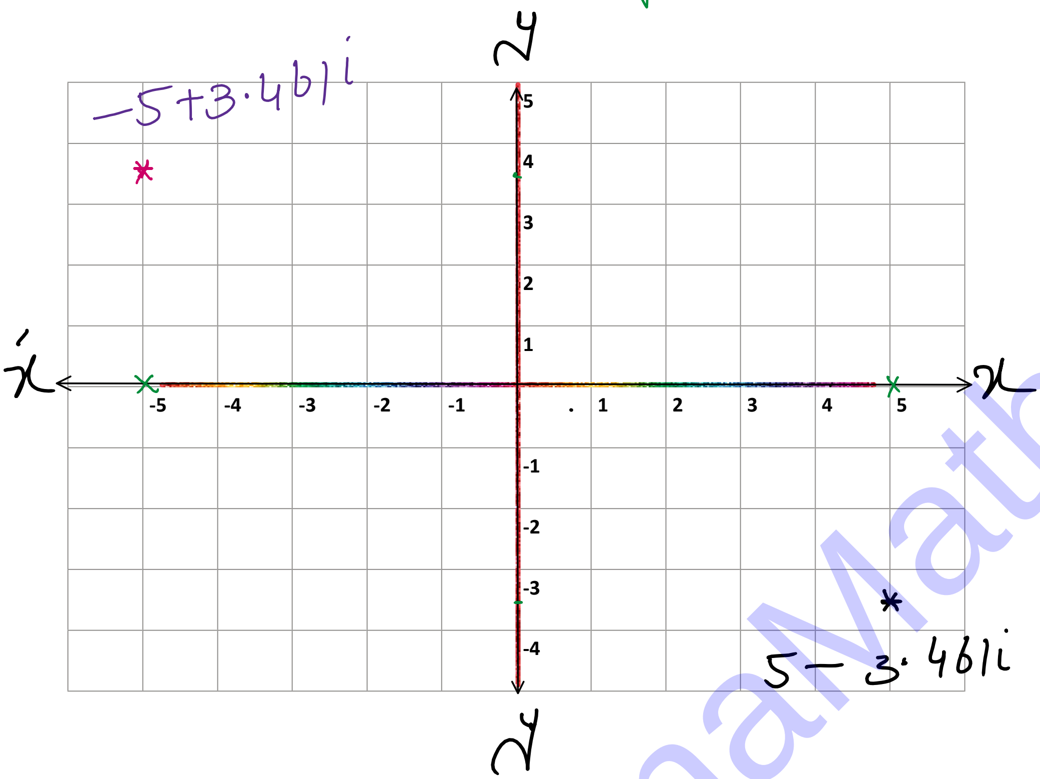
$$\boxed{\sqrt{z} = \pm\left(5 - i \cdot 2\sqrt{3}\right)}$$

$$\sqrt{z} = 5 - 2\sqrt{3}i \qquad \sqrt{z} = -(5 - 2\sqrt{3}i)$$

$\sqrt{z} = 5 - 3.464i$

$\sqrt{z} = -5 + 2\sqrt{3}i$

$\sqrt{z} = -5 + 3.461i$



9. **Find the value of x and y if $(-7+i)(x+iy)+(-1-5i)=i(11-i)$**

$(-7+i)(x+iy)+(-1-5i) = i(11-i)$

$(-7+i)(x+iy) - 1 - 5i = 11i - i^2$

$$(-7+i)(x+iy)-1-5i = 11i - i^2$$

$$\because i^2 = -1$$

$$(-7+i)(x+iy)-1-5i = 11i + 1$$

$$(-7+i)(x+iy) = 11i + 1 + 1 + 5i$$

$$(-7+i)(x+iy) = 2 + 16i$$

$$x+iy = \frac{2+16i}{-7+i}$$

$$x+iy = \frac{2+16i}{-7+i} \times \frac{-7-i}{-7-i}$$

$$x+iy = \frac{2(-7-i)+16i(-7-i)}{(-7)^2 - i^2}$$

$$x+iy = \frac{-14-2i-112i-16i^2}{49-(-1)}$$

$$x+iy = \frac{-14-114i-16(-1)}{49+1}$$

$$x+iy = \frac{-14-114i+16}{50}$$

$$x+iy = \frac{2-114i}{50}$$

$$x+iy = \frac{2}{50} - \frac{114i}{50}$$

$$x+iy = \frac{2}{50} - \frac{114i}{50}$$

$$x+iy = \frac{1}{25} - \frac{57}{25}i$$

$$x = \frac{1}{25} \qquad y = \frac{-57}{25}$$

10. Find the value of x and y if $(5-2i)(x+yi)+3 = i(11-i)-4i$

$$(5-2i)(x+yi)+3 = i(11-i)-4i$$

$$(5-2i)(x+yi) = 11i - i^2 - 4i - 3$$

$$(5-2i)(x+yi) = 7i + 1 - 3$$

$$(5-2i)(x+yi) = 7i - 2$$

$$x+yi = \frac{7i-2}{5-2i}$$

$$x+yi = \frac{7i-2}{5-2i} \times \frac{5+2i}{5+2i}$$

$$x+yi = \frac{7i(5+2i)-2(5+2i)}{(5)^2-(2i)^2}$$

$$x+yi = \frac{35i+14i^2-10-4i}{}$$

$$x+yi=\frac{35i+14i^2-10-4i}{25-4i^2}$$

$$x+yi=\frac{35i+14(-1)-10-4i}{25-4(-1)}$$

$$x+yi=\frac{31i-14-10}{25+4}$$

$$x+yi=\frac{-24+3i}{29}$$

$$x+yi=\frac{-24}{29}+\frac{3}{29}i$$

$x=\frac{-24}{29}$ $\quad$ $y=\frac{3}{29}$

11. Find the values of u and v if :

(i) $(u+iv)^2 = 20+21i$

$(u+iv)^2 = 20+21i$

Taking square root b.s

$u+iv=\pm\sqrt{20+21i}$

$|z|=\sqrt{x^2+y^2}=\sqrt{20^2+21^2}$

$$|z| = \sqrt{x^2+y^2} = \sqrt{20^2+21^2}$$

$$|z| = \sqrt{400+441} = \sqrt{841}$$

$$|z| = 29$$

$$\text{Square Root} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$u+iv = \pm\left(\sqrt{\frac{29+20}{2}} + \frac{i(21)}{|21|}\sqrt{\frac{29-20}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{49}{2}} + \frac{i(21)}{21}\sqrt{\frac{9}{2}}\right)$$

$$u+iv = \pm\left(\frac{7}{\sqrt{2}} + i\frac{3}{\sqrt{2}}\right)$$

$$\boxed{u = \pm\frac{7}{\sqrt{2}} \quad \Big| \quad v = \pm\frac{3}{\sqrt{2}}}$$

(ii) $(u+vi)^2 = 48 - 10i$

$$(u+vi)^2 = 48-10i$$

Taking Square Root b.s

$$u+vi = \pm\sqrt{48-10i}$$

$$|z| = \sqrt{x^2+y^2} = \sqrt{48^2+(-10)^2}$$

$|z| = \sqrt{x^2 + y^2} = \sqrt{48^2 + (-10)^2}$

$|z| = \sqrt{2304 + 100} = \sqrt{2404}$

$|z| = 2\sqrt{601}$

Square Root $= \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

$$\sqrt{z} = \pm\left(\sqrt{\frac{2\sqrt{601}+48}{2}} + \frac{i(-10)}{|-10|}\sqrt{\frac{2\sqrt{601}-48}{2}}\right)$$

$$\sqrt{z} = \pm\left(\sqrt{\frac{2(\sqrt{601}+24)}{2}} - \frac{i(10)}{10}\sqrt{\frac{2(\sqrt{601}-24)}{2}}\right)$$

$$\sqrt{z} = \pm\left(\sqrt{\sqrt{601}+24} - i\sqrt{\sqrt{601}-24}\right)$$

$$u = \pm\sqrt{\sqrt{601}+24}, \quad v = \mp\sqrt{\sqrt{601}-24}$$

12. If $z_1 = 4 + 5i$ and $z_2 = \alpha - 2i$, find the value of α such that $Re(z_1 z_2) = 20$.

$z_1 = 4+5i \quad z_2 = \alpha - 2i$

$z_1 z_2 = (4+5i)(\alpha - 2i)$

$= 4(\alpha - 2i) + 5i(\alpha - 2i)$

$= 4\alpha - 8i + 5\alpha i - 10i^2$

$= 4\alpha - 8i + 5\alpha i - 10i^2$

$\because i^2 = -1$

$= 4\alpha - 8i + 5\alpha i + 10$

$z_1 z_2 = 4\alpha + 10 + (5\alpha - 8)i$

$Re(z_1 z_2) = 20$

$4\alpha + 10 = 20$

$4\alpha = 20 - 10$

$4\alpha = 10$

$\alpha = \frac{10}{4}$

$\alpha = \frac{5}{2}$