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Exercise 1.3

Complex polynomials as a product of linear factors

A **complex polynomial** $P(z)$ is a polynomial function of the complex variable z with complex coefficients. It is expressed in the general form as

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

Where $a_n, a_{n-1}, \dots, a_1, a_0$ are complex numbers $\left(a_n \neq 0\right), and\ n \geq 0$ is an integer representing the degree of polynomial.

- A fundamental property of complex polynomials is that they can always be factored into a product of linear factors.

- According to the **fundamental theorem of algebra,** a polynomial of degree $n \geq 1$ has exactly n roots in complex number system C.

- **A corollary** to this theorem states that any polynomial $P(z)$ of degree n can be factored completely into a constant a and n linear factor over C in the form $\boldsymbol{P(z) = a(z - z_1)(z - z_2) \ldots (z - z_n)}$
  where $z_1, z_2, \ldots, z_n$ are complex roots of the polynomial.

- **If** $\boldsymbol{P(z)}$ is a polynomial function, the values of z that satisfy $P(z) = 0$ are called the **zeros of the function** $\boldsymbol{P(z)}$ and roots of the polynomial equation $P(z) = 0$

- The **Rational root theorem** is a mathematical tool used to find all possible rational roots of a polynomial equation with integer coefficients. According to rational root theorem:

  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every rational root $\frac{p}{q}$ (in simplest terms) satisfies:

(i) p is a factor of the constant term a_0.
(ii) q is a factor of the leading coefficient a_n

Solution of quadratic equations by completing the square

Completing square is a powerful and systematic method for solving quadratic equations.

1. Factorize the following:

(i) $a^2 + 4b^2$

$= a^2 - (-1)4b^2$

$\because i^2 = -1$

$= a^2 - i^2 4b^2$

$= a^2 - (2bi)^2$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (a - 2bi)(a + 2bi)$ Ans

(ii) $9a^2 + 16b^2$

$= 9a^2 - (-1)16b^2$ $\quad \because i^2 = -1$

$= 3^2a^2 - i^2 \cdot 4^2 b^2$

$= (3a)^2 - (4bi)^2$

$\because \quad a^2 - b^2 = (a-b)(a+b)$

$= (3a - 4bi)(3a + 4bi)$ Ans

(iii) $3x^2 + 3y^2$

$= 3[x^2 + y^2]$

$= 3[x^2 - (-1)y^2]$

$\because \quad i^2 = -1$

$= 3[x^2 - i^2 y^2]$

$= 3[x^2 - (iy)^2]$

$\because \quad a^2 - b^2 = (a-b)(a+b)$

$= 3(x - iy)(x + iy)$ Ans

(iv) $144x^2 + 225y^2$

$= 144x^2 - (-1)225y^2$

$= (12)^2x^2 - i^2(15)^2y^2$

$\because i^2 = -1$

$= (12x)^2 - (15yi)^2$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (12x - 15yi)(12x + 15yi)$

(v) $z^2 - 2iz - 1$

$= z^2 - 2iz + (-1)$

$\because i^2 = -1$

$= z^2 - 2iz + i^2$

$= (z)^2 - 2(z)(i) + (i)^2$

$$=(z)^2-2(z)(i)+(i)$$

$$\because\ a^2-2ab+b^2=(a-b)^2$$

$$=(z-i)^2$$

$$=(z-i)(z-i)\quad \text{Ans}$$

(vi) $z^2+6z+13$

$$=z^2+6z+13$$

$$=(z)^2+2(z)(3)+3^2-3^2+13$$

$$\because\ a^2+2ab+b^2=(a+b)^2$$

$$=(z+3)^2-9+13$$

$$=(z+3)^2+4$$

$$=(z+3)^2-(-1)4$$

$$\because\ i^2=-1$$

$\because i^2 = -1$

$= (z+3)^2 - i^2 4$

$= (z+3)^2 - (2i)^2$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (z+3-2i)(z+3+2i)$ Ans

(vii) $z^2 + 4z + 5$

$= (z)^2 + 2(z)(2) + 2^2 - 2^2 + 5$

$\because a^2 + 2ab + b^2 = (a+b)^2$

$= (z+2)^2 - 4 + 5$

$= (z+2)^2 + 1$

$= (z+2)^2 - (-1)$

$$= (z+2)^2 - (-1)$$

$\because i^2 = -1$

$$= (z+2)^2 - i^2$$

$\because a^2 - b^2 = (a-b)(a+b)$

$$= (z+2-i)(z+2+i)$$

(viii) $2z^2 - 22z + 65$

$$= 2\left[z^2 - 11z + \frac{65}{2}\right]$$

$$= 2\left[z^2 - 2(z)\left(\frac{11}{2}\right) + \left(\frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + \frac{65}{2}\right]$$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$$= 2\left[\left(z - \frac{11}{2}\right)^2 - \frac{121}{4} + \frac{65}{2}\right]$$

$$= 2\left[\left(z - \frac{11}{2}\right)^2 - \frac{121 + 130}{4}\right]$$

$$= 2\left[\left(z-\frac{11}{2}\right)^2+\frac{9}{4}\right]$$

$$= 2\left[\left(z-\frac{11}{2}\right)^2-(-1)\frac{9}{4}\right]$$

$\because i^2 = -1$

$$= 2\left[\left(z-\frac{11}{2}\right)^2-i^2\frac{9}{4}\right]$$

$$= 2\left[\left(z-\frac{11}{2}\right)^2-\left(\frac{3i}{2}\right)^2\right]$$

$\because a^2 - b^2 = (a-b)(a+b)$

$$= 2\left(z-\frac{11}{2}-\frac{3i}{2}\right)\left(z-\frac{11}{2}+\frac{3i}{2}\right) \text{ Ans.}$$

2. Factorize the following polynomial into its linear factors:

(i) $z^3 + 8$

$$= z^3 + 2^3$$

$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$\because \quad a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$=(z+2)(z^2-z(2)+2^2)$$

$$=(z+2)(z^2-2z+4)$$

$$=(z+2)\left[(z)^2-2(z)(1)+1^2-1^2+4\right]$$

$$\because \quad a^2-2ab+b^2=(a-b)^2$$

$$=(z+2)\left[(z-1)^2-1+4\right]$$

$$=(z+2)\left[(z-1)^2+3\right]$$

$$=(z+2)\left[(z-1)^2-(-1)3\right]$$

$$\because \quad i^2=-1$$

$$=(z+2)\left[(z-1)^2-i^2(\sqrt{3})^2\right]$$

$$=(z+2)\left[(z-1)^2-(i\sqrt{3})^2\right]$$

$$\because \quad a^2-b^2=(a-b)(a+b)$$

$$=(z+2)(z-1-i\sqrt{3})(z-1+i\sqrt{3}) \quad \text{Ans}$$

(ii) $z^3 + 27$

$= z^3 + 3^3$

$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (z+3)(z^2 - z(3) + 3^2)$

$= (z+3)[z^2 - 3z + 9]$

$= (z+3)\left[z^2 - 2(z)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 9\right]$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 - \frac{9}{4} + 9\right]$

$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 - \frac{9+36}{4}\right]$

$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 + \frac{27}{4}\right]$

$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 - (-1)\frac{27}{4}\right]$

$\because i^2 = -1$

$\because i^2 = -1$

$$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 - i^2 \frac{9 \times 3}{4}\right]$$

$$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 - i^2 \cdot \frac{3^2 (\sqrt{3})^2}{2^2}\right]$$

$$= (z+3)\left[\left(z - \frac{3}{2}\right)^2 - \left(\frac{3\sqrt{3}}{2} i\right)^2\right]$$

$\because a^2 - b^2 = (a-b)(a+b)$

$$= (z+3)\left(z - \frac{3}{2} - \frac{3\sqrt{3}}{2} i\right)\left(z - \frac{3}{2} + \frac{3\sqrt{3}}{2} i\right)$$

$$= (z+3)\left[z - \left(\frac{3 + 3\sqrt{3} i}{2}\right)\right]\left[z - \left(\frac{3 - 3\sqrt{3} i}{2}\right)\right]$$ Ans.

(iii) $z^3 - 2z^2 + 16z - 32$

$$= z^2 \cdot z - 2z^2 + 16z - 16 \times 2$$

$$= z^2(z-2) + 16(z-2)$$

$$= (z-2)(z^2+16)$$

$$= (z-2)[z^2 - (-1)16]$$

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$$\because i^2 = -1$$

$$= (z-2)\left[z^2 - i^2 \cdot 4^2\right]$$

$$= (z-2)\left[z^2 - (4i)^2\right]$$

$$\because a^2 - b^2 = (a-b)(a+b)$$

$$= (z-2)(z-4i)(z+4i) \text{ Ans}$$

(iv) $z^4 + 21z^2 - 100$

$$= z^4 + 25z^2 - 4z^2 - 100$$

$$= z^2(z^2+25) - 4(z^2+25)$$

$$= (z^2+25)(z^2-4)$$

$$= \left[z^2 - (-1)25\right]\left[z^2 - 4\right]$$

$$\because i^2 = -1$$

$$= \left[z^2 - i^2 5^2\right]\left[z^2 - 4\right]$$

$= [z^2 - i^2 5^2][z^2 - 2^2]$

$= [z^2 - (i5)^2][z^2 - 2^2]$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (z - i5)(z + i5)(z - 2)(z + 2)$ Ans.

(v) $z^4 - 16$

$= (z^2)^2 - 4^2$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (z^2 - 4)(z^2 + 4)$

$= (z^2 - 4)[z^2 - (-1)4]$

$\because i^2 = -1$

$= [z^2 - 4][z^2 - i^2 4]$

$= [z^2 - 2^2][z^2 - (i2)^2]$

$$= [z^2 - 2^2][z^2 - (i2)^2]$$

$$= (z-2)(z+2)(z-i2)(z+i2) \text{ Ans.}$$

(vi) $z^4 + 3z^2 - 4$

$$= z^4 + 4z^2 - z^2 - 4$$

$$= z^2(z^2+4) - 1(z^2+4)$$

$$= (z^2+4)(z^2-1)$$

$$= [z^2 - (-1)4](z^2 - 1)$$

$$= [z^2 - i^2 4][z^2 - 1]$$

$$= [z^2 - (2i)^2][z^2 - 1^2]$$

$$= (z-2i)(z+2i)(z-1)(z+1) \text{ Ans.}$$

(vii) $z^4 + 5z^2 + 6$

$= z^4 + 3z^2 + 2z^2 + 6$

$= z^2(z^2 + 3) + 2(z^2 + 3)$

$= (z^2 + 3)(z^2 + 2)$

$= (z^2 - (-1)3)(z^2 - (-1)2)$

$\because i^2 = -1$

$= [z^2 - i^2(\sqrt{3})^2][z^2 - i^2(\sqrt{2})^2]$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (z - i\sqrt{3})(z + i\sqrt{3})(z - i\sqrt{2})(z + i\sqrt{2})$ Ans.

(viii) $z^4 + 7z^2 - 144$

$= z^4 + 16z^2 - 9z^2 - 144$

$= z^2(z^2+16) - 9(z^2+16)$

$= (z^2+16)(z^2-9)$

$= (z^2-(-1)16)(z^2-9)$

$\because i^2 = -1$

$= (z^2 - i^2 16)(z^2-9)$

$= [z^2 - (i4)^2][z^2 - 3^2]$

$\because a^2 - b^2 = (a-b)(a+b)$

$= (z - i4)(z+i4)(z-3)(z+3)$ Ans.

3. Find the roots of $z^4 + 7z^2 - 144 = 0$ and hence express it as a product of linear factors.

$z^4 + 16z^2 - 9z^2 - 144 = 0$

$z^2(z^2+16) - 9(z^2+16) = 0$

$(z^2+16)(z^2-9) = 0$

$z^2 + 16 = 0$, $z^2 - 9 = 0$

$z^2 - (-1)16 = 0$ $\because i^2 = -1$

$z^2 - i^2 16 = 0$ $z^2 - 3^2 = 0$

$z^2 - (i4)^2 = 0$ $(z-3)(z+3) = 0$

$\because a^2 - b^2 = (a-b)(a+b)$

$(z - i4)(z + i4) = 0$, $z - 3 = 0$ $z + 3 = 0$

$z - i4 = 0$, $z + i4$ $z = 3$ $z = -3$

$z = i4$, $z = -i4$

$z = i4$, $z = -i4$

Product of Roots $= (z - i4)(z + i4)(z - 3)(z + 3)$

4. Solve the following complex quadratic equation by completing square method:

(i) $2z^2 - 3z + 4 = 0$

$$2\left(z^2 - \frac{3}{2}z + 2\right) = 0$$

$$z^2 - \frac{3}{2}z + 2 = 0$$

$$z^2 - \frac{3}{2}z = -2$$

$$(z)^2 - 2(z)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 = -2 + \left(\frac{3}{4}\right)^2$$

$$\because a^2 - 2ab + b^2 = (a-b)^2$$

$$\left(z - \frac{3}{4}\right)^2 = -2 + \frac{9}{16}$$

$$\left(z - \frac{3}{4}\right)^2 = \frac{-32 + 9}{16}$$

$$\left(z - \frac{3}{4}\right) = \frac{-32 + 9}{16}$$

$$\left(z - \frac{3}{4}\right)^2 = -\frac{23}{16}$$

Taking sq. root b.s

$$z - \frac{3}{4} = \pm\sqrt{\frac{-23}{16}}$$

$$z - \frac{3}{4} = \pm \frac{\sqrt{23}\sqrt{-1}}{\sqrt{16}}$$

$$\because i^2 = -1 \; , \; i = \sqrt{-1}$$

$$z - \frac{3}{4} = \pm \frac{\sqrt{23}\, i}{4}$$

$$z = \frac{3}{4} \pm \frac{\sqrt{23}\, i}{4}$$

$$z = \frac{3 \pm \sqrt{23}\, i}{4}$$ Ans.

(ii) $z^2 - 6z + 30 = 0$

$z^2 - 6z = -30$

$z^2 - 2(z)(3) + 3^2 = -30 + 3^2$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$(z-3)^2 = -30 + 9$

$(z-3)^2 = -21$

Taking sq. root b.s

$z - 3 = \pm\sqrt{-21}$

$z = 3 \pm \sqrt{-21}$

$z = 3 \pm \sqrt{21}\, i$

$\because i = \sqrt{-1}$

(iii) $3z^2 - 18z + 50 = 0$

$$3\left(z^2 - 6z + \frac{50}{3}\right) = 0$$

$$z^2 - 6z + \frac{50}{3} = 0$$

$$z^2 - 6z = -\frac{50}{3}$$

$$(z)^2 - 2(z)(3) + 3^2 = 3^2 - \frac{50}{3}$$

$$\because a^2 - 2ab + b^2 = (a-b)^2$$

$$(z-3)^2 = 9 - \frac{50}{3}$$

$$(z-3)^2 = \frac{27-50}{3}$$

$$(z-3)^2 = \frac{-23}{3}$$

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Taking square roots

$$z-3 = \pm\sqrt{\frac{-23}{3}}$$

$$z-3 = \pm\sqrt{\frac{23}{3}}\sqrt{-1}$$

$$z-3 = \pm\sqrt{\frac{23}{3}}\,i \qquad \because i=\sqrt{-1}$$

$$z = 3 \pm \sqrt{\frac{23}{3}}\,i$$

(iv) $z^2 + 4z + 13 = 0$

$$z^2 + 4z = -13$$

$$(z)^2 + 2(z)(2) + 2^2 = 2^2 - 13$$

$$\because a^2 + 2ab + b^2 = (a+b)^2$$

$$(z+2)^2 = 4 - 13$$

$(z+2)^2 = 4 - 13$

$(z+2)^2 = -9$

Taking sq. root b.s

$\sqrt{(z+2)^2} = \pm\sqrt{-9}$

$z+2 = \pm\sqrt{9}\sqrt{-1}$ $\quad \because i = \sqrt{-1}$

$z+2 = \pm 3i$

$\boxed{z = -2 \pm 3i}$

(v) $2z^2 + 6z + 9 = 0$

$2\left(z^2 + 3z + \frac{9}{2}\right) = 0$

$z^2 + 3z + \frac{9}{2} = 0$

$$z^2 + 3z + \frac{9}{2} = 0$$

$$z^2 + 3z = -\frac{9}{2}$$

$$z^2 + 2(z)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2 - \frac{9}{2}$$

$\because \; a^2 + 2ab + b^2 = (a+b)^2$

$$\left(z + \frac{3}{2}\right)^2 = \frac{9}{4} - \frac{9}{2}$$

$$\left(z + \frac{3}{2}\right)^2 = \frac{9-18}{4}$$

$$\left(z + \frac{3}{2}\right)^2 = \frac{-9}{4}$$

Taking sq. root b.s

$$\sqrt{\left(z + \frac{3}{2}\right)^2} = \pm\sqrt{\frac{-9}{4}}$$

$$z + \frac{3}{2} = \pm\sqrt{\frac{9}{4}}\sqrt{-1}$$

$$z+\frac{3}{2}=\pm\sqrt{\frac{9}{4}}\sqrt{-1}$$

$\because i=\sqrt{-1}$

$$z+\frac{3}{2}=\pm\frac{3}{2}i$$

$$z=-\frac{3}{2}\pm\frac{3}{2}i$$

$$z=\frac{-3\pm 3i}{2}$$

(vi) $3z^2-5z+7=0$

$$3\left(z^2-\frac{5}{3}z+\frac{7}{3}\right)=0$$

$$z^2-\frac{5}{3}z+\frac{7}{3}=0$$

$$z^2-\frac{5}{3}z=-\frac{7}{3}$$

$$z^2-2(z)\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)^2=\left(\frac{5}{6}\right)^2-\frac{7}{3}$$

$$a^2-2ab+b^2=(a-b)^2$$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$$\left(z - \frac{5}{6}\right)^2 = \frac{25}{36} - \frac{7}{3}$$

$$\left(z - \frac{5}{6}\right)^2 = \frac{25 - 84}{36}$$

$$\left(z - \frac{5}{6}\right)^2 = \frac{-59}{36}$$

Taking sq. root b.s

$$\sqrt{\left(z - \frac{5}{6}\right)^2} = \pm\sqrt{-\frac{59}{36}}$$

$$z - \frac{5}{6} = \pm \frac{\sqrt{59}\sqrt{-1}}{\sqrt{36}}$$

$$z - \frac{5}{6} = \pm \frac{\sqrt{59}\, i}{6}$$

$$z = \frac{5}{6} \pm \frac{\sqrt{59}\, i}{6}$$

$5 \pm \sqrt{59}\, i$

$$z = \frac{5 \pm \sqrt{59}\,i}{6}$$

5. Solve the following equations:
(i) $2z^4 - 32 = 0$

$$2(z^4 - 16) = 0$$

$$z^4 - 16 = 0$$

$$(z^2)^2 - 4^2 = 0$$

$$\because a^2 - b^2 = (a-b)(a+b)$$

$$(z^2 - 4)(z^2 + 4) = 0$$

$$z^2 - 4 = 0 \qquad z^2 + 4 = 0$$

$$z^2 = 4 \qquad z^2 = -4$$

Taking sq. root b.s

$z = \pm\sqrt{4}$, $\quad z = \pm\sqrt{-4}$

$z = \pm 2$ $\quad z = \pm 2i$

$\because i = \sqrt{-1}$

S.S $= \{\pm 2, \pm 2i\}$

(ii) $3z^5 - 243z = 0$

$3z(z^4 - 81) = 0$

$3z = 0$ $\quad z^4 - 81 = 0$

$\boxed{z = 0}$ $\quad (z^2)^2 - 9^2 = 0$

$\because a^2 - b^2 = (a-b)(a+b)$

$(z^2 - 9)(z^2 + 9 = 0$

$z^2 - 9 = 0$ $\quad z^2 + 9 =$

$z^2 = 9$, $\quad z^2 = -9$

Taking sq. root b.s

Taking sq. root b.s

$z = \pm\sqrt{9}$, $z = \pm\sqrt{-9}$

$z = \pm 3$ $\quad$ $z = \pm 3i$

$\because \; i = \sqrt{-1}$

$$\boxed{S.S = \{0, \pm 3, \pm 3i\}}$$

(iii) $5z^5 - 5z = 0$

$5z(z^4 - 1) = 0$

$5z = 0$ $\quad$ $z^4 - 1 = 0$

$\boxed{z = 0}$ $\quad$ $(z^2)^2 - 1^2 = 0$

$(z^2 - 1)(z^2 + 1) = 0$

$\because a^2 - b^2 = (a-b)(a+b)$

$$z^2 - 1 = 0 \quad , \quad z^2 + 1 = 0$$

$$z^2 = 1 \qquad z^2 = -1$$

Taking sq. root b.s

$$z = \pm\sqrt{1} \quad , z = \pm\sqrt{-1}$$

$$z = \pm 1 \quad , z = \pm i$$

$$\boxed{S.S = \{0, \pm 1, \pm i\}}$$

(iv) $z^3 - 5z^2 + z - 5 = 0$

$$z^2(z - 5) + 1(z - 5) = 0$$

$$(z - 5)(z^2 + 1) = 0$$

$$z - 5 = 0 \qquad z^2 + 1 = 0$$

$\boxed{z = 5}$

$z^2 = -1$

Taking sq. root b/s

$\sqrt{z^2} = \pm\sqrt{-1}$

$z = \pm i$

$\because i = \sqrt{-1}$

$S.S = \{5, \pm i\}$

(v) $4z^4 - 25z^2 + 21 = 0$

$4z^4 - 4z^2 - 21z^2 + 21 = 0$

$4z^2(z^2 - 1) - 21(z^2 - 1) = 0$

$(z^2 - 1)(4z^2 - 21) = 0$

$z^2 - 1 = 0 \qquad 4z^2 - 21 = 0$

$z^2 = 1 \qquad 4z^2 = 21$

$z^2 = 1 \qquad 4z^2 = 21$

$z^2 = 1 \qquad z^2 = \frac{21}{4}$

Taking sq. root.

$z = \pm\sqrt{1} \,, \qquad z = \pm\sqrt{\frac{21}{4}}$

$z = \pm 1 \qquad z = \pm \frac{\sqrt{21}}{2}$

$S.S = \left\{ \pm 1, \pm \frac{\sqrt{21}}{2} \right\}$

(vi) $z^3 + z^2 + z + 1 = 0$

$z^2(z+1) + 1(z+1) = 0$

$(z+1)(z^2+1) = 0$

$z+1=0$

$\boxed{z=-1}$

$z^2+1=0$

$z^2=-1$

Taking sq. root b.s

$z=\pm\sqrt{-1}$

$z=\pm i \quad \because i=\sqrt{-1}$

$S.S=\{-1, \pm i\}$

6. Find a polynomial of degree 3 with zeros $3, -2i, 2i$ and satisfying $P(1) = 20$.

Let z be the complex variable

As $3, -2i, 2i$ are zeros of Polynomial

Then

$z=3, \quad z=-2i, \quad z=2i$

$z = 3, \quad z = -2i, \quad z = 2i$

$z - 3 = 0 \quad z + 2i = 0 \quad z - 2i = 0$

By using Fundamental theorem of Algebra

$$P(z) = a(z - z_1)(z - z_2)(z - z_3)$$

$$P(z) = a(z-3)(z+2i)(z-2i) \quad \text{--- (1)}$$

By given condition

$$P(1) = 20$$

$$P(1) = a(1-3)(1+2i)(1-2i)$$

$$20 = a(-2)[1^2 - (2i)^2]$$

$$20 = a(-2)[1 - 4i^2]$$

$$20 = a(-2)[1 - 4(-1)]$$

$$20 = a(-2)[1 + 4]$$

$$20 = a(-2)[1+4]$$

$$20 = a(-2)(5)$$

$$20 = a(-10)$$

$$\frac{20}{-10} = a$$

$$\boxed{-2 = a}$$ Put in ①

$$P(z) = -2(z-3)(z+2i)(z-2i)$$

$\because a^2 - b^2 = (a-b)(a+b)$

$$P(z) = -2(z-3)[z^2 - (2i)^2]$$

$$= -2(z-3)[z^2 - 4i^2]$$

$\because i^2 = -1$

$$= -2(z-3)(z^2+4)$$

$$= -2(z^3 + 4z - 3z^2 - 12)$$

$= -2(z^2 + 4z - 3z - 12)$

$= -2(z^3 - 3z^2 + 4z - 12)$

$$\boxed{P(z) = -2z^3 + 6z^2 - 8z + 24}$$

7. Find a polynomial of degree 4 with zeros $2i, -2i, 1, -1$ and satisfying $P(2) = 240$.

Let z be the Complex variable

As $2i, -2i, 1, -1$ are zeros then

$z = 2i, \quad z = -2i, \quad z = 1, \quad z = -1$

$z - 2i = 0 \quad z + 2i = 0 \quad z - 1 = 0, \quad z + 1 = 0$

By using Fundamental theorem of Algebra

of Algebra

$$P(z) = a(z-z_1)(z-z_2)(z-z_3)(z-z_4)$$

$$P(z) = a(z-2i)(z+2i)(z-1)(z+1) \quad \text{(1)}$$

By given condition.

$$P(2) = 240$$

$$P(2) = a(2-2i)(2+2i)(2-1)(2+1)$$

$$240 = a[2^2-(2i)^2](1)(3)$$

$$240 = a[4-4i^2](3)$$

$$240 = a[4+4](3) \qquad \because i^2=-1$$

$$240 = a(8)(3)$$

$$240 = a(24)$$

$$\frac{240}{24} = a$$

$$\frac{240}{24} = a$$

$$\boxed{10 = a}$$ put in (i)

$$P(z) = 10(z-2i)(z+2i)(z-1)(z+1)$$

$$\because a^2 - b^2 = (a-b)(a+b)$$

$$P(z) = 10[z^2 - (2i)^2][z^2 - 1^2]$$

$$\because i^2 = -1$$

$$P(z) = 10[z^2 - 4i^2][z^2 - 1]$$

$$= 10[z^2 + 4][z^2 - 1]$$

$$= 10[z^4 - z^2 + 4z^2 - 4]$$

$$= 10[z^4 + 3z^2 - 4]$$

$$\boxed{P(z) = 10z^4 + 30z^2 - 40}$$

8. Find a polynomial of degree 4 with zeros $4, -4, 1+i, 1-i$ and satisfying $P(2) = 72$

Let z be the complex variable.

As $4, -4, 1+i, 1-i$ are zeros

then

$z = 4, \quad z = -4, \quad z = 1+i, \quad z = 1-i$

$z - 4 = 0, \quad z + 4 = 0, \quad z - (1+i) = 0, \quad z - (1-i) = 0$

$z - 1 - i = 0, \quad z - 1 + i = 0$

By using Fundamental theorem of algebra.

$$P(z) = a(z - z_1)(z - z_2)(z - z_3)(z - z_4)$$

$$P(z) = a(z-4)(z+4)(z-1-i)(z-1+i) \quad (1)$$

By given condition

By given Condition

$$P(2) = 72$$

$$P(2) = a(2-4)(2+4)(2-1-i)(2-1+i)$$

$$72 = a(-2)(6)(1-i)(1+i)$$

$$72 = a(-12)(1^2 - i^2)$$

$$72 = a(-12)(1+1) \qquad \because i^2 = -1$$

$$72 = a(-12)(2)$$

$$72 = a(-24)$$

$$\frac{72}{-24} = a$$

$\boxed{-3 = a}$ put in (1)

$$P(z) = -3(z-4)(z+4)(z-1-i)(z-1+i)$$

$$P(z) = -3(z-4)(z+4)(z-1-i)(z-1+i)$$

$$p(z) = -3[z^2-4^2][z^2 - z + \cancel{zi} - z + 1 - \cancel{i} - \cancel{zi} + \cancel{i} - i^2].$$

$$\because i^2 = -1$$

$$P(z) = -3[z^2-16][z^2-2z+1+1]$$

$$= -3[z^2-16][z^2-2z+2]$$

$$= -3[z^4-2z^3+2z^2-16z^2+32z-32]$$

$$= -3[z^4-2z^3-14z^2+32z-32]$$

$$P(z) = -3z^4+6z^3+42z^2-96z+96$$