Oblique Asymptotes

$$f(x) = \frac{x^2+1}{x+1}$$

$$\begin{array}{r|l} x+1 & x^2+1 \\ & \underline{x^2+x} \\ & -x+1 \\ & \underline{-x-1} \\ & 2 \end{array} \qquad x-1+\frac{2}{x+1}$$

$y = x-1$ is oblique asymptotes.

line

$$\lim_{x\to\infty} \frac{x^3+x^2+1}{5x^4+2x+1}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^3}}{5x^{\cancel{4}}}$$

$$= \frac{1}{5} \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty}$$

$$\lim_{x\to\infty} \frac{x^2+5x}{2x^2+6}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}}{2\cancel{x^2}} = \frac{1}{2}$$

Curvilinear Asymptotes

$$f(x) = \frac{x^3+1}{x+2}$$

$$\begin{array}{r|l} x+2 & x^3+1 \\ & \underline{x^3+2x^2} \\ & -2x^2+1 \\ & \underline{-2x^2-4x} \\ & 4x+1 \\ & \underline{4x+8} \\ & -7 \end{array} \qquad x^2-2x+4$$

$y = x^2-2x+4$

Curvilinear Asymptote

Continuity at a point

Def $x = a$

① $f(a)$ is defined.

② $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$ i.e $\lim\limits_{x \to a} f(x)$ exits.

③ $\lim\limits_{x \to a} f(x) = f(a)$.

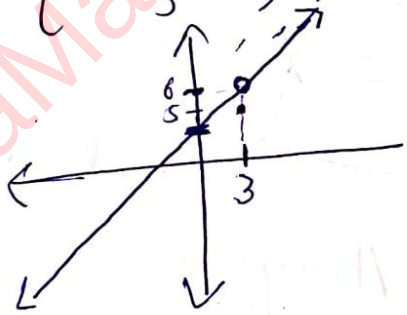
Example Check the continuity of the follo'g functions at $x = 3$.

$$f(x) = \frac{x^2-9}{x-3} \;;\; g(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases} ,\; h(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

$$\frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$$

$\Rightarrow f(3)$ is not defined

$\Rightarrow f(x)$ is discontinuous at $x=3$.



$g(3) = 5 \rightarrow$ ①

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x-3}$$

$$= 6 \rightarrow ②$$

$\lim\limits_{x \to 3} g(x) = g(3)$

$g(x)$ is discontinuous at $x = 3$.

Continuity of a Polynomial

Example Check the continuity of $f(x) = |x|$.

$$|x| = \begin{cases} -x, & x < 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$$

f is continuous on $(-\infty, 0)$ and $(0, +\infty)$ being a polynomial function.

At $x = 0$ $f(0) = 0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} |x| = 0$$

$$\therefore \lim_{x \to 0} f(x) = f(0)$$

f is continuous at $x = 0$.

f is continuous everywhere.

Continuity on an interval

A function f is said to be continua on $[a,b]$ if it is continua at each point of $[a,b]$.

$$\underset{x \to a^+}{Lt} f(x) = f(a)$$

$$\underset{x \to b^-}{Lt} f(x) = f(b)$$

Check

Continuity of $f(x) = \sqrt{9-x^2}$ on $[-3,3]$.

First we check the continuity on $(-3,3)$.

Let "c" be an arbitrary point

$$f(c) = \sqrt{9-c^2} \longrightarrow (1)$$

$$\underset{x \to c}{Lt} f(x) = \underset{x \to c}{Lt} \sqrt{9-x} = \sqrt{9-c^2} \longrightarrow (2)$$

From (1) & (2), $\underset{x \to c}{Lt} f(x) = f(c)$, i.e.

$f(x)$ is continua at c. Hence $f(x)$ is continua on $(-3,3)$.

At $x=-3$, $\underset{x \to -3^+}{Lt} f(x) = \underset{x \to -3^+}{Lt} \sqrt{9-x^2} = 0$

$f(-3) = 0$

At $x=3$, $\underset{x \to 3^-}{\lim} f(x) = \underset{x \to 3^-}{\lim} \sqrt{9-x^2} = 0 = f(3)$

Ex 2.6.

Q.9

$$\lim_{x\to\infty} \frac{\sin 2x}{x}$$

As we know that $\because -1 \le \sin 2x \le 1$

$$\frac{-1}{x} \le \frac{\sin 2x}{x} \le \frac{1}{x}$$

By Sandwich theorem $\therefore \lim_{x\to\infty} \frac{\sin 2x}{x} = 0$

Q.13

(a) $f(x) = \frac{2x+3}{5x+7}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x+3}{5x+7}$$

$$= \lim_{x\to\infty} \frac{\frac{2x}{x}+\frac{3}{x}}{\frac{5x}{x}+\frac{7}{x}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{3}{x}}{5+\frac{7}{x}}$$

$$= \frac{2}{5}$$

(b) Same as (a)

Q. 21

$$= \lim_{x\to\infty} \frac{3x^7+5x^2-1}{6x^3-7x+3}$$

$$= \frac{3x^4+5/x-\frac{1}{x^3}}{6-\frac{7}{x^2}+\frac{3}{x^3}} = \infty$$

because $\lim_{x\to\infty} 3x^4 = \infty$

23

$$\lim_{x\to\infty} \sqrt{\frac{8x^2-3}{2x^2+x}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{8-\frac{3}{x^2}}{2+\frac{1}{x}}}$$

$$= \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

37 $\lim_{x\to 0^+} \frac{1}{3x} = \infty$

38 $\lim_{x\to 0^-} \frac{5}{2x} = -\infty$

49.

$$\lim_{x \to \frac{\pi}{2}^-} \tan x$$

$$= \infty$$

53.

$$\lim \frac{1}{x^2-4}$$

(a) $x \to 2^+$ (b) $x \to 2^-$

(c) $x \to -2^+$ (d) $x \to -2^-$

(a) $\lim_{x \to 2^+} \frac{1}{x^2-4} = \lim_{x \to 2^+} \frac{1}{(x+2)(x-2)}$

$= \infty$

(b) $\lim_{x \to 2^-} \frac{1}{(x+2)(x-2)} = -\infty$

(c) $-\infty$

(d) $\lim_{x \to -2^-} \frac{1}{(x+2)(x-2)}$

$= \infty$