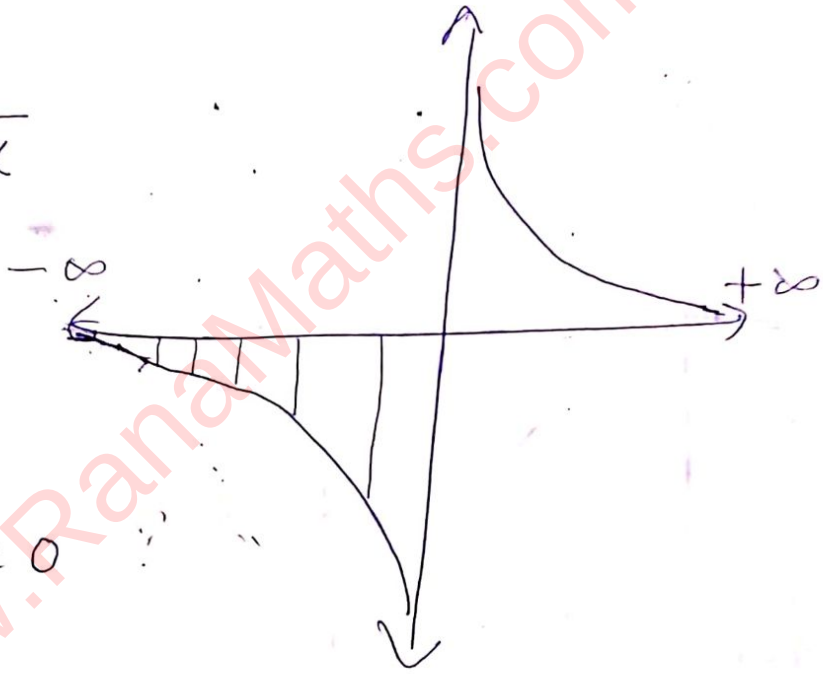
Limits

$\lim_{x \to 0} x^2 + 2x + 1$, $\lim_{x \to 1} x + 1$.

$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

Limits at infinity $-\infty, +\infty$

$f(x) = \frac{1}{x}$

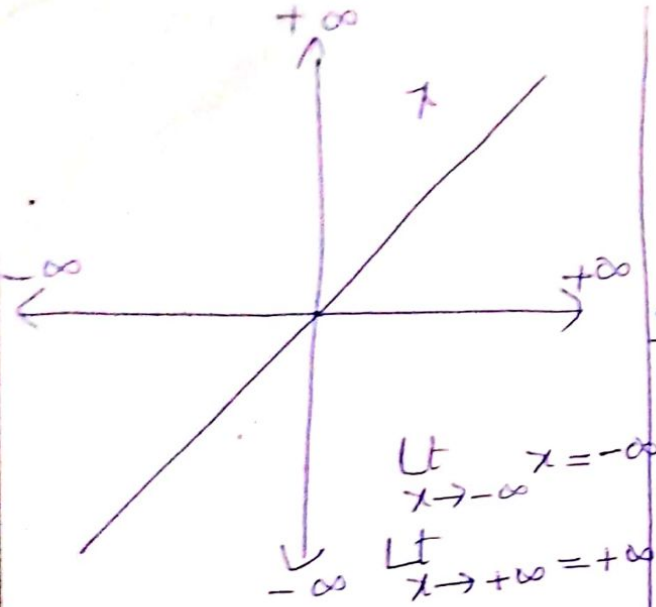


$\lim_{x \to -\infty} \frac{1}{x} = 0$

$\lim_{x \to +\infty} \frac{1}{x} = 0$

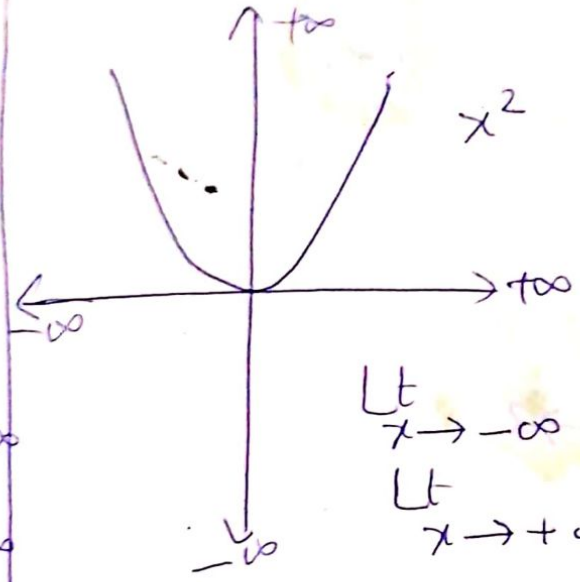
$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \pm\infty} \frac{1}{x^2}, \lim_{x \to \pm\infty} \frac{1}{x^3}, \ldots = 0$$



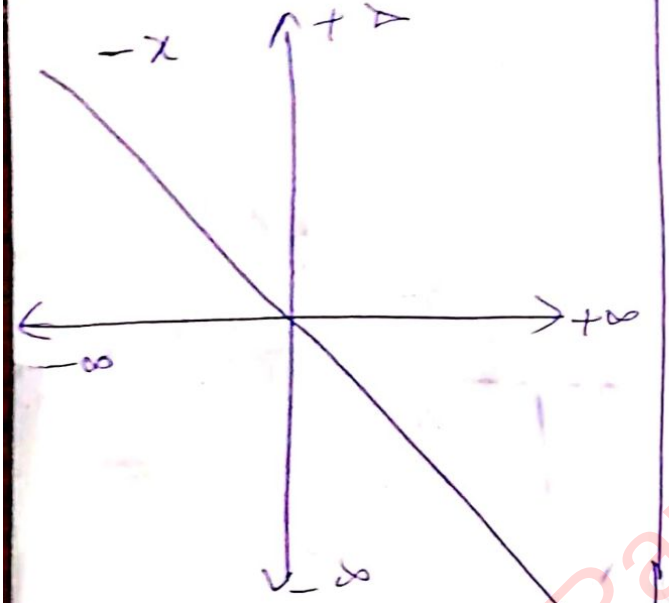
$$\lim_{x \to -\infty} x = -\infty$$

$$\lim_{x \to +\infty} x = +\infty$$



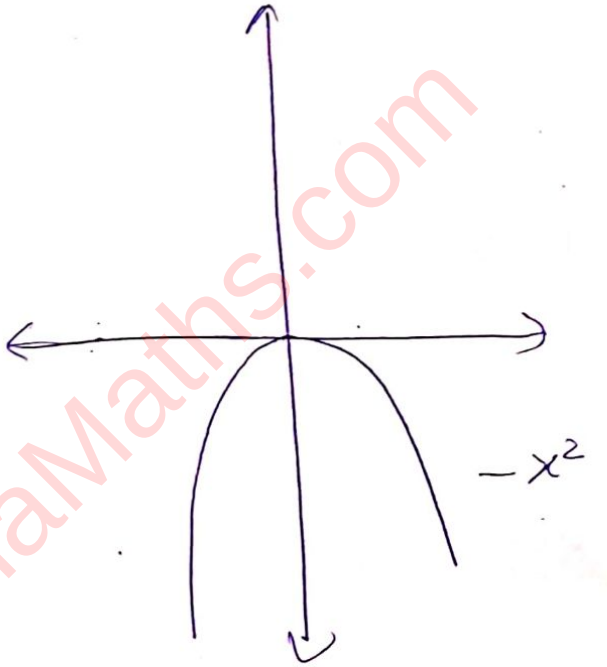
$$\lim_{x \to -\infty} x^2 = +\infty$$

$$\lim_{x \to +\infty} x^2 = +\infty$$



$$\lim_{x \to -\infty} -x = +\infty$$

$$\lim_{x \to +\infty} -x = -\infty$$



$$\lim_{x \to -\infty} -x^2 = -\infty$$

$$\lim_{x \to +\infty} -x^2 = -\infty$$

Laws of Limits at infinity

$$\lim_{x \to \infty} \frac{x^2+1}{x^2+2x+1}$$

$$= \lim_{x \to \infty} \frac{x^2/x^2 + 1/x^2}{x^2/x^2 + 2x/x^2 + 1/x^2}$$

$$= \lim_{x \to +\infty} \frac{1 + \frac{1}{x^2}}{1 + \frac{2}{x} + \frac{1}{x^2}}$$

$$= \frac{\lim_{x \to \infty} 1 + \lim_{x \to \infty} \frac{1}{x^2}}{\lim_{x \to \infty} 1 + \lim_{x \to \infty} \frac{2}{x} + \lim_{x \to \infty} \frac{1}{x^2}}$$

$$= \frac{1+0}{1+0+0} = 1$$

$$\lim_{x \to \infty} \frac{x^2 + 1}{x^3 + 3x + 1}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{1}{x^3}}{1 + \frac{3}{x^2} + \frac{1}{x^3}}$$

$$= \frac{0 + 0}{1 + 0 + 0} = 0$$

$y = 0$ is horizontal asymptote

$$\lim_{x \to +\infty} \frac{x^3+1}{x^2+1}$$

$$= \lim_{x \to +\infty} \frac{\frac{x^3}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{x + 1/x^2}{1 + \frac{1}{x^2}}$$

$\therefore \lim_{x \to \infty} x = \infty$

$$= +\infty$$

$$\therefore \lim_{x \to \infty} \frac{\sqrt{x^2+2}}{3x-6}$$

(4) $\therefore \sqrt{x^2} = |x|$

$$= \lim_{x \to \infty} \frac{\frac{\sqrt{x^2+2}}{|x|}}{\frac{3x-6}{|x|}}$$

$$|x| = \begin{cases} -x, & x<0 \\ 0, & x=0 \\ x, & x>0 \end{cases}$$

$$= \lim_{x \to \infty} \frac{\frac{\sqrt{x^2+2}}{\sqrt{x^2}}}{\frac{3x}{x} - \frac{6}{x}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{\frac{x^2+2}{x^2}}}{3-\frac{6}{x}} = \lim_{x \to \infty} \frac{\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}}$$

$$= \frac{1}{3}$$

$$\lim_{x \to \infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2+2}\,/|x|}{3x-6\,/|x|}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{x^2+2}{x^2}}}{\frac{3x}{|x|}-\frac{6}{|x|}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{\frac{3x}{-x}-\frac{6}{-x}}$$

$$= \frac{1}{-3+0} = -\frac{1}{3}$$

$y = -\frac{1}{3}$ is hrz. asymptote

Asymptote

Asymptote of a function (curve or line) is a curve or a line such that the distance b/w both of them tends to zero as they tend to infinity.

1. Horizontal Asymptote
2. Vertical Asymptotes
3. Oblique Asymptotes
4. Curvilinear Asymptotes

(1) Horizontal Asymptote

$$\frac{1}{x}$$

$$\lim_{x \to \infty} \frac{1}{x} = 0, \qquad \lim_{x \to -\infty} \frac{1}{x} = 0$$

$y = 0$ is horizontal asymptote.

(2) Vertical Asymptote

$$\frac{x-1}{x^2-1} \quad , \quad \frac{1}{x}$$

$$= \frac{x-1}{(x-1)(x+1)}$$

$$= \frac{1}{x+1}$$

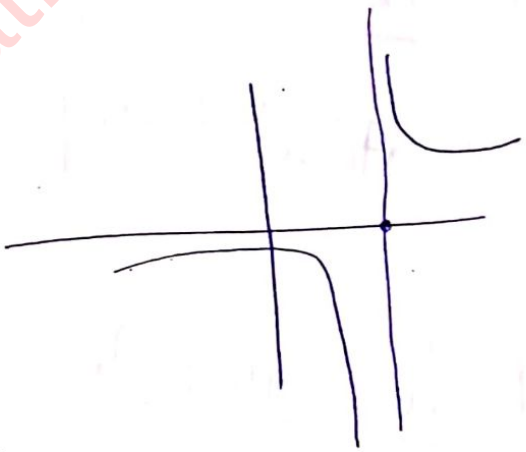
$$x+1=0$$

$$\boxed{x=-1}$$

$$\boxed{x=0}$$

Example:

$y = \frac{1}{x-3}$



$D(-\infty,3)\cup(3,\infty)$

$R(-\infty,0)\cup(0,\infty)$

$y = \frac{1}{x+2} + 7$