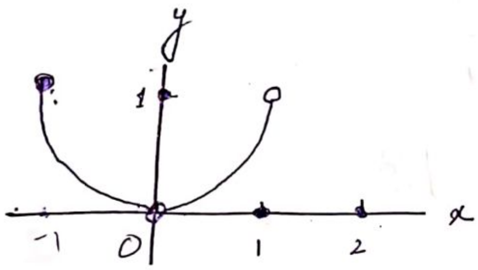
Exercise 2.4   Lecture 07

(1)



(a) $\lim_{x\to -1^+} f(x) = 1$ T   (b) $\lim_{x\to 0^-} f(x) = 0$. T

(c) $\lim_{x\to 0^-} f(x) = 1$ F   (d) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$. T

(3) $$f(x) = \begin{cases} 3-x, & x<2 \\ x/2+1, & x>2 \end{cases}$$

(a) Find $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$

Sol:-

$$\lim_{x\to 2^+} f(x) = \frac{2}{2}+1 = 2$$

$$\lim_{x\to 2^+} f(x) = 3-2 = 1$$

(b) Does $\lim_{x\to 2} f(x)$ exist.

No. $\lim_{x\to 2} f(x)$ does not exist

because $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

(c)

$$\lim_{x\to 4^-} f(x) = \frac{4}{2}+1 = 3.$$

$$\lim_{x\to 4^+} f(x) = \frac{4}{2}+1 = 3$$

(d)

Yes exist because

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$$

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(11)

$$\lim_{x \to 0.5^-} \sqrt{\frac{x+2}{x+1}}$$

$$= \sqrt{\frac{3/2}{1/2}} = \sqrt{3}$$

(18) (a) $\lim_{x \to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$

$$= \lim_{x \to 1^+} \frac{\sqrt{2x}\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \sqrt{2}$$

(b) $\lim_{x \to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$

$$= \lim_{x \to 1^-} \frac{\sqrt{2x}\cancel{(x-1)}}{-\cancel{(x-1)}}$$

$$= -\sqrt{2}$$

(19)

(a) $\lim_{\theta \to 3^+} \frac{\lfloor \theta \rfloor}{\theta}$

$$= \frac{3}{3} = 1$$

(b) $\lim_{\theta \to 3^-} \frac{\lfloor \theta \rfloor}{\theta}$

$$= \frac{2}{3}$$

21 $\lim_{\theta \to 0} \frac{\sin\sqrt{2}\theta}{\sqrt{2}\theta}$

Let $\sqrt{2}\theta = x$

$$= \lim_{x \to 0} \frac{\sin x}{x} = 1$$

(22) $\lim_{t \to 0} \frac{\sin kt}{t}$

$= \lim_{t \to 0} \frac{k \sin kt}{kt}$ $\quad \because \theta = kt$

$= k \lim_{\theta \to 0} \frac{\sin \theta}{\theta}$

$= k \cdot 1 = k.$

27// $\lim_{x \to 0} \frac{x \operatorname{cosec} 2x}{\cos 5x}$

$= \lim_{x \to 0} \left(\frac{x}{\sin 2x} \cdot \frac{1}{\cos 5x} \right)$

$= \left(\frac{1}{2} \lim_{x \to 0} \frac{2x}{\sin 2x} \right) \left(\lim_{x \to 0} \frac{1}{\cos 5x} \right)$

$= \frac{1}{2}(1) \cdot 1$

$= \frac{1}{2}$ Ans

37// $\lim_{\theta \to 0} \theta \cos \theta$

$= 0$

Q.42

$$\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta}$$

$$= \lim_{\theta \to 0} \frac{\theta \dfrac{\cos 4\theta}{\sin 4\theta}}{\sin^2\theta \dfrac{\cos^2 2\theta}{\sin^2 2\theta}}$$

$$= \lim_{\theta \to 0} \frac{\theta \cos 4\theta \sin^2 2\theta}{\sin^2\theta \cos^2 2\theta \sin 4\theta}$$

$$= \lim_{\theta \to 0} \frac{\theta \cos 4\theta \left(2\sin\theta\cos\theta\right)^2}{\sin^2\theta \cos^2 2\theta \sin 4\theta}$$

$$= \lim_{\theta \to 0} \frac{\theta \cos 4\theta \, 4\cancel{\sin^2\theta} \cos^2\theta}{\cancel{\sin^2\theta} \cos^2 2\theta \sin 4\theta}$$

$$= \lim_{\theta \to 0} \frac{4\theta \cos 4\theta \cos^2\theta}{\cos^2 2\theta \sin 4\theta}$$

$$= \left(\lim_{\theta \to 0} \frac{4\theta}{\sin 4\theta}\right) \lim_{\theta \to 0} \frac{\cos 4\theta \cdot \cos^2\theta}{\cos^2 2\theta}$$

$$= 1(1) = 1$$

Limits at Infinity:- The value of $f(x)$ eventually get as close as we like to number L as x increases without bound. Then we write

$$\lim_{x \to +\infty} f(x) = L \quad \text{or}$$

$$f(x) \to L \quad \text{as } x \to +\infty$$

Similarly $\Rightarrow$ Negative

Example:- Find $\lim_{x \to +\infty} \frac{3x+5}{6x-8}$

Divided by highest power of Denom

$$= \lim_{x \to +\infty} \frac{\frac{3\cancel{x}}{\cancel{x}} + 5/x}{6\cancel{x}/\cancel{x} - 8/x}$$

$$= \lim_{x \to +\infty} \frac{3 + 5/x}{6 - 8/x}$$

$$= \frac{3}{6} = \frac{1}{2}$$

(2)

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

Dividing highest power of denomi: and using that $\sqrt{x^2} = |x|$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{|x|} \Big/ \frac{3x-6}{|x|}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2+2}/\sqrt{x^2}}{\frac{3x-6}{|x|}}$$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{1 + 2/x^2}}{-3 + \frac{6}{x}} =$$

$$= -1/3$$

(3)

$$\lim_{x \to +\infty} \sqrt{x^6+5x^3} - x^3$$

$$= \lim_{x \to +\infty} \sqrt{x^6+5x^3} - x^3 \times \frac{\sqrt{x^6+5x^3}+x^3}{\sqrt{x^6+5x^3}+x^3}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^6}+5x^3-\cancel{x^6}}{\sqrt{x^6+5x^3}+x^3}$$

$$= \lim_{x \to +\infty} \frac{5x^3/x^3}{\sqrt{\frac{x^6}{x^6}+\frac{5x^3}{x^6}}+1}$$

$$= \frac{5}{\sqrt{1+0}+1}$$

$$= \frac{5}{1+1} = 5/2$$

Limits of Polynomial:-

$$\lim_{x \to +\infty} (7x^5-4x^3+2x-2)$$

$$= \infty.$$

"Asymptotes"

Horizantal Asymptote:-

A line $y = b$

$$\lim_{x \to \infty} f(x) = b$$

or

$$\lim_{x \to -\infty} f(x) = b$$

Example:-

$$f(x) = \frac{5x^2 + 8x - 3}{3x^2 + 2}$$

Divided by highest power

$$= \frac{5\cancel{x^2}/\cancel{x^2} + 8x/x^2 - 3/x^2}{3x^2/x^2 + 2/x^2}$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{5 + 8/x - 3/x^2}{3 + 2/x^2}$$

$$= 5/3 \,, \quad \lim_{x \to -\infty} f(x) = 5/3$$

Vertical Asymptotes:-

A line $x = a$

$$\lim_{x \to a^+} f(x) = \pm\infty \quad , \quad \lim_{x \to a^-} f(x) = \pm\infty$$

Simplest form; $\boxed{\text{denominator} = 0}$

$$f(x) = \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)}$$

$$f(x) = \frac{1}{x+1}$$

$$x + 1 = 0$$

$$\boxed{x = -1}$$

Find vertical and horizontal asymptotes:-

$$y = \frac{x+3}{x+2}$$

vertical,

$x+2 = 0$, $x = -2$ (V.A)

horizontal Asymptote

$$y = \frac{x+3}{x+2} = \frac{x/x + 3/x}{x/x + 2/x}$$

$$= \lim_{x \to +\infty} \frac{1 + 3/x}{1 + 2/x} = 1$$