Lecture 03

Combining Functions; Shifting and Scaling Graphs

Sum, Differences, Products and Quotients:-

* Like numbers, functions can be added, subtracted, multiplied and divided to produce new function (except where the denominator is zero).

If f and g are functions, for the every x that belongs to the domain of both f and g (that is $x \in D(f) \cap D(g)$. we define the functions $f+g$, $f-g$ and fg by formulas

$$(f+g)x = f(x) + g(x)$$

$$(f-g)x = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x).$$

* Notice that + sign on the left hand represents the operation of addition of functions; whereas the + on right hand side represents the addition of two real numbers $f(x)$ and $g(x)$

At any point $D(f) \cap D(g)$ at which $g(x) \neq 0$, we can define

$$\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)} \quad (\text{where } g(x) \neq 0)$$

* Functions can also be multiplied by constants. If c is real no, then cf can defined for all x in the domain of f by

$$(cf(x)) = c\,f(x)..$$

Example:-

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = \sqrt{1-x}$$

$$\swarrow \qquad\qquad\qquad \downarrow$$

$D(f) = [0, \infty)$.

$D(g) = (-\infty, 1]$

∴ The points common to these domains are the points in

$$[0,\infty) \cap (-\infty, 1] = [0,1]$$

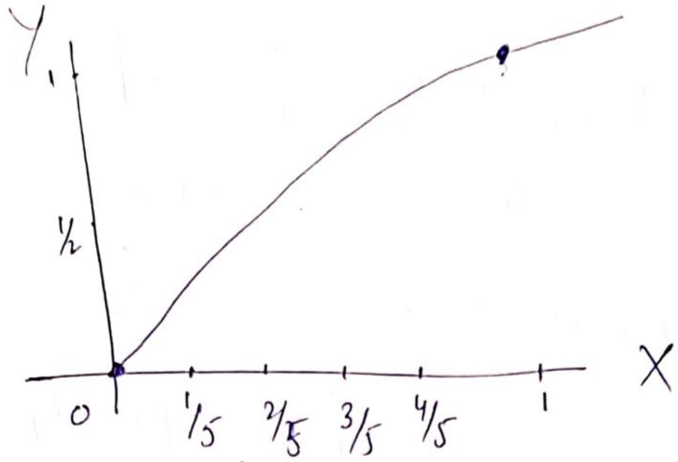
Function	Formula	Domain
$f+g$	$(f+g)(x) = \sqrt{x} + \sqrt{1-x}$	$[0,1]$
$f-g$	$(f-g)(x) = \sqrt{x} - \sqrt{1-x}$	$[0,1]$
$f \cdot g$	$(f \cdot g)x = f(x)g(x) = \sqrt{x(1-x)}$	$[0,1]$
$g-f$	$(g-f)x = \sqrt{1-x} - \sqrt{x}$	$[0,1]$
f/g	$f/g(x) = \dfrac{f(x)}{g(x)} = \sqrt{\dfrac{x}{1-x}}$	$[0,1)$ ($x=1$ excluded)
g/f	$g/f(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{1-x}}{x}$	$(0,1]$ ($x=0$ excluded)

Graphically Representation:

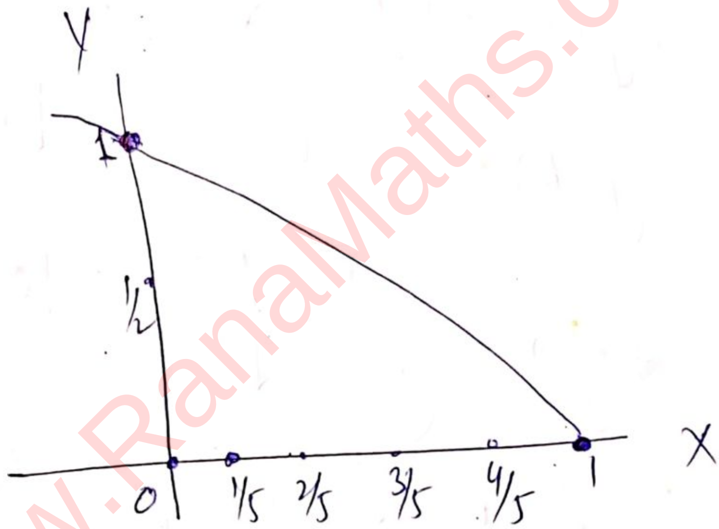
$$f(x) = \sqrt{x}$$

$$g(x) = \sqrt{1-x}$$

(i) $f(x) = \sqrt{x}$

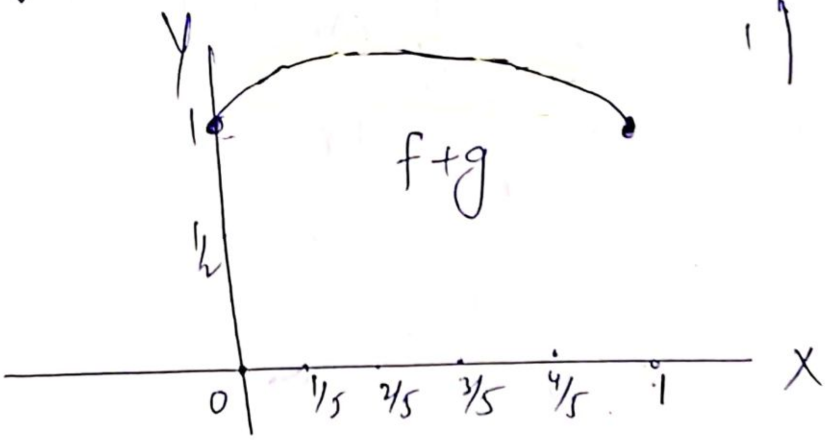


(ii) $g(x) = \sqrt{1-x}$



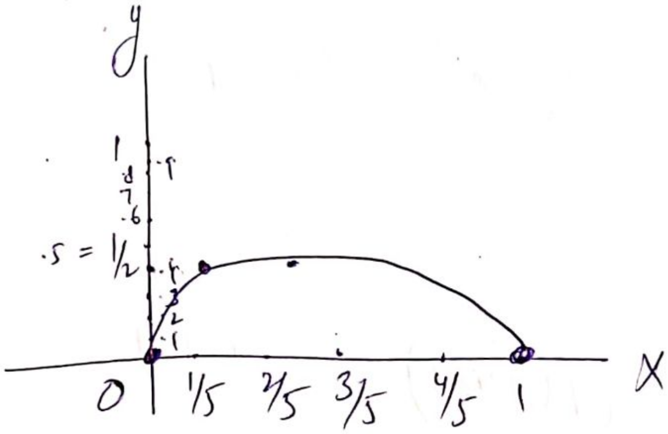
(iii) $f+g(x) = \sqrt{x} + \sqrt{1-x}$

x	$(f+g)(x)$
0	1
1	1



(iv)

$f \cdot g(x) = \sqrt{x(1-x)}$



Composite Functions:-

composition is another method of combining functions

Def:-

If f and g are functions, the composite function $f \circ g$ (f composed with g) is defined by

$$(f \circ g)x = f(g(x))$$

Example:- If $f(x) = \sqrt{x}$ and $g(x) = x+1$. Find

(a) $f \circ g(x)$ (b) $(g \circ f)(x)$ (c) $(f \circ f)(x)$

(d) $(g \circ g)(x)$

(a)

$(f \circ g)(x) = f(g(x)) = \sqrt{x+1}$

(b)

$(g \circ f)(x) = g(f(x)) = \sqrt{x} + 1$.

(c)

$(f \circ f)(x) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$

(d) $(g \circ g)(x) = g(g(x)) = g(x) + 1$

$= x+1+1$

$= x+2$

(a) Domain $[-1, \infty)$

(b) $[0, \infty)$

(c) $[0, \infty)$

(d) $(-\infty, \infty)$

Shifting a Graph of Function:-

A common way to obtain a new function from a existing one is by adding a constent to each output of existing function, or to its input variable.

The graph of the new function is the graph of the original function shifted vertically or horizontally.

Shift Formulas:-

Vertical shifts

$$y = f(x) + k$$

if $k > 0$ shifts the graph up.

$|k|$ units

shifts it down if $k < 0$

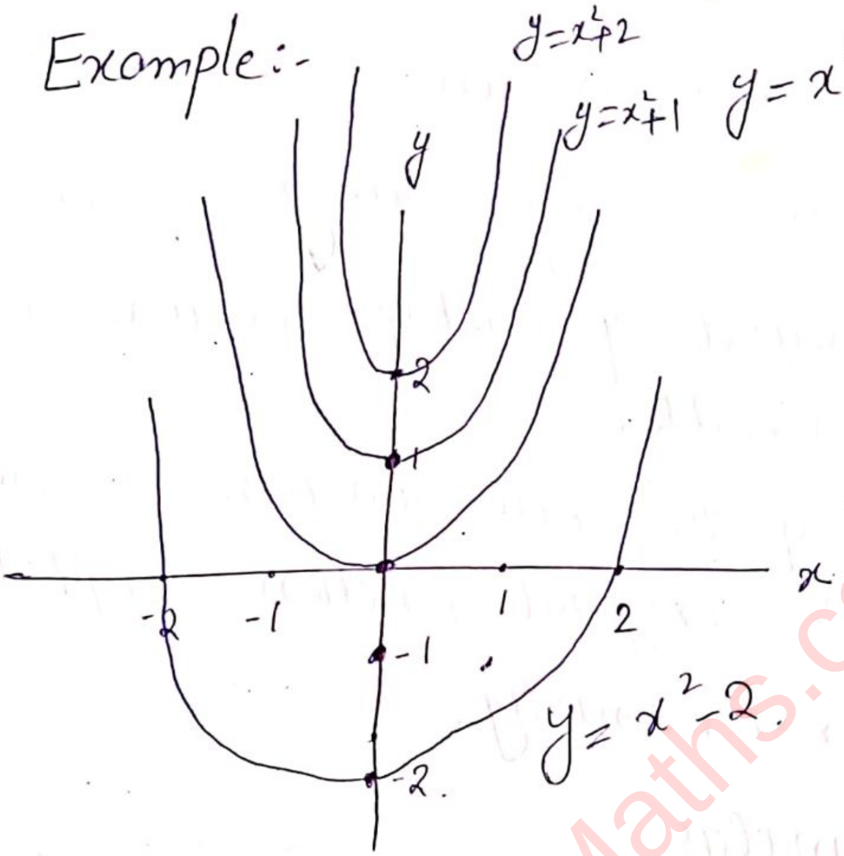
Horizontal shifts

$$y = f(x+h)$$

$h > 0$ shifts the graph left

$h<0$ shifts it right $|h|$ units

Example:-



$y=x^2+1$

$y=x^2+2$

For horizontal shifts

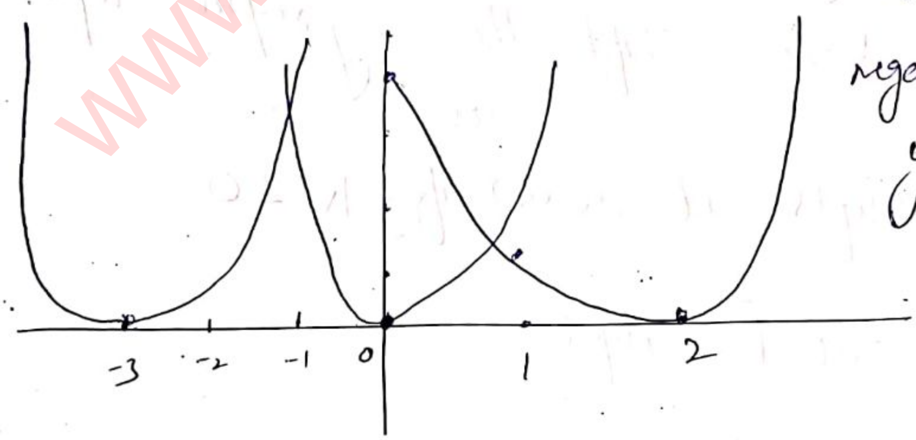
$y=(x+3)^2$ positive

$y=x^2$

$y=(x-2)^2$

negative

$y=(x-2)^2$.



Vertical and Horizontal Scaling and reflecting Formulas:-

(1) For $c > 1$, the graph is scaled

* $y = cf(x)$ stretches graph vertically by a factor c.
* $y = \frac{1}{c}f(x)$ compresses the graph vertically by factor c.
* $y = f(cx)$ compress the graph horizantally by a facter c.
* $y = f(x/c)$ stretches the graph horizontally by facter c.

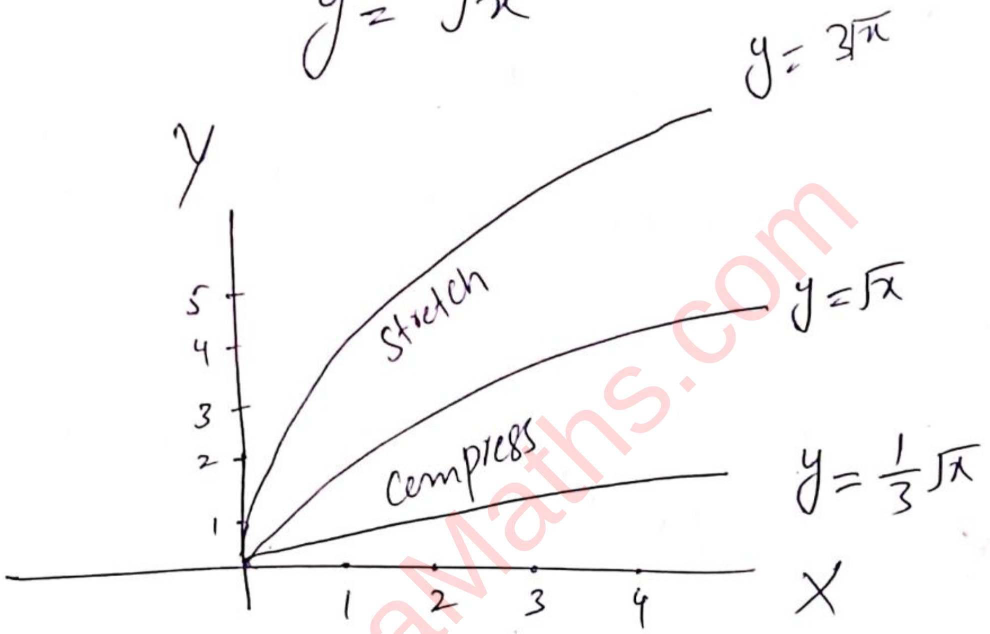
(2) For $c = -1$ the graph reflected.

* $y = -f(x)$ reflects the graph across x-axis

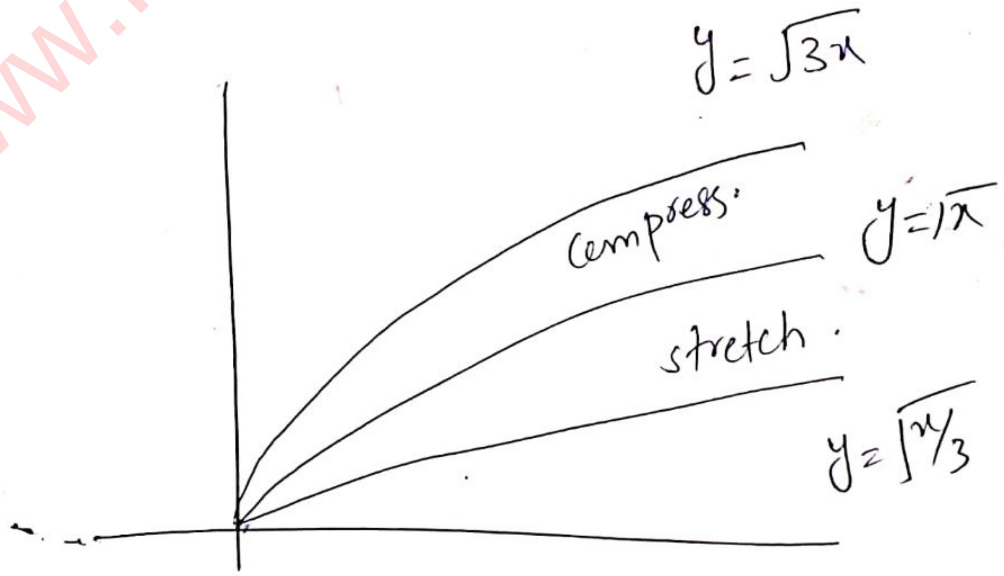
* $y = f(-x)$ reflects the graph across y-axis

Example:-

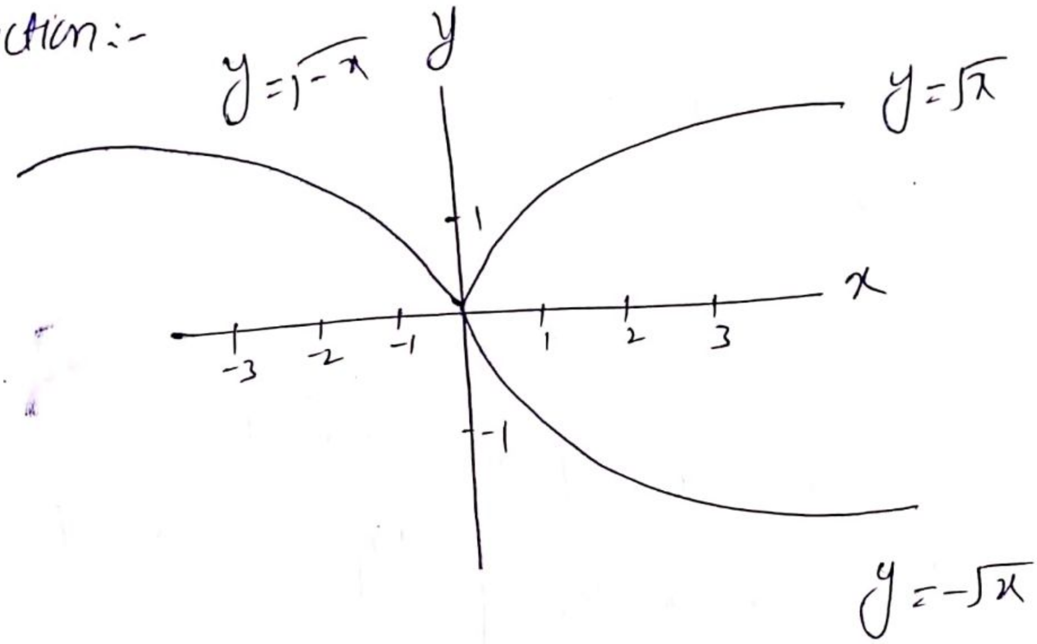
$y = \sqrt{x}$



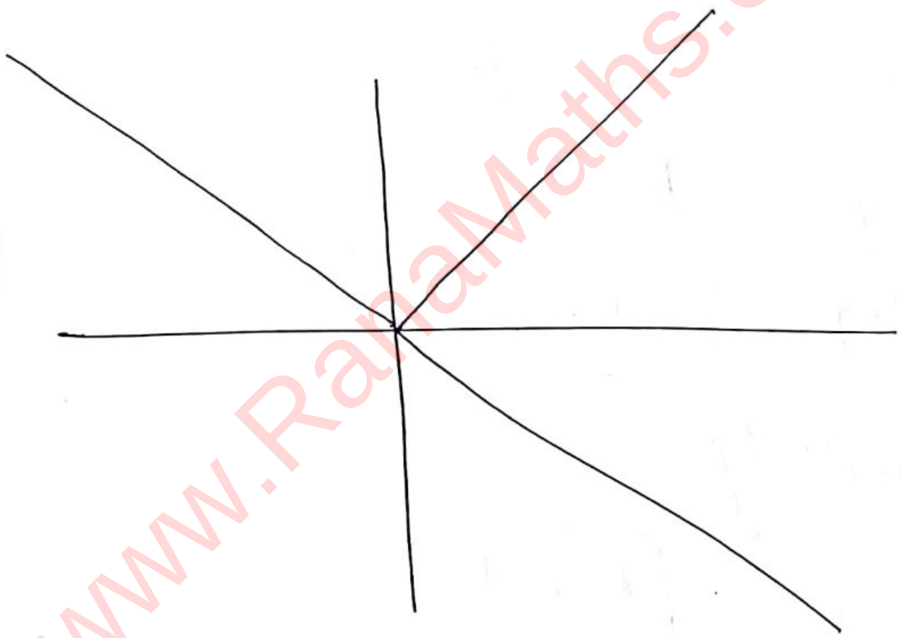
$y = \sqrt{x}$



Reflection:-



$y = x$



$y = -x$

$y = -x$

$y = f(-x)$

Exercise 1.2

(1)

f , g , f+g , f.g.

$f(x) = x$, $g(x) = \sqrt{x-1}$

$D_f = -\infty < x < \infty$ | $D_g = x \geq 1$

$R_f = -\infty < y < \infty$ | $R_g = y \geq 0$

$f+g = x + \sqrt{x-1}$

D_{f+g} $x \geq 1$

R_{f+g} : $y \geq 1$

Similarly

$f.g = x\sqrt{x-1}$

$D_{f.g} = x \geq 1$

$R_{f.g} = y \geq 0.$

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(a)

$f(x) = x - 1$

$g(x) = \frac{1}{x+1}$

$f(g(\frac{1}{3})) = ?$

$f(g(x)) = \frac{1}{x+1} - 1$

$= \frac{\cancel{1} - x - \cancel{1}}{x+1}$

$f(g(x)) = \frac{-x}{x+1}$

$f(g(\frac{1}{3})) = \frac{-\frac{1}{3}}{-\frac{1}{3}+1} = \frac{-\frac{1}{3}}{\frac{-1+3}{3}} = \frac{-\frac{1}{\cancel{3}}}{\frac{2}{\cancel{3}}}$

$= -2$

(6) (b)

$$g(f(1/2)) = ?$$

$$g(f(x)) = \frac{1}{x \cancel{-1} \cancel{+1}}$$

$$= \frac{1}{x}$$

$$g(f(\tfrac{1}{2})) = \frac{1}{1/2} = 2$$

Q.7// fogoh

$f(x) = 5x+8,\ g(x) = 6x-7,\ h(x) = 3x$

$(f\circ g\circ h)(x) = f(g(h(x))) = ?$

$$g(h(x)) = 6(3x) - 7$$

$$= 18x - 7$$

$$f(g(h(x))) = 5(18x-7) + 8$$

$$= 90x - 35 + 8$$

$$= 90x - 27$$

Scaling and reflecting Formulas:-

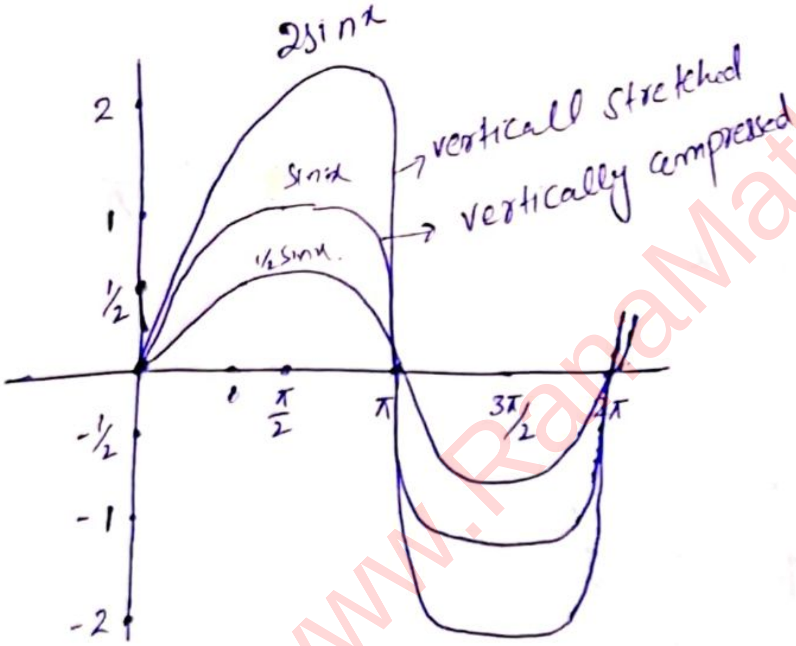
Lecture 3

$y = \sin x$

x	$2\sin x$
0	0
$\frac{\pi}{2}$	2
π	0
$\frac{3\pi}{2}$	-2
2π	0

$y = 2\sin x$

$y = \frac{1}{2}\sin x$



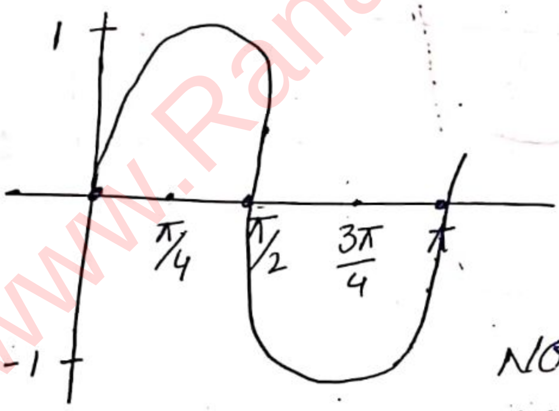
x	$\frac{1}{2}\sin x$
0	0
$\frac{\pi}{2}$	$\frac{1}{2}$
π	0
$\frac{3\pi}{2}$	$-\frac{1}{2}$
2π	0

Horizotally:-

Now

$y = \sin x.$

$y = \sin 2x$ Period $\frac{2\pi}{2} = \pi$



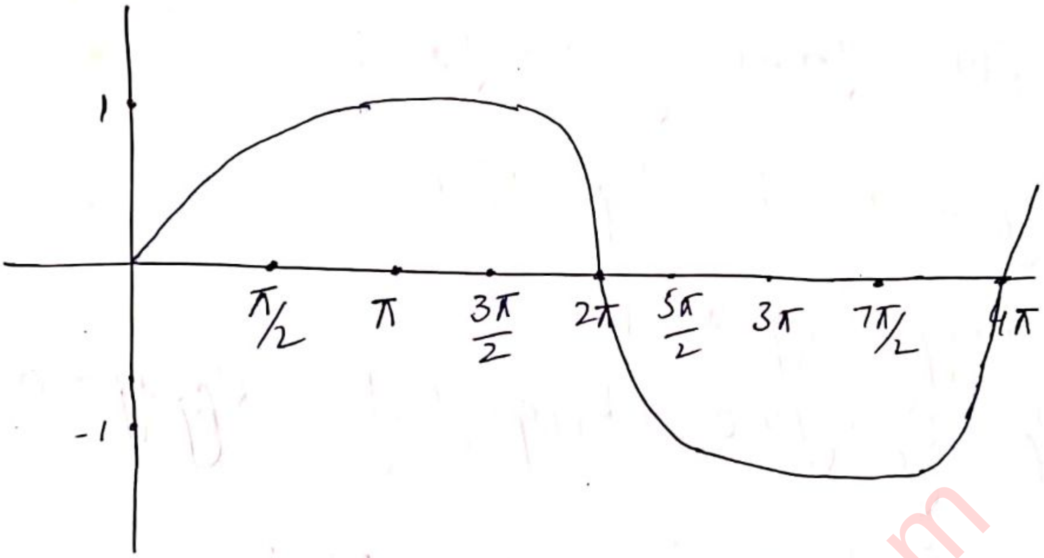
Now

$y = \sin \frac{1}{2} x$

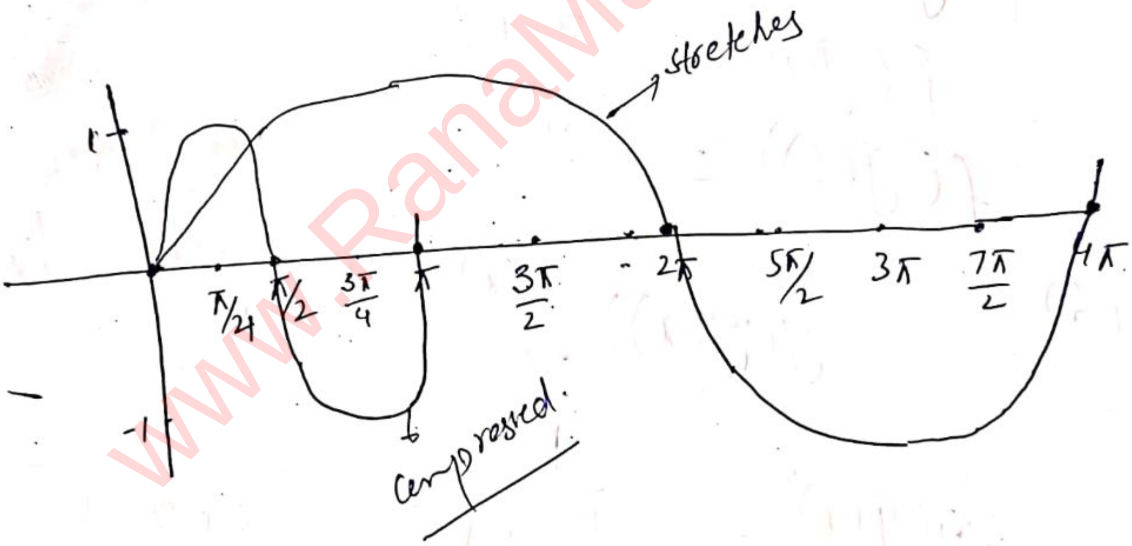
Period will become $\frac{2\pi}{1/2} = 4\pi$

Lecture 3

$\sin \frac{1}{2}x$



Comparison :-



Exercise 1.2

(2) $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{x-1}$

$D_f = x \geq -1$, $D_g = x \geq 1$

$D_{f+g} = D_{fg}$ $x \geq 1$

$R_f = R_g$; $y \geq 0$, $R_{f+g} = y \geq \sqrt{2}$, R_{fg} $y \geq 0$

(5) $f(x) = x+5$, $g(x) = x^2-3$

(a) $f(g(0)) = ?$

$f(g(x)) = x^2-3+5$

$f(g(x)) = x^2+2$

$f(g(0)) = 2$

(6) $f(x) = x-1$, $g(x) = 1/x+1$

$g(g(x)) \Rightarrow$

$$g(g(x)) = \frac{1}{\frac{1}{x+1}+1} = \frac{1}{\frac{1+x+1}{x+1}}$$

$$= \frac{x+1}{x+2}$$

(7) $f \circ g \circ h = ?$

Lecture 3.

$f(x) = x+1, \; g(x) = 3x, \; h(x) = 4-x$

$$f(g(h(x))) = f(3(4-x))$$
$$= f(12-3x)$$
$$= 12-3x+1$$
$$= 13-3x.$$

(15)

x	-2	-1	0	1	2
$f(x)$	1	0	-2	1	2
$g(x)$	2	1	0	-1	0

$f(g(1)) = 0, \quad f(f(-1)) = -2.$

(19) $f(x) = \dfrac{x}{x-2}, \quad g(x) = ?$

$$(f \circ g)(x) = x.$$
$$f \circ g(x) = x.$$
$$\frac{g(x)}{g(x)-2} = x.$$

$$g(x) = (g(x) - 2)\,x.$$

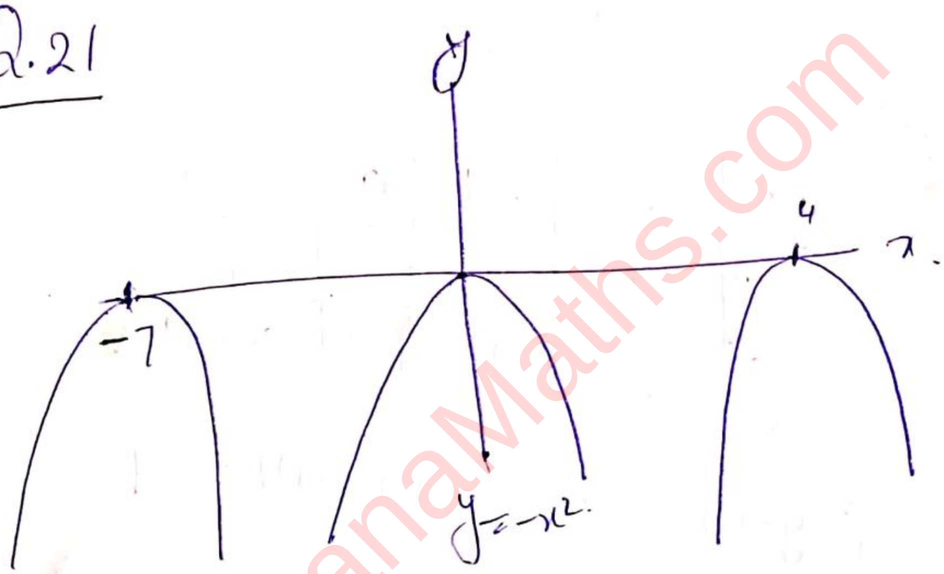
$$g(x) = x\,g(x) - 2x.$$

$$x\,g(x) - g(x) = 2x.$$

$$(x-1)\,g(x) = 2x.$$

$$g(x) = \frac{2x}{x-1}$$

Q.21



$y = -(x+7)^2$, $y = -(x-4)^2$

Lecture 3

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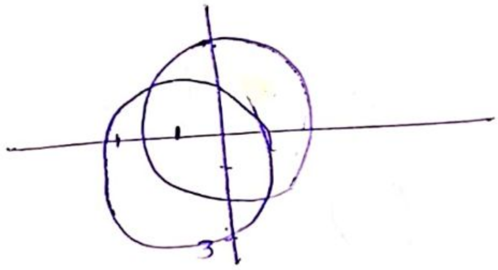
$x^2+y^2=49$.



Down 3

left = 2

$y=f(x)$

$(x+2)^2+(y+3)^2=49$.

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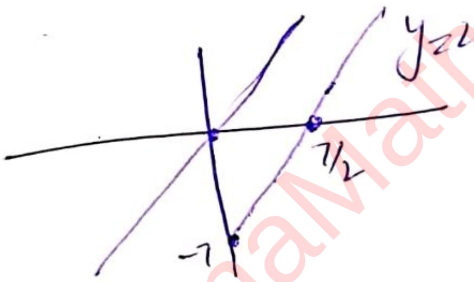
$y=2x-7$

$y=2x-7+7$

$y=2x$.



Scaling

$y=x^2-1$ stretched vertically by factor of 3.

$y=c(f(x))$

$=3(x^2-1)$

$y=3x^2-3$

(66) $y = 1 - x^3$ stretched horizontally by a factor 2.

$$y = 1 - \left(\frac{x}{2}\right)^3$$
$$= 1 - x^3/8$$

(65) $y = 1 - x^3$ compressed by 3

$$y = 1 - (3x)^3$$
$$= 1 - 27x^3$$