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(1) Find the absolute values of function on $[1, 6]$. (1)

$$f(x) = 4x^3 - 39x^2 + 90x + 2$$

$$f'(x) = 12x^2 - 78x + 90 = 0$$

$$6x^2 - 39x + 45 = 0$$

$$2x^2 - 13x + 15 = 0$$

$$\cancel{2x^2 - 15x + 2x + 15 = 0}$$

$$\cancel{x(2x}$$

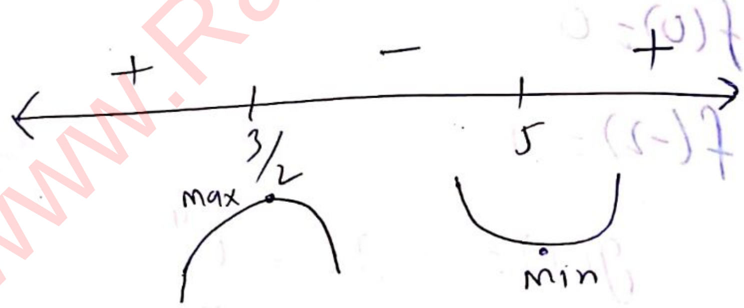
$$\cancel{x(2x - 15) =}$$

$$2x^2 - 10x - 3x + 15 = 0$$

$$2x(x-5) - 3(x-5) = 0$$

$$(x-5)(x-3)$$

$$x = 3/2 \text{ and } x = 5$$



X	Y
1	57
3/2	62.75 max
5	−23 min
6	2

$$f(1) = 4(1)^3 - 39(1)^2 + 90(1) + 2$$

$$= 4 - 39 + 90 + 2$$

$$= 57$$

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$f(1.5) = 62.75$

$f(5) = -23$

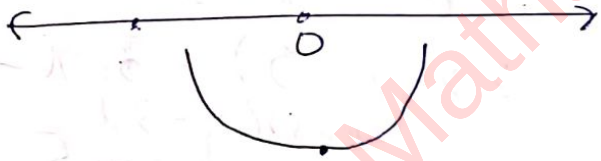
$f(6) = 2$

(2)

$f(x) = x^2 \qquad [-2, 1]$

$f'(x) = 2x$

$x = 0$



$f(1) = 1$

$f(0) = 0$

$f(-2) = 4$

(3) $g(t) = 8t - t^4 \qquad [-2, 1]$

$g'(t) = 8 - 4t^3$

$g'(1) = 0 \Rightarrow 4t^3 = 8, \; t^3 = 2$

$t = 2^{1/3} > 1$

$g(-2) = 8(-2) - (-2)^4$

$= -16 - 16$

$= -32$

$g(1) = 7$

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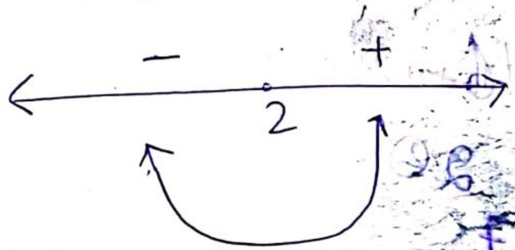
Examples:- (1)

$f(x) = x^2 - 4x$ Identify local maximum and minimum value of function

$f(x) = x^2 - 4x$

$f'(x) = 2x - 4 = 2(x-2)$

$x = 2.$



we have local minimum at

$x = 2$

$f(2) = 4 - 4(2) = -4$

minimum value $= (2, -4)$

(2)

$f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 2(3x^2) + 3(2x) - 12$

$= 6x^2 + 6x - 12$

$= 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

$$\boxed{x = -2, \quad x = 1}$$

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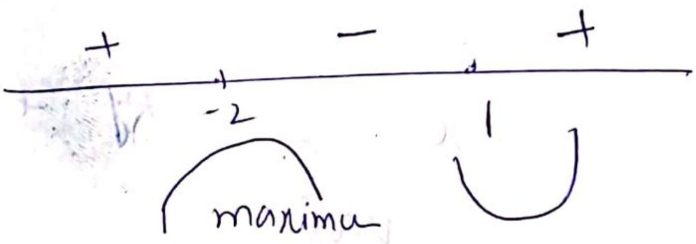


local Maximum at -2

local minimum at $x=1$

$$f(-2) = 2(-2)^3+3(-2)^2-12(-2)$$

$$= -16+12+24$$

$$= 20$$

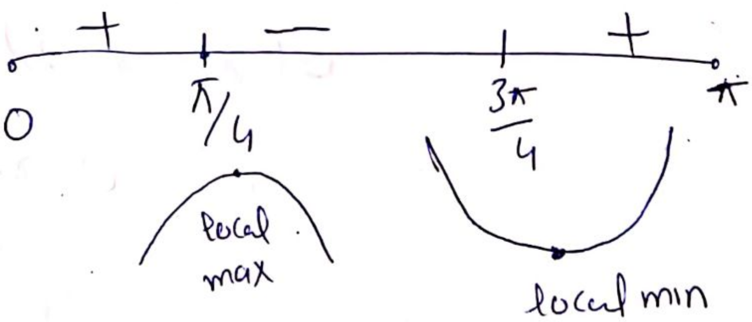
$$f(1) = -7$$

(53) (a) $f(x)=\sin 2x \qquad 0 \le x \le \pi$

$$f'(x)=2\cos 2x$$

$$2\cos 2x = 0, \quad \cos 2x = 0$$ (5)

C.P are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$



~~local maxim~~

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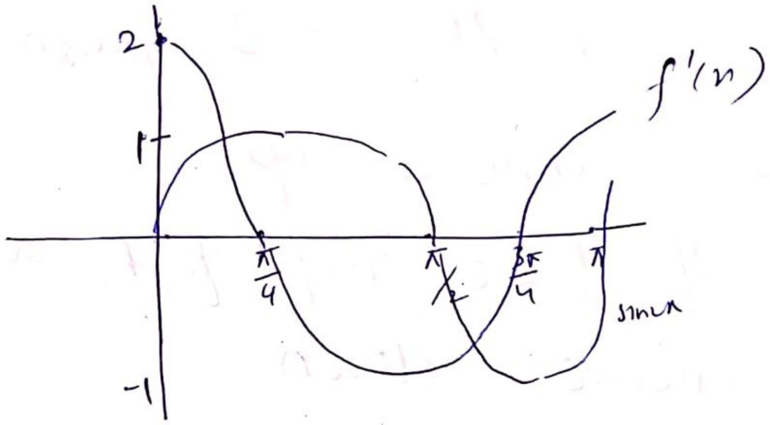
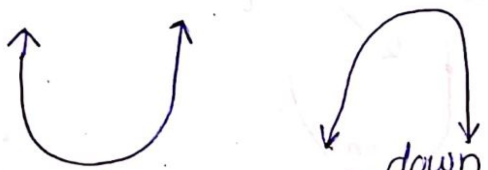
(b)



Concavity :-



inflection

$f'' = +$ $\qquad$ $f'' = -1$ $\qquad$ $f'' = 0$

$f' \uparrow$ $\qquad$ $f' \downarrow$

Definition :-

(a) Concave up on open interval I if f' is increasing on I

(b) Concave down on an open interval I if f' is decreasing on I

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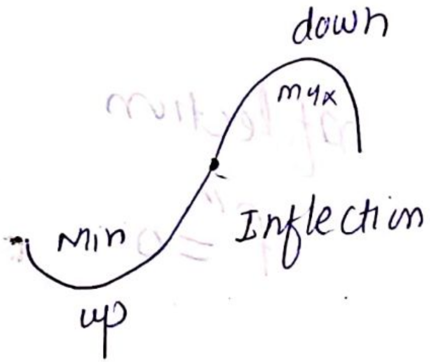
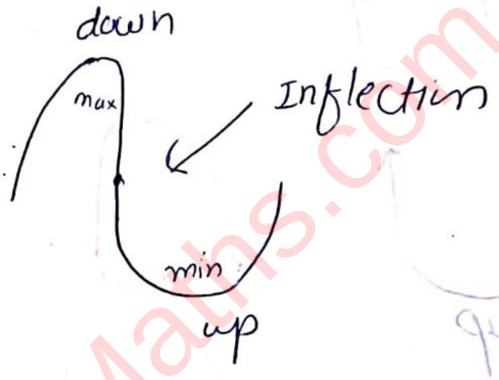
The 2nd Derivative Test:-

1. If $f'' > 0$ on I, graph of f over I is concave up
2. If $f'' < 0$, graph of f over I is concave down.

Explaination:-





Example:-

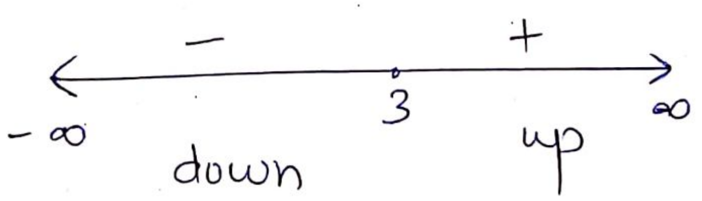
$$f(x) = x^3 - 9x^2 + 7x$$

$$f'(x) = 3x^2 - 18x + 7$$

$$f''(x) = 6x - 18$$

$$= 6(x-3)$$

$$x = 3$$



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(4)

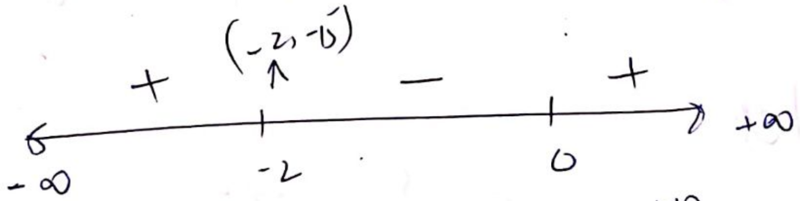
- Concave down $(-\infty, 3)$
- concave up $(3, \infty)$

Inflection point $x = 3$

$$f(3) = 3^3 - 9(3)^2 + 7(3)$$
$$= 27 - 81 + 21$$
$$= -33$$
$$(3, -33)$$

(2)

$$f(x) = x^4 + 4x^3 + 1$$
$$f'(x) = 4x^3 + 12x^2$$
$$f''(x) = 12x^2 + 24x$$
$$12x^2 + 24x = 0$$
$$12x(x+2) = 0$$
$$x = 0 \;,\; x = -2$$



Concave up down up

$(0, 1)$

C. up $(-\infty, -2) \cup (0, \infty)$

C. down $(-2, 0)$

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$f(0) = (0)^4 + 4(0) + 1$

$= 1$

$(0, 1)$

$f(-2) = (-2)^4 + 4(-2)^3 + 1$

$= 16 - 32 + 1$

$= -15$

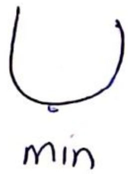
Second Derivative Test for Local Extrema:-

local max $\rightarrow c$

$f'(c) = 0$ and $f''(c) < 0$

Local min $\rightarrow c$

$f'(c) = 0$ and $f''(c) > 0$



Examples:

$f(x) = 2x^3 - 12x^2$

$f'(x) = 6x^2 - 24x$

$f''(x) = 12x - 24$

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$f'(x) = 0$

$6x^2 - 24x = 0$

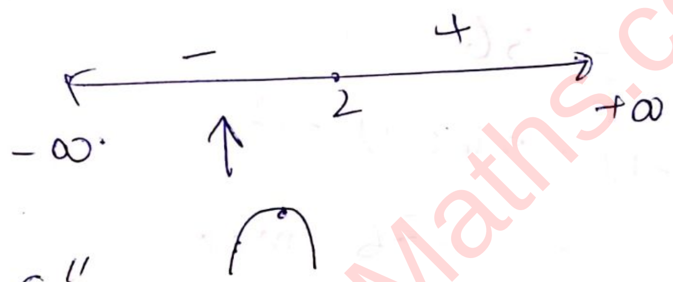
$6x(x-4) = 0$

$\boxed{x = 0}$ $\boxed{x = 4}$ → min

max

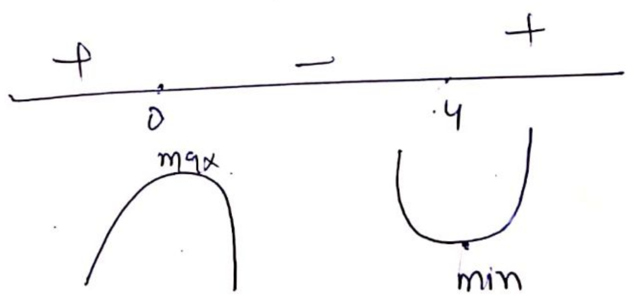
$f''(x) = 0$

$x = 2$



$f''(0) < 0$

$f''(x) > 0$



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(2)

$$f(x) = 4x^3 - 6x^2 - 24x + 1$$
$$f'(x) = 12x^2 - 12x - 24$$
$$f''(x) = 24x - 12$$

$12(x^2 - x - 2) = 0$

$12(x-2)(x+1)$

$\boxed{x=2}$, $\boxed{x=-1}$

$f''(x) = 0$

$24x - 12 = 0$

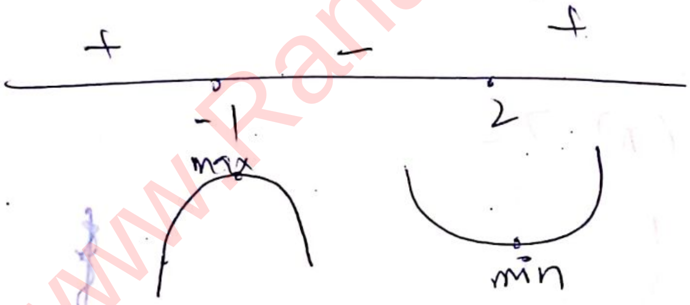
$12(x-1)$

$\boxed{x=1}$

$f''(2) = 36$ min

$f''(-1) = 24(-1) - 12$

$= -36$ max



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