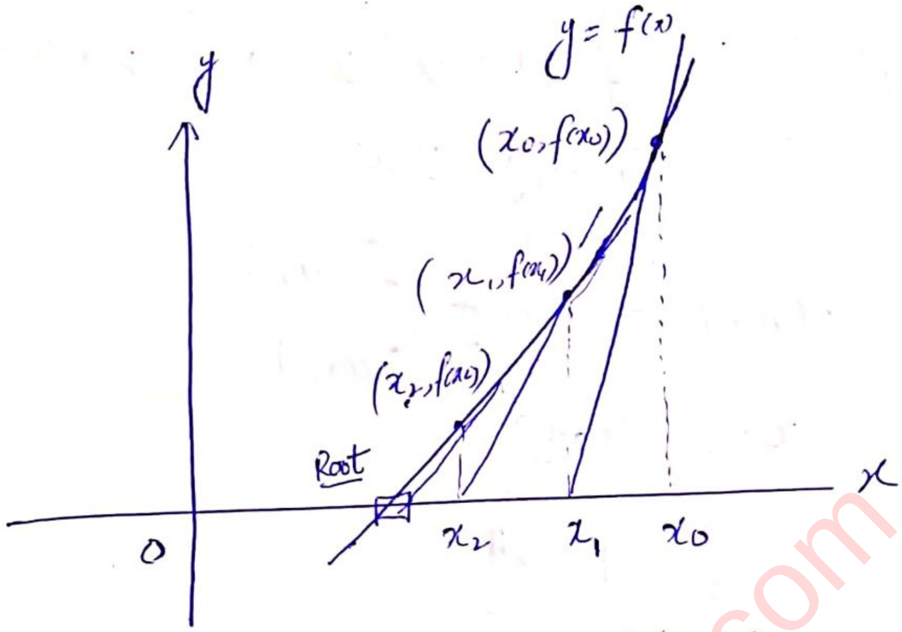
Newtons Method:-



$$y = f(x_n) + f'(x_n)(x - x_n)$$

when crosses x-axis by setting $y=0$

$$0 = f(x_n) + f'(x_n)(x - x_n)$$

$$-f(x_n) = f'(x_n)(x - x_n)$$

$$x - x_n = \frac{-f(x_n)}{f'(x_n)}$$

$$x = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{If } f'(x_n) \neq 0$$

Examples:-

1. Find the positive root of equation $f(x) = x^2 - 2 = 0$ $\quad x_0 = 1$

$$f'(x) = 2x$$

$$x_{n+1} = x_n \pm \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 \pm \frac{f(x_0)}{f'(x_0)}$$

$$= 1 + \frac{1}{2}$$

$$= 1.5$$

$$x_2 = x_1 + \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 + (-)$$

$$= 1.41667$$

$$x_3 = x_2 + (-)$$

$$= 1.41422$$

error

$$|x_n - x_{n+1}|$$

Example. 2:-

$\cos x = x$ has

a solution between 0 and 1. use Newton method the approximate it.

Sol:-

$\cos x = x$

$\cos x - x = 0 \Rightarrow x - \cos x$

$f(x) = \cancel{\cos x - x}\ x - \cos x.$

$f'(x) = \cancel{-\sin x - 1}\ 1 + \sin x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_0 = 0, \quad x_1 = 1$

$x_2 = .750363868$

$x_3 = .739112891$

$x_4 = .739085133$

$x_5 \approx .739085133$.

Minimum error:.

Exercise 4.6 (1)

$f(x) = x^2 + x - 1$, $x_0 = 1$, $x_0 = -1$

find x_2

(10)

Apply Newton's Method to $f(x) = x^{1/3}$ with $x_0 = 1$ and Calculate x_1, x_2 and x_3, x_4. Find formula for $|x_n|$. what happens $|x_n|$ as $n \to \infty$? Draw a picture what is going on?

Sol:-

$$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

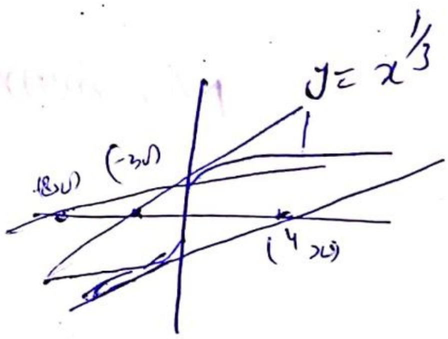
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - 3 = -2$$

$x_2 = 4$, $x_3 = -8$, $x_4 = 16$ So on.

$$|x_n| = 2|x_{n-1}|$$

as $n \to \infty$ $|x_n| \to \infty$



Q.9 Show that $h > 0$, applying Newton's method to

$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ \sqrt{-x} & x < 0 \end{cases}$$

leads to $x_1 = -h$ if $x_0 = h$ and to $x_1 = h$ if $x_0 = -h$. Draw a picture that shows what is going on.

Sol:-

$x_0 = h > 0$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= h - \frac{\sqrt{h}}{\frac{1}{2\sqrt{h}}} = h - \sqrt{h}(2\sqrt{h}) = -h$$

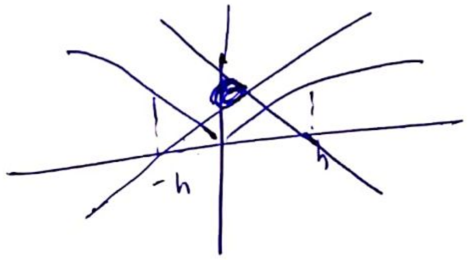
if $x_0 = -h < 0$

$f(x) = \sqrt{-x}$

$= -\frac{1}{2\sqrt{x}}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -h - \frac{\sqrt{h}}{-\frac{1}{2\sqrt{h}}}$$

$$= -h + 2h = h.$$



Antiderivatives:-

1

The process of recovering a function $F(x)$ from its derivative $f(x)$ is called Antidifferentiation.

* $x^2 \longrightarrow 2x$ and also $x^2+1 \longrightarrow 2x$

Examples:-

(1)

$$f(x) = 2x$$

$$F(x) = x^2$$

(2) $f(x) = 3x^2$. that Satisfies $F(1) = -1$

Sol:-

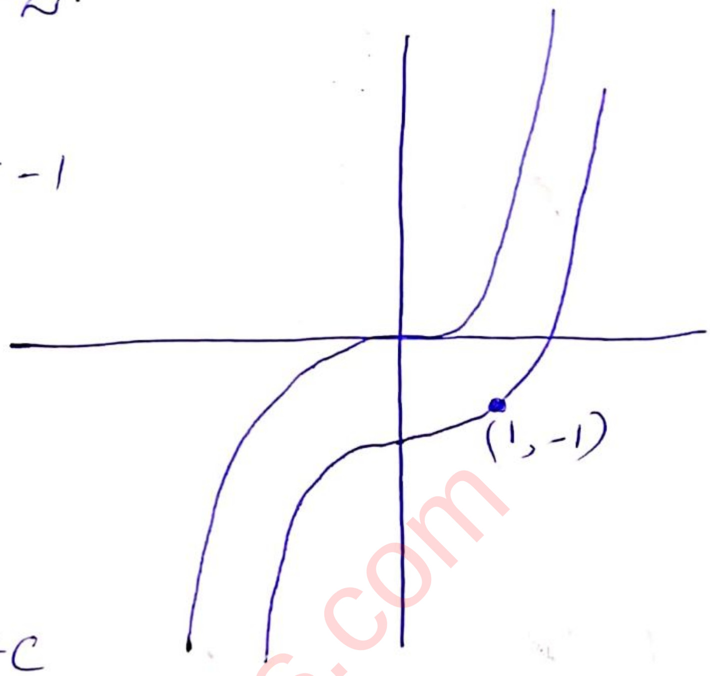
$$F(x) = \frac{3x^{2+1}}{3} + C$$

$$F(x) = x^3 + C$$

$$F(1) = (1)^3 + C$$

$$-1 = 1 + C \Rightarrow \boxed{C = -2}$$

$F(x) = x^3 - 2.$



Functions	General Anti
1. x^n	$\frac{x^{n+1}}{n+1} + C \quad n \neq -1$
2. $\sin kx$	$-\frac{\cos kx}{k} + C$
3. $\cos kx$	$\frac{\sin kx}{k} + C$
4. $\sec^2 kx$	$\frac{\tan kx}{k} + C$
5. $\csc^2 kx$	$-\frac{\cot kx}{k} + C$
6. $\sec kx \tan kx$	$\frac{1}{k} \sec kx + C$
7. $\csc kx \cot kx$	$-\frac{\csc kx}{k} + C$

Example :- 3

(1) $f(x) = x^5$

$F(x) = \frac{x^6}{6} + C$

(2) $i(x) = \cos\frac{x}{2}$

$I(x) = 2\sin\frac{x}{2} + C$

National University of Computer and Emerging Sciences

Subject: Calculus and Analytical Geometry(MT-1003)

Semester FA-21 Date: 20-10-2021 Time allowed: 20 minutes

Student Name: __________ Student ID: __________

Q#1 Evaluate (i) $\lim_{x\to\infty} \frac{x^2+1}{x^3+1}$ (1) (ii) $\lim_{x\to\infty} \frac{\sqrt{x^2+8}}{3x-14}$ (2)

Q#2 Check continuity of $f(x) = \sqrt{4-x^2}$ on [-2,2] (3)

Q#3 (i) $y = x^3 - 2x + 7$, $x = -3$. Find the slope of the curve at point indicated. (1)

(ii) $\frac{x+1}{x^2-1}$, Find vertical asymptote (2)

(iii) $f(x) = \frac{x^2+1}{x-1}$, What is oblique asymptote (2)

Best of luck

Antiderivative linearity rules:-

1. Constant Multiple Rule.

$$Kf(x) \rightarrow KF(x) + C$$

2. Negative Rule:-

$$-f(x) \rightarrow -F(x) + C$$

3. Sum or Difference Rule:

$$f(x) \pm g(x) \rightarrow F(x) \pm G(x) + C$$

Example:

$$f(x) = 3\sqrt{x} + \sin 2x.$$

$$F(x) = 2x^{3/2} - \frac{1}{2}\cos 2x + C.$$

Initial value Problems and Differential Equations:-

$$\frac{dy}{dx} = f(x)$$

Differential Equation

$y(x_0) = y_0$

$y(x)$ has value y_0 at $x = x_0$.

Example:- (103)

The standard equation for the position s of body moving with a constant acceleration a along a coordinate line is

$$s = \frac{a}{2}t^2 + v_0 t + s_0$$

v_0 and s_0 are body's velocity and position. Derive the equation by solving initial value problem

Differential equation $\dfrac{d^2s}{dt^2} = a$.

I.C $\dfrac{ds}{dt} = v_0$ and $s = s_0$ when $t = 0$.

Best of luck

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Sol:-

$$\frac{d^2s}{dt^2} = a$$

$$\frac{ds}{dt} = at + c \qquad \text{at } \frac{ds}{dt} = v_0 \,, \; t = 0$$

$$\boxed{v_0 = c}$$

$$\frac{ds}{dt} = at + v_0$$

$$s = \frac{at^2}{2} + v_0 t + c_1$$

$$s = s_0 \rightarrow t = 0$$

$$\boxed{s_0 = c_1}$$

$$s = \frac{at^2}{2} + v_0 t + s_0$$

Indefinite Integrals:-

Examples:-

$$\int (x^2 - 2x + 5)\,dx$$

$$= \frac{x^3}{3} - x^2 + 5x + C$$