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Applied optimization :-

Examples: ① Find two numbers whose sum is 60 and whose product is maximum?

Sol:-

$$S = x + y \quad -①$$

$$P = xy \quad -② \longrightarrow \text{objective}$$

$$60 = x + y$$

$$60 - x = y$$

$$P = x(60 - x)$$

$$P = 60x - x^2$$

$$P' = 60 - 2x$$

$$2x = 60$$

$$\boxed{x = 30}$$

$$60 - 30 = y$$

$$\therefore y = 30$$

$$P = 30(30)$$

$$\boxed{P = 900}$$

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X	Y	S	P
10	50	60	50
20	40	60	800
30	30	60	900
31	29	60	899
32	28	60	896
37	23	60	851

2. Find two numbers whose difference is 40 and whose product is a minimum what is value of minimum product?

Sol:-

$$d = y - x$$

$$P = xy$$

$$40 = y - x$$

$$y = 40 + x$$

$$P = x(40 + x)$$

$$P = 40x + x^2$$

$$P' = 40 + 2x$$

$$\boxed{x = -20}$$

$$\boxed{y = 20}$$

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$P = (-20)(20) = -400$

Exercise

Q:10 A $1125\,ft^3$ open-top rectangular tank with a square base x ft on a side and y ft deep is to built with its top flush with ground to catch runoff water. The costs associated with the tank involve not only the material from which the tank is made but also an excavation charge proportional to the product xy.

(a) If the total cost is

$$C = 5(x^2 + 4xy) + 10xy$$

what values of x and y will minimize it?

(b) Give a possible scenario for the cost function in part (a)

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Sol:- (a) The volume of tank $= 1125 ft^3$ ...

$$yx^2 = 1125$$

$$y = \frac{1125}{x^2} \quad \text{---} (1)$$

The cost of building tank is ...

$$C(x) = 5x^2 + 30xy$$

$$C(x) = 5x^2 + 30x\frac{1125}{x^2}$$

$$C'(x) = 10x - \frac{33750}{x^2}$$

$$C'(x) = 0$$

$$\boxed{\begin{aligned} x &= 0 \\ x &= 15 \end{aligned}}$$

$C''(15) > 0$ at $x = 15$ we have minimum. The value of $x = 15$ putting in

$$y = \frac{1125}{225} = \frac{75}{15} = 5$$

$x = 15, y = 5$, minimize the cost