Lecture 02.

Common Functions:-

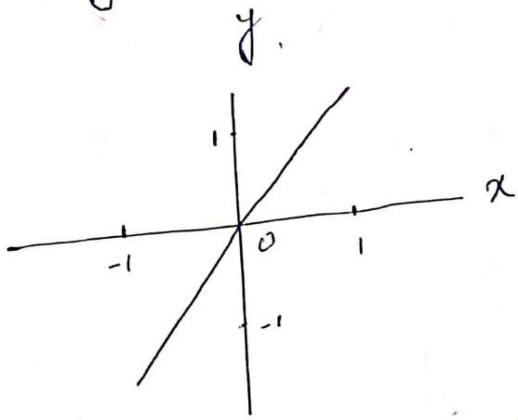
Power Functions:-

A function $f(x) = x^a$, where a is constant, called power function.

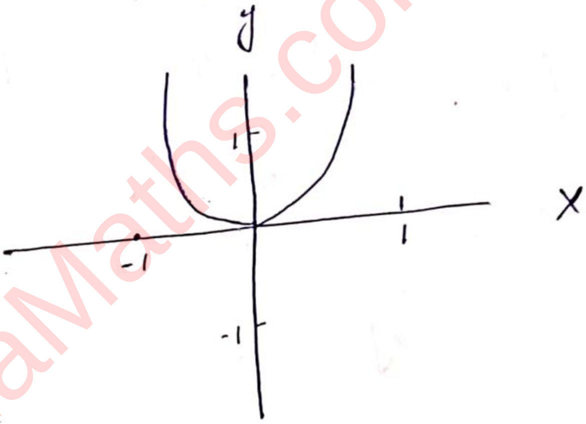
There are several important cases to consider:

(a) $f(x) = x^a$ with $a = n$, a positive Integer

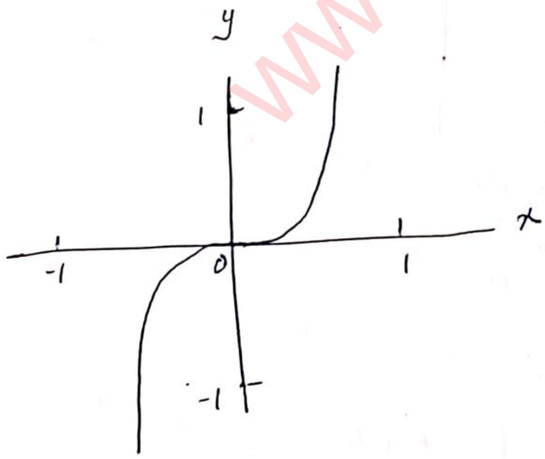
1- $y = x$



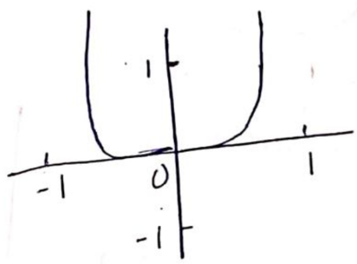
2- $y = x^2$



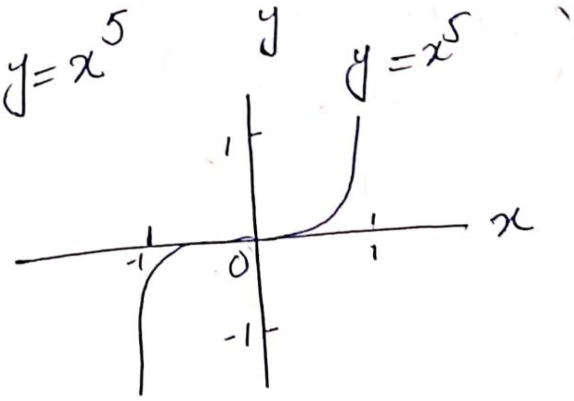
3- $y = x^3$



4- $y = x^4$



5- $y = x^5$

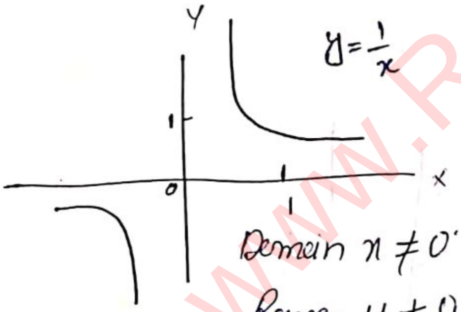


* The power n gets larger, the curves tend to flatten towards the x-axis on the Interval $(-1,1)$ and rise more steeply for $|x|>1$.
* The even-powered functions are decreasing on the Interval $(-\infty,0]$ and increasing on $[0,\infty)$.
* The odd-powered functions are increasing over the entire real line $(-\infty,\infty)$.

(b) $f(x) = x^a$ with $a=-1$ or $a=-2$.

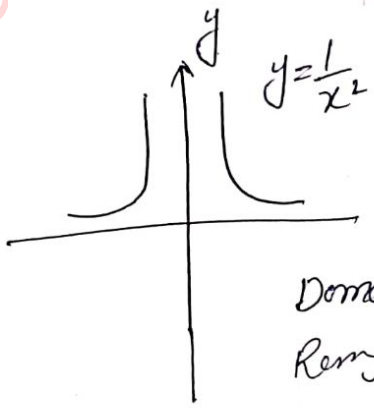
$\Rightarrow f(x) = x^{-1} = 1/x$ and $g(x) = x^{-2} = \frac{1}{x^2}$

$y = \frac{1}{x}$, $xy=1$ $\qquad\qquad$ $y = 1/x^2$



Domain $x \neq 0$
Range $y \neq 0$

* Symmetric about origin
* Decreasing on $(-\infty, 0)$ and $(0,\infty)$



Domain $x \neq 0$
Range $y > 0$

* Symmetric about y-axis
* Increasing on $(-\infty, 0)$ and decreasing $(0,\infty)$

c) $a = \frac{1}{2}, \frac{1}{3}, 3/2$ and $2/3$ Recal. $x^{3/2} = (x^{1/2})^3 \Rightarrow x^{2/3} = (x^{1/3})^2$

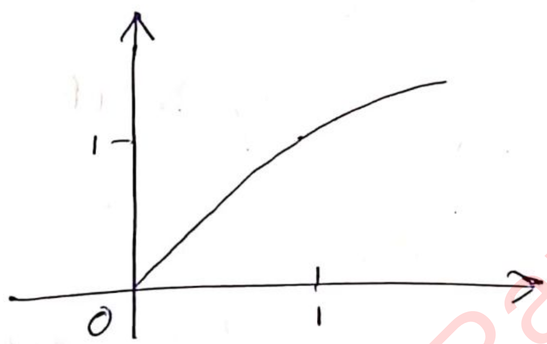
$f(x) = x^a$

$f(x) = x^{1/2}$ and $g(x) = x^{1/3} = \sqrt[3]{x}$

are Square root and cube root functions.

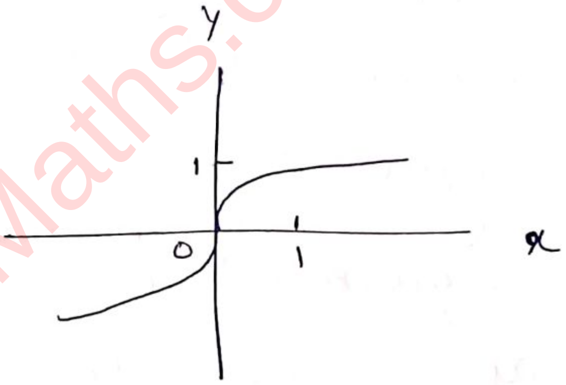
* The domain of square root function is $[0, \infty)$.
* cube root defined over all real x.

$y = \sqrt{x}$



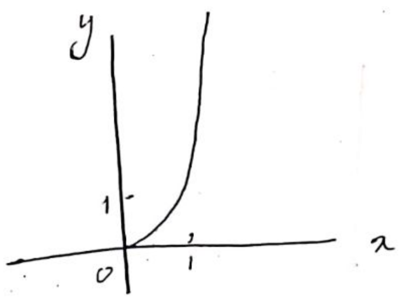
Dom: $0 \le x < \infty$
Ran: $0 \le y < \infty$

$y = \sqrt[3]{x}$



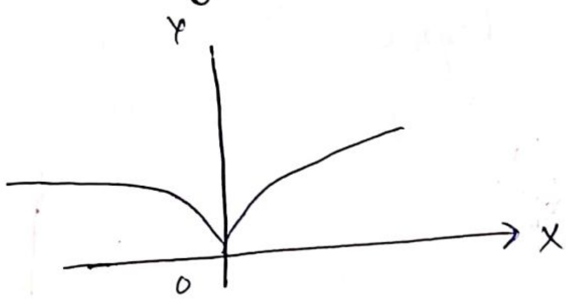
D: $-\infty < x < \infty$
R: $-\infty < y < \infty$

$y = x^{3/2}$



D: $0 \le x < \infty$
R: $0 \le y < \infty$

$y = x^{2/3}$



Dom: $-\infty < x < \infty$
R: $0 \le y < \infty$

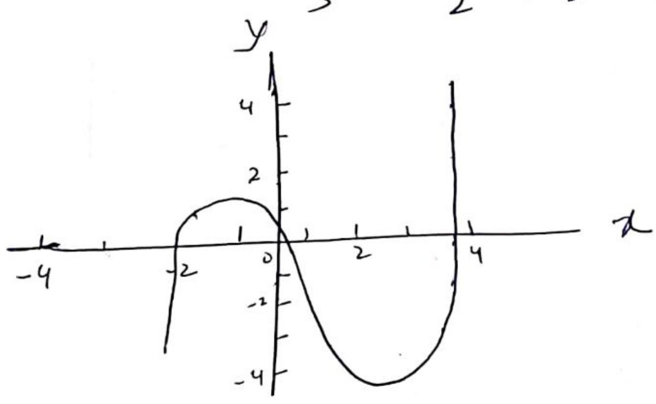
Polynomials:- A function p is a polynomial if

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are real constants (called coefficients of the polynomial). All the polynomials have domain $(-\infty, \infty)$. If the leading coefficient $a_n \neq 0$, then n is called the degree of polynomial. Linear functions with $m \neq 0$ are polynomials of degree 1.

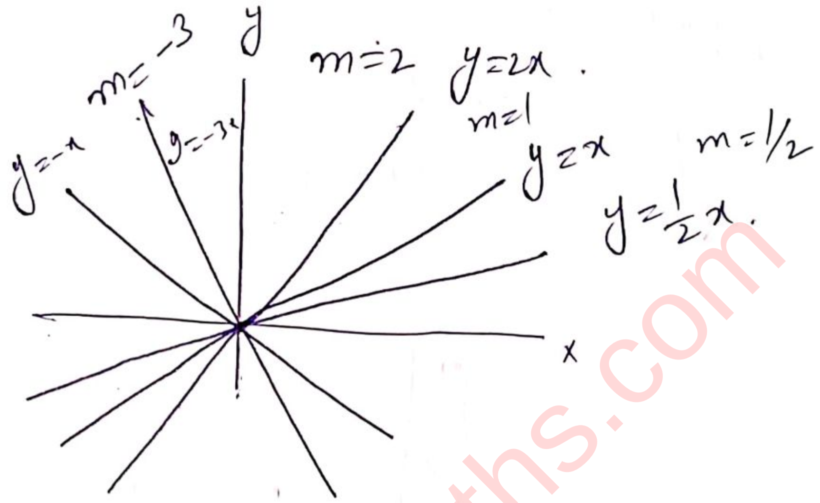
Polynomials of degree 2, usually written as $p(x) = ax^2 + bx + c$ are called quadratic functions. Likewise, cubic functions are polynomials $p(x) = ax^3 + bx^2 + cx + d$ of degree 3

Graph $\quad y = \dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x + \dfrac{1}{3}$



Linear Function:-

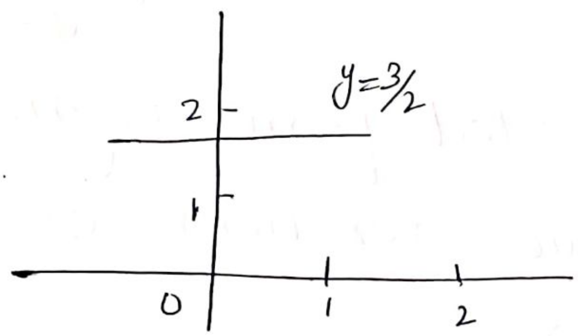
A function of the form $f(x) = mx + b$, where m and b are fixed constants is called linear function



when $b = 0$, so these lines passes through origin.

when $b=0$ and $m=1 \rightarrow$ identity function

when $m=0$, the function is said to be constant.



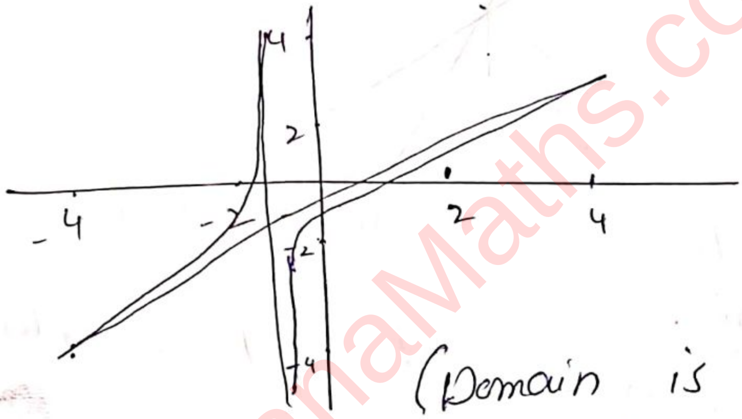
Rational function:-

$$f(x) = p(x)/q(x)$$

p and q are polynomials.

$q(x) \neq 0$

$$y = \frac{2(x^2) - 3}{7x + 3}$$



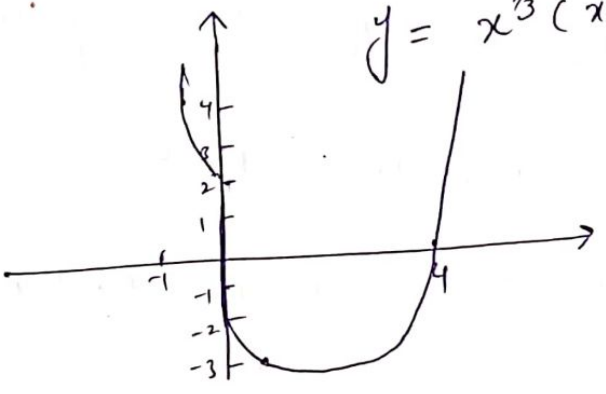
(Domain is set of all real numbers)

Algebraic Functions:-

Any function constructed from polynomials using algebraic operations (addition, −, ÷, and taking roots) lies in the class of algebraic functions

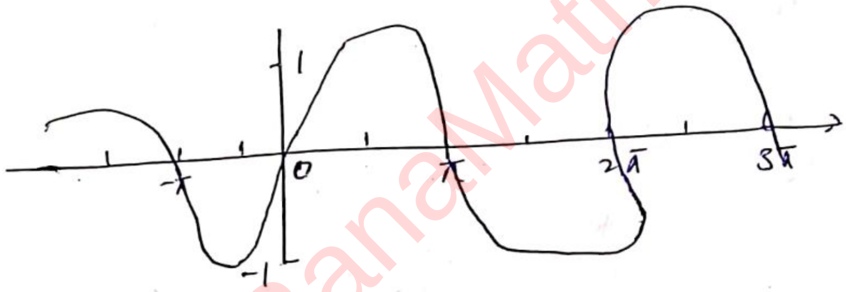
* All rational functions are algebraic.

$y = x^{1/3}(x-4)$

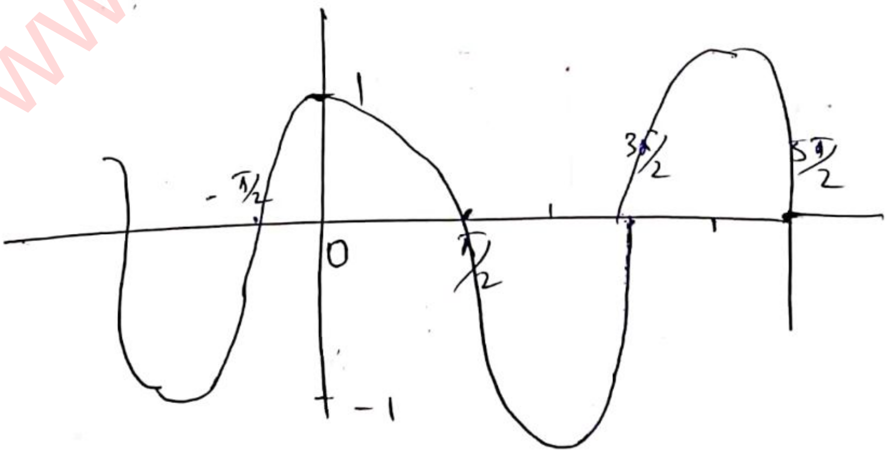


Trignometric Functions:-

(a) $f(x) = \sin x$.



(b) $f(x) = \cos x$.



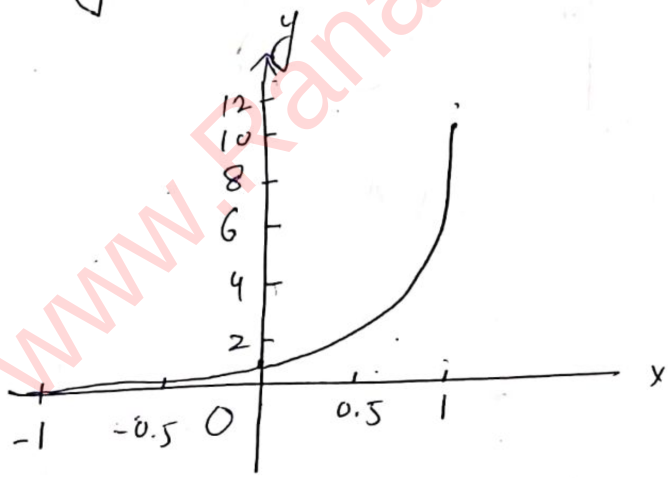
Exponential Functions:-

A function of the form $f(x) = a^x$ where $a > 0$ and $a \neq 0$ is called exponential function (with base a)

* Domain $(-\infty, \infty)$
* Range $(0, \infty)$

So an exponential function never assumes the value 0

$y = 10^x$



Logrithmic Functions:-

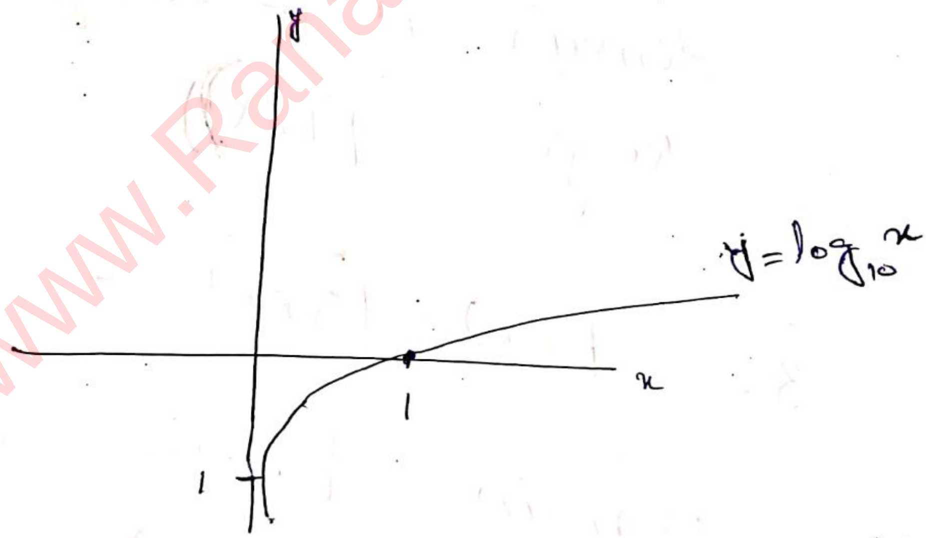
These are the functions $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant.

They are Inverse of exponential functions

* D is $(0, \infty)$
* R is $(-\infty, \infty)$

$$y = \log_{10} x$$

$$10^y = x$$



Trascendental Functions:-

These functions are not algebraic. They include the trignometric, Inverse trignometric, exponential and logrithmic functions and many more. The catenary is one example of transcendental function.

Exercise 1.1

1- $f(x) = 1+x^2$

domain is $(-\infty, \infty)$

Range $= [1, \infty)$

3- $F(x) = \sqrt{5x+10}$

Domain $= [-2, \infty)$

Range $=$ $[0, \infty)$

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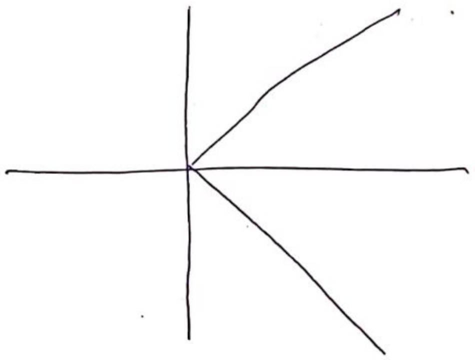
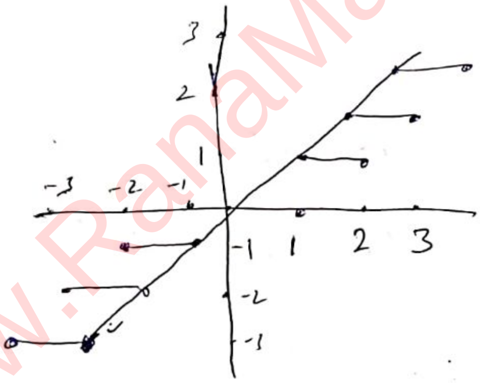
$|y| = x$ Explain not graph of a function.



34- $\lfloor x \rfloor = \lceil x \rceil$ iff only integers.

36- $f(x) = \begin{cases} \lfloor x \rfloor & x \geq 0 \\ \lceil x \rceil & x < 0 \end{cases}$

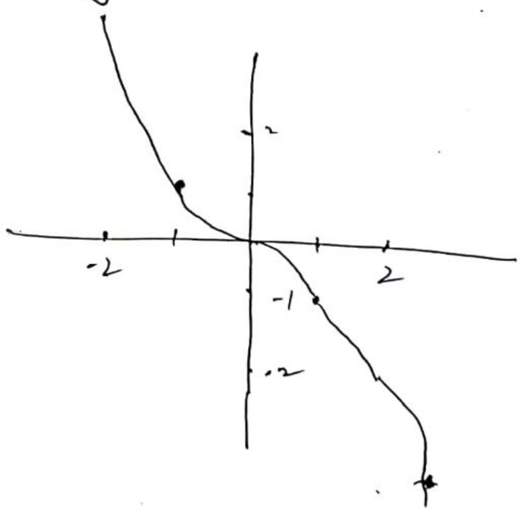


Increasing, Decreasing Function

$y = -x^3$

D. $-\infty < x < \infty$

I. nowhere.



even-odd function:-

(50) $f(x) = x^2 + x$

$f(-x) = (-x)^2 + -x$

$= x^2 - x$

$= -(-x^2 + x)$

neither

54 $g(x) = x/x^2-1$

$g(-x) = \dfrac{-x}{x^2-1}$

$= -\left(\dfrac{x}{x^2-1}\right) \Rightarrow$ Odd.

64.

K = 12,960 Joules.

V = 18 m/sec what K when v = 10 m/se.

$K = CV^2$

$12960 = C(18)^2$

$\boxed{C = 40}$

$k = CV^2$

$= 40(10)^2$

$= 40(100)$

$= 4000$ Joules

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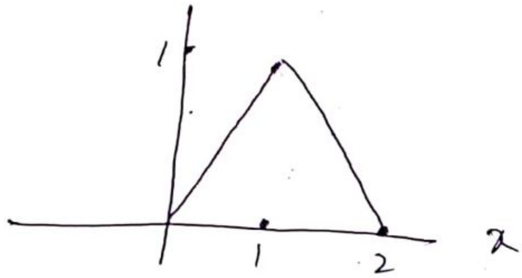
$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 < x \le 2 \end{cases}$$



x	0	1	2
y	0	1	0

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$$r = \frac{k}{s}$$

or

$$r \propto \frac{1}{s}$$

$$r = k/s$$

$$6 = k/4$$

$$\boxed{k = 24}$$

$$r = \frac{k}{s}$$

$$10 = \frac{24}{s}$$

$$s = \frac{24}{10}$$

$$\boxed{s = 2.4}$$