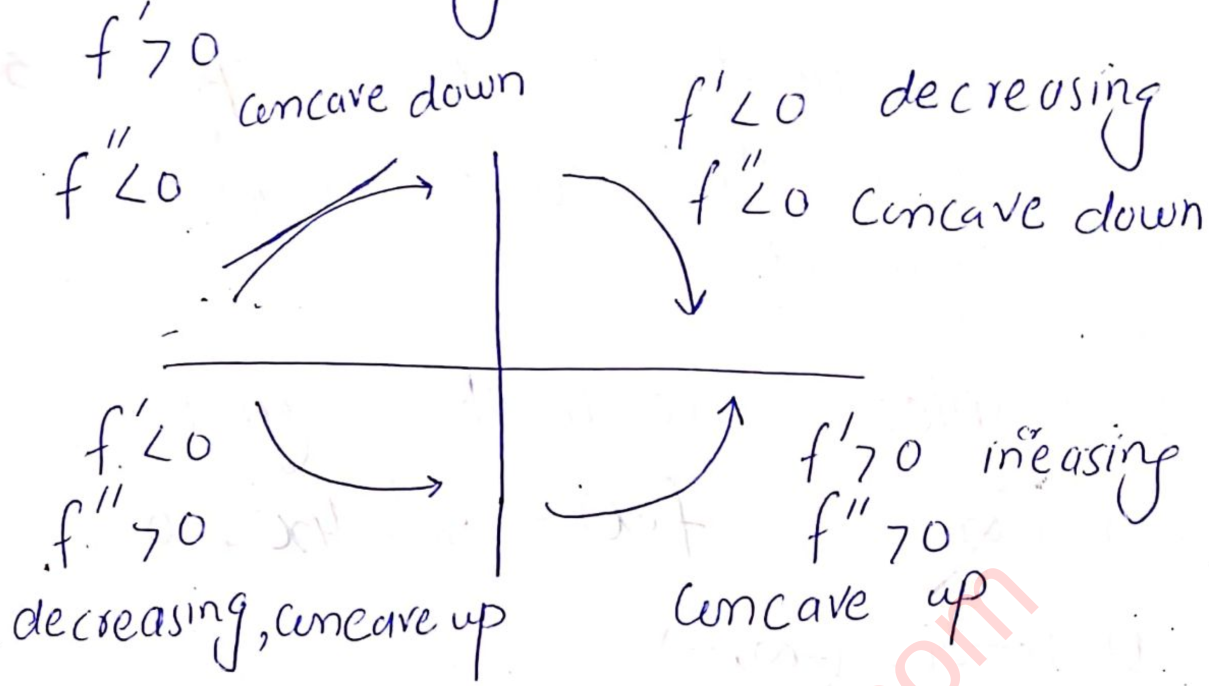
Curve Sketching:-



Procedure:

1- Identify the domain of f.

2. Find the derivatives y' & y''

3- Find the critical points of f if any and identify the function behaviour at each one.

4- Find where the curve increasing/decreasing

5- Find the points of inflection, if any occur and determine the concavity.

6- Identify asymptotes that may exists

7- Plot key-points, such as intercepts and the points found in step 3-5 and sketch the curve together with any asymptote

Example:- Sketch the graph of function $f(x) = x^4 - 4x^3 + 10$

① Domain $(-\infty, \infty)$

② $f(x) = x^4 - 4x^3 + 10$

$f'(x) = 4x^3 - 12x^3$

Critical point $f'(x) = 0$

$4x^3 - 12x^2 = 0$

$x^2(4x - 12) = 0$

$4x^2(x - 3) = 0$

$x = 0 \quad , \quad x = 3$

Critical points $0, 3$

$-\infty \quad\quad 0 \quad\quad 3 \quad\quad \infty$

$(-\infty, 0) \quad\quad (0, 3) \quad\quad (3, +\infty)$

$(-\infty, 0)$

$f'(x) = 4x^3 - 12x^2$

$f'(-1) = 4(-1)^3 - 12(-1)^2$

$= -4 - 12 = -16 < 0$

Decreasing

on $(0, 3)$

$f'(x) = 4(1) - 12(1)$

$= 4 - 12$

$= -8 < 0$

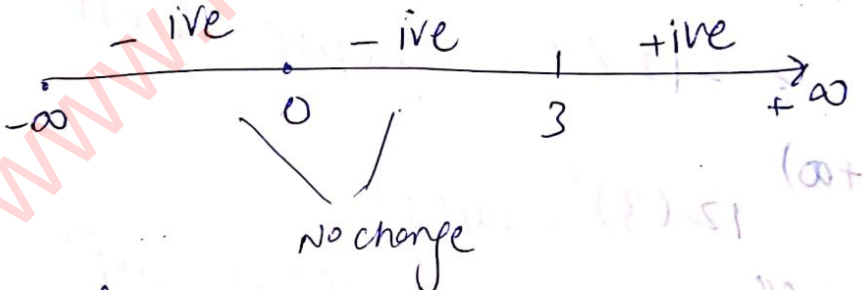
Decreasing

on $(3, +\infty)$

$f'(x) = 4(4)^3 - 12(4)^2$

$= > 0$ Increasing

$f'(x)$



No relative exteremum at $x = 0$

At $x = 3$

relative max is 17

$f''(x) = 12x^2 - 24x$

$f''(x) = 12x(x-2)$

$x = 0, \quad x = 2$ Inflection points

$f''(x) = 12x^2 - 24x$

$\Rightarrow (-\infty, 0)$

$= 12(-1) - 24(-1)$

$= -12 + 24 > 0$

$f''(x) > 0$ concave up

$(0, 2)$

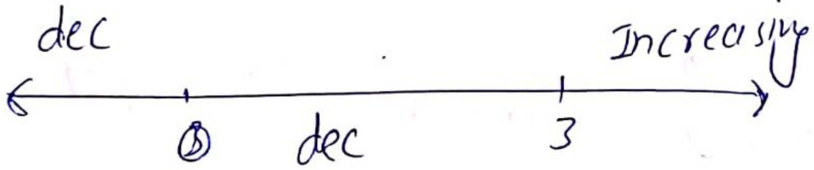
$= 12(1) - 24(1)$

$= -12 < 0$ concave down

$(2, +\infty)$ $12(3)^2 - 24(3)$

$f''(x) > 0$ concave up

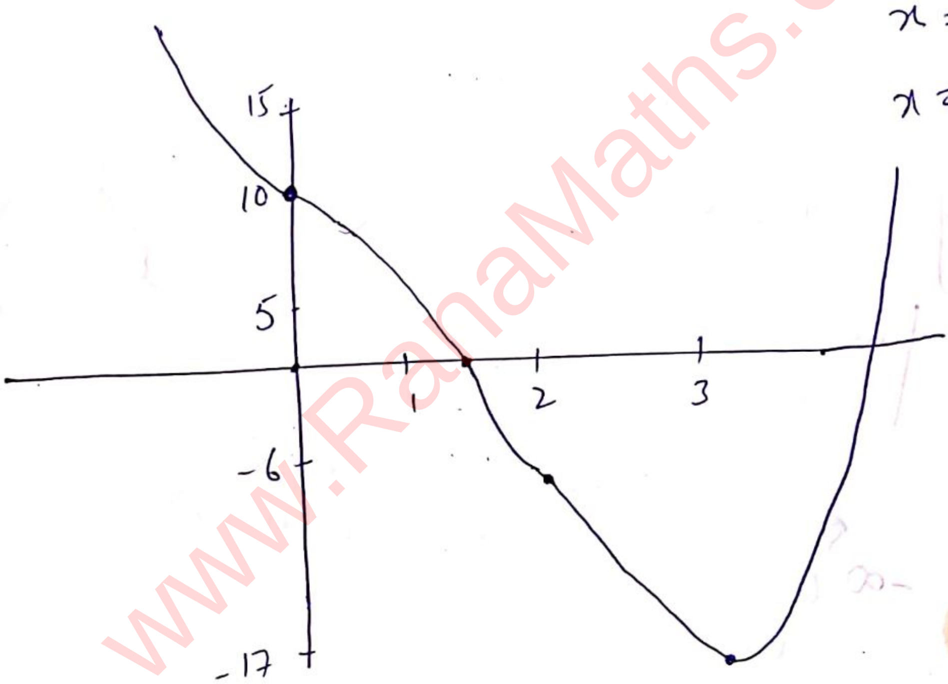
$f'(x)$



$f''(x)$

$x < 0$	$0 < x < 2$	$2 < x < 3$	$3 < x$
decreasing	decreasing	decreasing	increasing
concave up	concave down	concave up	concave up

Intercepts:
$x=0,\ y=10$
$x=3,\ y=-17$
$x=2,\ y=-6$



Example 2:- Sketching the general shape of graph, knowing y'

$$y' = 2 + x - x^2$$

Critical points

$$2+x-x^2=0$$

$$x=-1, \quad x=2$$

$$(-\infty,-1), (-1,2), (2,\infty)$$

f'

+ive increasing

-ive

-ive -1   2

decreasing   decreasing

Local max = +2

Local min at $x=-1$

$$y'' = 1-2x$$

$$\boxed{x=\frac{1}{2}}$$

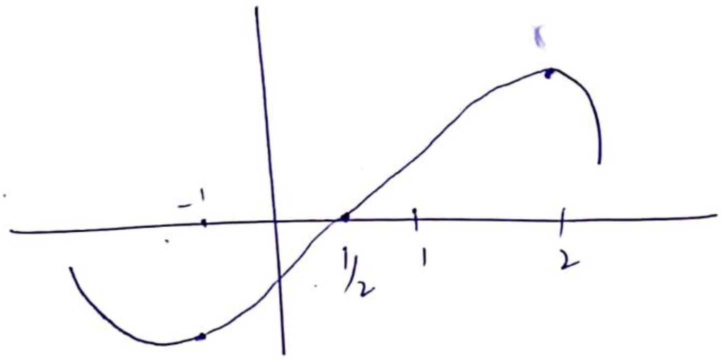
$-\infty$ $\frac{1}{2}$ $+\infty$

$-\infty$ Concave up $\frac{1}{2}$. Concave down

Concave up   concave up   decreasing

$-\infty$ -1 $\frac{1}{2}$ 2 ∞

decreasing concave down   Concave down.



Plotting Rational Function:-

Sketch graph of

$$f(x) = \frac{x^2+4}{2x}$$

Domain $(-\infty, 0) \cup (0, \infty)$

$$f'(x) = \frac{2x(2x) - (x^2+4)(2)}{(2x)^2}$$

$$= \frac{4x^2 - 2x^2 - 8}{4x^2}$$

$$= \frac{2x^2 - 8}{4x^2}$$

$$f'(x) = \frac{1}{2} + \frac{2}{x^2}$$

$$f''(x) = \frac{4x}{x^4} = \frac{4}{x^3}$$

$\frac{1}{2} + \frac{2}{x^2}$ $\quad x^2 \cdot \frac{-2(2x)}{x^4}$

Critical points:

$x = \pm 2$ ($x = 0$ is not a critical point because it is not function domain)

+ive   -ive   +ive

$-\infty$ — -2 — 0 — 2 — ∞

increasing

Relative max at $x = -2$

min at $x = 2$.

Also from 2nd derivative test

(-2)

$f''(x) = \frac{4}{x^3} < 0$ relative max.

(2)

$f''(x) > 0$ relative min

point of Inflection :-

NO Inflection point

$f''(x) < 0$ whenever $x < 0$

$f''(x) > 0$ when $x > 0$

$(-\infty, 0)$ concave ~~up~~ down

$(0, +\infty)$ concave ~~down~~ up

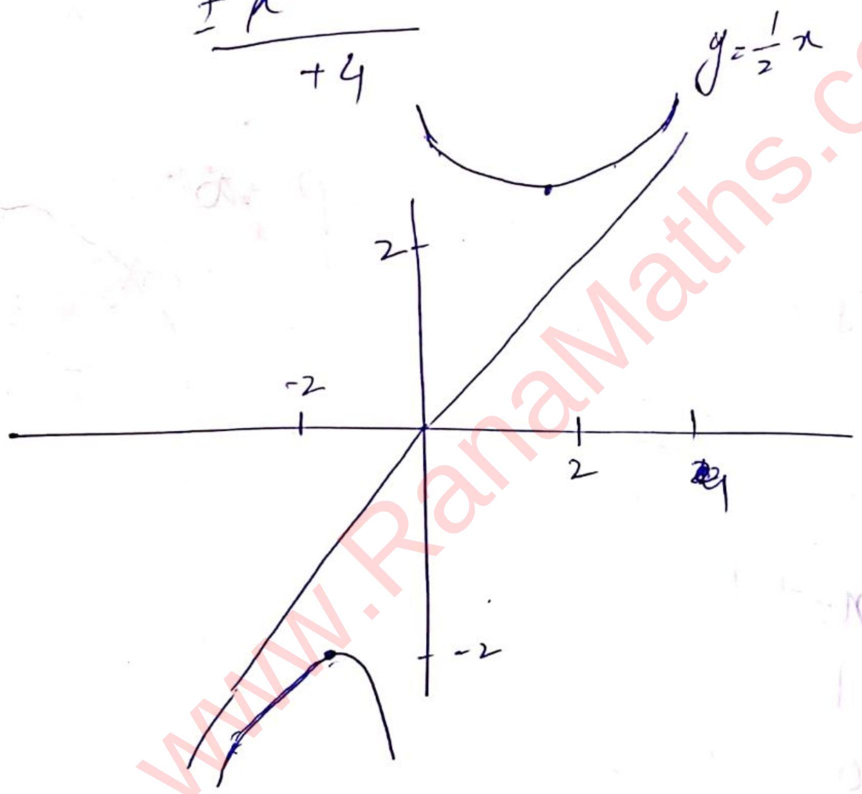
Asymptotes:-

$$f(x) = \frac{x}{2} + \frac{2}{x}$$

$$= \frac{x^2+4}{2x}$$

$x=0$ vertical Asymptote.

$$2x \,\overline{\big)\, x^2 + 4}^{\;\frac{1}{2}x}$$
$$\underline{\pm x^2}$$
$$+4$$



91 Example:-

$$y = -\frac{x^2-2}{x^2-1}$$

Domain $x \neq \pm 1$

Domain $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$$y' = \frac{-2x}{(x^2-1)^2} \qquad C.P\ x=0$$

f'

$-\infty$	A	-1	B	0	C	1	D	∞
	+ive		+ive		-ive		-ive	

$$y'' = \frac{6x^2+2}{(x^2-1)^3}$$

No inflection point. Maximum $\boxed{\text{at } x=0,-2}$

$-\infty$		-1		0		1		$+\infty$
	Inc		Inc		Dec		Dec	
	up		down		down		up	

Asymptotes :-

$$y = \frac{-x^2-2}{x^2-1}$$

V.A $\Rightarrow$ $x = \pm 1$

H.A $= -1$

$x=0$, $y=-2$