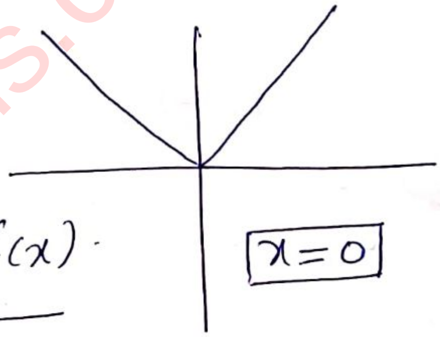
Points of Non-differentiability:-

1. Discontinuous ✓
2. Corner
3. Points of vertical Tangency
4. Cusp

Corner :-

$$f(x) = |x|$$



$\boxed{x=0}$

$$f'(x) = \underset{h \to 0}{Lt} \frac{f(x+h) - f(x)}{h}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$f'(0^-) = \underset{h \to 0^-}{Lt} \frac{|0+h| - |0|}{h}$$

$$= \underset{h \to 0^-}{Lt} \frac{|h|}{h} = \frac{-h}{h} = -1$$

$$f'(0^+) = 1$$

Points of vertical Tangency:-

$\frac{dy}{dx} \rightarrow$ slope of tangent

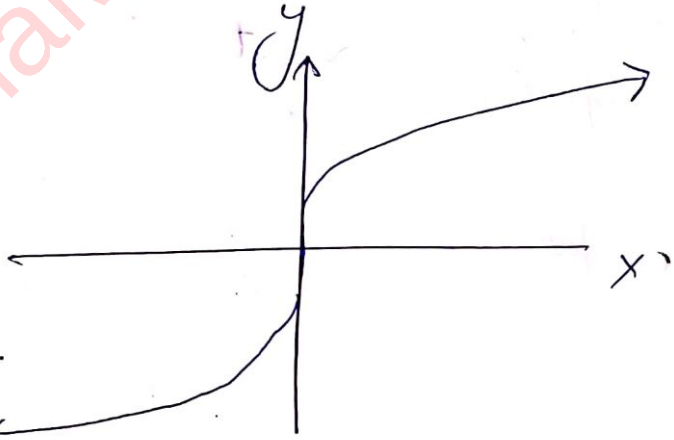
$$\frac{dy}{dx} = \infty$$

We say that a continuous curve $y = f(x)$ has vertical tangent at x if

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \pm\infty$$

Example.

$$y = x^{1/3}$$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^{1/3} - x^{1/3}}{h}$$

$$f'(0) = \underset{h\to 0}{Lt} \frac{h^{1/3}}{h} = \underset{h\to 0}{Lim} \frac{1}{h^{2/3}} = \infty$$

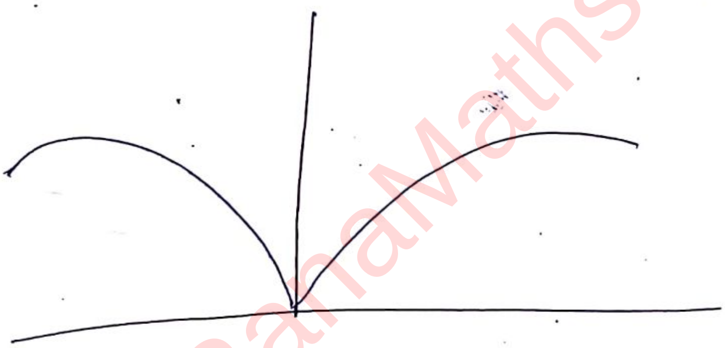
$1-1/3 \quad 2$

$$= \underset{h\to 0^-}{Li} \frac{1}{h^{2/3}} = \infty$$

$$= \underset{h\to 0^+}{Lt} \frac{1}{h^{2/3}} = \infty$$

Cusp:-

$$y = x^{2/3}$$



Example: $y = x^{2/3}$

$$y' = \underset{h\to 0}{Lt} \frac{(x+h)^{2/3} - x^{2/3}}{h}$$

$$y'(0) = \underset{h\to 0}{Lt} \frac{h^{2/3}}{h} = \underset{h\to 0}{Lt} \frac{1}{h^{1/3}}$$

$$y'(0^+) = \underset{h\to 0^+}{Lt} \frac{1}{h^{1/3}} = \infty$$

$$y'(0^-) = \underset{h\to 0^-}{Lt} \frac{1}{h^{1/3}} = -\infty$$