Related Rates :-

Q.6 if $x = y^3 - y$ and $\frac{dy}{dt} = 5$ then what is $\frac{dx}{dt}$ when $y = 2$?

Solution :-

$$\frac{dx}{dt} = 3y^2\frac{dy}{dt} - \frac{dy}{dt}, \quad y = 2, \ \frac{dy}{dt} = 5$$

$$\frac{dx}{dt} = 3(2)^2(5) - 5$$

$$= 60 - 5$$

$$= 55$$

Q.11 of the original 24m edge length x of a cube decreases at the rate of 5m/min, when $x = 3m$ at what rate does the cube's

(a) surface area change?

(b) volume change?

(a) $S = 6x^2$

$\frac{dx}{dt} = -5 m/min$

$$\frac{ds}{dt} = 12x\frac{dx}{dt}$$

$$= 12(3)(-5)$$

$$= -180 m^2/min$$

(b) $V = x^3$

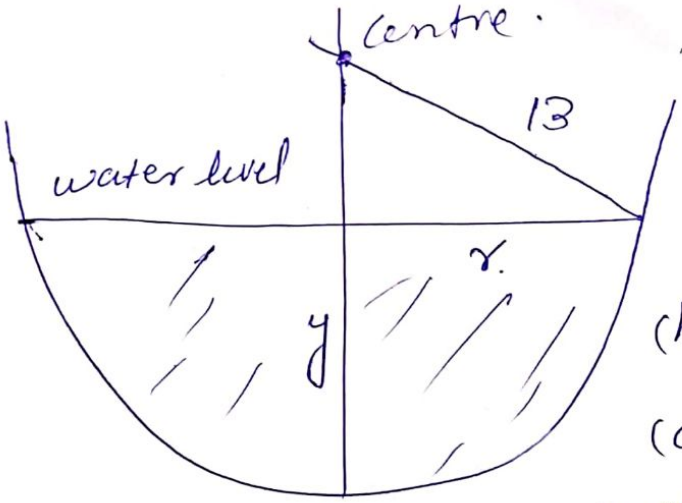
$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

$$= 3(3)^2(-5)$$

$$= 3(9)(-5)$$

$$= -135 m^3/min$$

29 Water is flowing at the rate of $6 m^3/min$ from a reservoir shaped like a hemispherical bowl of radius 13m, shown here in profile. Answer the following questions, given that the volume of water in a spherical bowl of radius R in $V = (\pi/3)y^2(3R - y)$ when the water is y meter deep



(a) At what rate is the water level changing when water 8m deep.
(b) radious of water
(c) At what radious r changing when water 8m deep?

Sol:- (a)

$$V = \frac{\pi}{3} y^2 (3R - y).$$

$$\frac{dv}{dt} = \frac{\pi}{3}\left[2y(3R-y) + y^2(-1)\right]\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{\pi}{3}\left[6Ry - 3y^2\right]\frac{dy}{dt}$$

$$\frac{dy}{dt} = \left[\frac{\pi}{3}(6Ry - 3y^2)\right]^{-1}\frac{dv}{dt}$$

$R = 13$ and $y = 8$

$$\frac{dy}{dt} = \left[\frac{\pi}{3}(6(13)(8) - 192)\right]^{-1}(-6)$$

$$= \left[\frac{\pi}{3}\left(\frac{144}{432}\right)\right]^{-1}(-6)$$

$$= \frac{1}{144\pi}(-6) = -\frac{1}{24\pi}$$

(b) $r^2+(13-y)^2 = 169$

$r^2 + 169 + y^2 - 26y = 169$

$r^2 = 26y - y^2$

$r = \sqrt{26y-y^2}$ m.

(c) $\frac{dr}{dt} = \frac{1}{2}(26y-y^2)^{-\frac{1}{2}}(26-2y)\frac{dy}{dt}$

$\frac{dr}{dt} = \frac{13-y}{\sqrt{26y-y^2}}\frac{dy}{dt}$

$= \frac{13-8}{\sqrt{26(8)-64}}\left(-\frac{1}{24\pi}\right)$

$= \frac{5}{12}\left(-\frac{1}{24\pi}\right)$

$= \frac{-5}{288\pi}$ m/min