Derivative of Logrithmic Functions:-

(i) $\lim_{v \to 0} (1+v)^{\frac{1}{v}} = e$

(ii) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

A derivative for the general logrithmic function $\log_b x$ is

(iii) $\frac{d}{dx}[\log_b x] = \frac{d}{dx}\left[\frac{\ln x}{\ln b}\right]$

$= \ln b \frac{1}{x(\ln b)^2} = \frac{1}{\ln b . x}$

$\frac{d}{dx}[\log_b x] = \frac{1}{\ln b . x}$, $x > 0$

(iv) $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$

(v) $\frac{d}{dx}[\log_b u] = \frac{1}{\ln b . u} \cdot \frac{du}{dx}$

Examples:-

(1) Find $\frac{d}{dx}\left[\ln(x^2+1)\right]$.

$$= \frac{1}{x^2+1}\frac{d}{dx}(x^2+1)$$

$$= \frac{1}{x^2+1}(2x+0)$$

$$= \frac{2x}{x^2+1}$$

(2) $\frac{d}{dx}\left[\ln\left(\frac{x^2\sin x}{\sqrt{1+x}}\right)\right]$

$$= \frac{d}{dx}\left[\ln x^2 + \ln \sin x - \ln(1+x)^{\frac{1}{2}}\right]$$

$$= \frac{d}{dx}\left[2\ln x + \ln \sin x - \frac{1}{2}\ln(1+x)\right]$$

$$= \left[2\cdot\frac{1}{x} + \frac{1}{\sin x}\cos x - \frac{1}{2(1+x)}(1)\right]$$

$$= \frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{2(1+x)}$$

$$= \frac{2}{x} + \cot x - \frac{1}{2(1+x)}$$

(3) Find derivative of $\ln|x|$ for $x \neq 0$. (2)

Solution:

Case $x > 0$:- In this case $|x| = x$, so

$$\frac{d}{dx}(\ln|x|) = \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Case $x < 0$: $|x| = -x$

$$\frac{d}{dx}(\ln|x| = \frac{d}{dx}\ln(-x)$$

$$= \frac{1}{-x}\frac{d}{dx}(-x)$$

$$= \frac{-1}{x}(-1)$$

So $$= \frac{1}{x}$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x} \quad x \neq 0$$

(4) $$y = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}$$

Sol:-

$$\ln y = \ln(x^2 \cdot \sqrt[3]{7x-14}) - \ln(1+x^2)^4$$

$$\ln y = \ln x^2 + \ln(7x-14)^{1/3} - 4\ln(1+x^2)$$

$$\ln y = 2\ln x + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$$

Taking Derivative on both sides

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{1}{3(7x-14)}(7) - \frac{4}{1-x^2}(2x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{7}{21x-42} - \frac{8x}{1-x^2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{\cancel{7}}{\cancel{7}(3x-6)} - \frac{8x}{1-x^2}$$

$$\frac{dy}{dx} = y\left[\frac{2}{x} + \frac{1}{3x-6} - \frac{8x}{1-x^2}\right]$$

$$\frac{dy}{dx} = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}\left(\frac{2}{x} + \frac{1}{3x-6} - \frac{8x}{1-x^2}\right)$$

Derivatives of Exponential Functions:-

(i) $\frac{d}{dx}[b^x] = \ln b \cdot b^x$

$= b^x \cdot \ln b$

(ii) $\frac{d}{dx}[e^x] = e^x$

(iii) $\frac{d}{dx}[b^u] = b^u \ln b \frac{du}{dx}$

(iv) $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

Examples:-

(i) $\frac{d}{dx}[2^x]$

$= 2^x \cdot \ln 2$

(ii) $\frac{d}{dx}[e^{\cos x}]$

$= e^{\cos x} \cdot \frac{d}{dx}(\cos x)$

$= e^{\cos x}(-\sin x)$

$= -(\sin x)e^{\cos x}$

(iii)

$$y = (x^2+1)^{\sin x}$$

$$\ln y = \ln(x^2+1)^{\sin x} = \sin x \ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\sin x . \ln(x^2+1))$$

$$= \sin x \frac{1}{x^2+1}(2x) + \ln(x^2+1)\cos x.$$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{2x\sin x}{x^2+1} + (\cos x)\ln(x^2+1)$$

$$\frac{dy}{dx} = (x^2+1)^{\sin x}\left[\frac{2x\sin x}{x^2+1} + (\cos x)\ln(x^2+1)\right]$$

Derivative of Inverse Trigonometric Functions:-

(i) $$\frac{d}{dx}[\sin^{-1}u] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

(ii) $$\frac{d}{dx}[\cos^{-1}u] = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

(iii) $\frac{d}{dx}\left[\tan^{-1}u\right] = \frac{1}{1+u^2} \cdot \frac{du}{dx}$

(iv) $\frac{d}{dx}\left[\cot^{-1}u\right] = -\frac{1}{1+u^2} \frac{du}{dx}$

(v) $\frac{d}{dx}\left[\sec^{-1}u\right] = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$

(vi) $\frac{d}{dx}\left[\operatorname{cosec}^{-1}u\right] = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$

Example:-

$$y = \sec^{-1}(e^x)$$

$$\frac{dy}{dx} = \frac{1}{|e^x|\sqrt{(e^x)^2-1}} \frac{d}{dx}(e^x)$$

$$\frac{dy}{dx} = \frac{1}{\cancel{e^x}\sqrt{e^{2x}-1}} \cdot \cancel{e^x}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{e^{2x}-1}}$$