Calculus and Analytical Geometry

* Real Numbers
* Intervals
* Functions
* Domain
* Range
* Applications
* Graphs

Functions:- A function f from a set D to a set Y is a rule that assigns a unique value $f(x)$ in Y to each x in D.

We can write

out put

$$y = f(x)$$

Input

Independent.

dependent

* The area of circle depends on the radious of circle.
* The distence an object travels depends upon the time

Domain: The set D of all possible input values is called domain of function

Range:-
The set of all output values of $f(x)$ as x varies throughout D is called the range of function

* Domain and Range are often sets of real numbers in Cuculus.

* $y = x^2, x \geq 0$ restricted.

Range is $[0, \infty)$

as
$y = x^2, x \geq 2$

Range is $[4, \infty)$ or $\{x^2 / x \geq 2\}$.

* When the range of function is a set of real numbers, the function is said to be real valued.

$$x \xrightarrow[\text{(Domain)}]{\text{Input}} \boxed{f} \xrightarrow[\text{(Range)}]{\text{output}} f(x)$$

Example:-

(i) $y = x^2$

Domain is $(-\infty, \infty)$

Range is $[0, \infty)$

(ii) $y = \frac{1}{x}$

Domain is $(-\infty, 0) \cup (0, \infty)$

Range is $(-\infty, 0) \cup (0, \infty)$

(iii) $y = \sqrt{x}$

Domain is $[0, \infty)$

Range is $[0, \infty)$

(iv) $y = \sqrt{4-x}$

Domain is $(-\infty, 4]$

Range is $[0, \infty)$

(vi) $\frac{x^2-4}{x-2}$

$= x+2$

$D = (-\infty, \infty)$

$R = (-\infty, \infty)$

Sin	$\mathbb{R}$	$[-1, 1]$
$\cos\theta$	$\mathbb{R}$	$[-1, 1]$
$\tan\theta$	$\frac{\pi}{2} + n\pi$	$\mathbb{R}$
Cot	$n\pi$	$\mathbb{R}$
Sec	$\frac{\pi}{2} + n\pi$	$(-\infty, -1] \cup [1, \infty)$
cosec	$n\pi$	$(-\infty, -1] \cup [1, \infty)$

(v) $y = \sqrt{1-x^2}$

Domain is $[-1, 1]$

Range $[0, 1]$

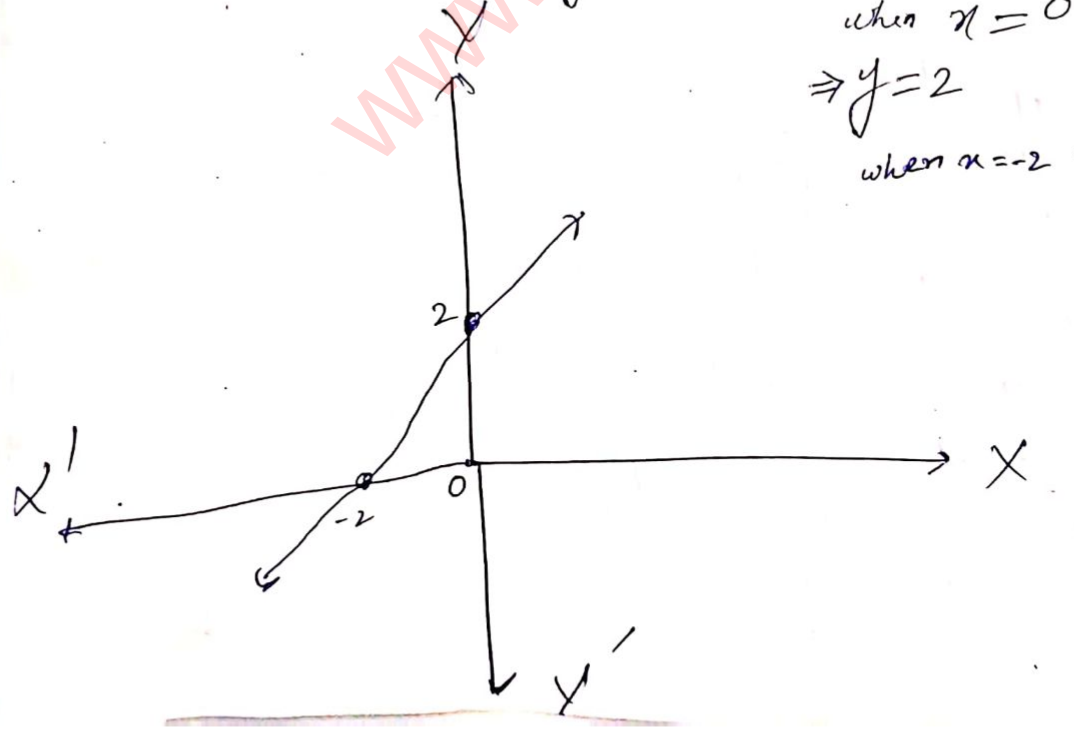
Graphs of Functions:-

If f is a function with domain D, its graph consists of the points in the cartesian plane whose coordinates are the input-output pairs for f. In set notation, the graph is

$$\{x, f(x) / x \in D\}$$

Example:- $y = f(x) = x + 2$.

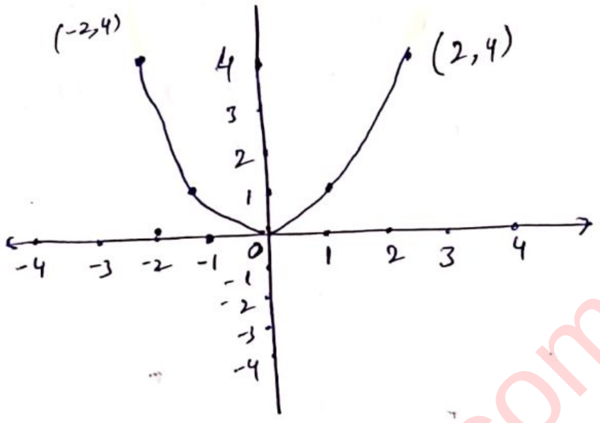
when $x = 0$
$\Rightarrow y = 2$
when $x = -2 \Rightarrow y = 0$



Example:-

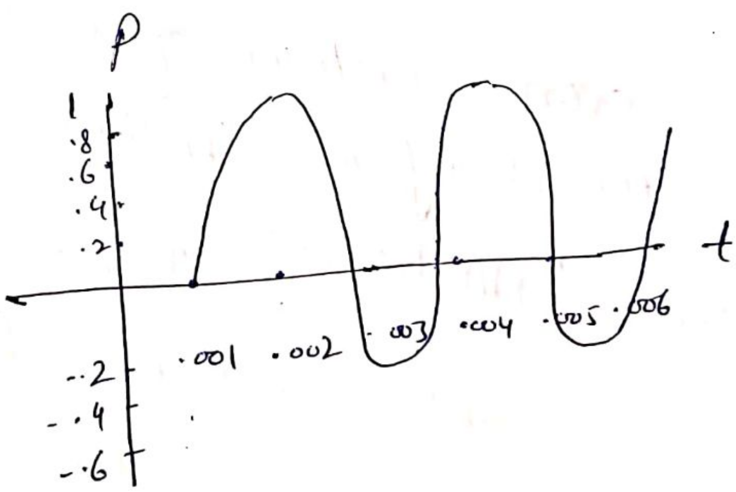
Graph of function $y=x^2$ over the Interval $[-2,2]$

x	$y=x^2$
-2	4
-1	1
0	0
1	1
2	4



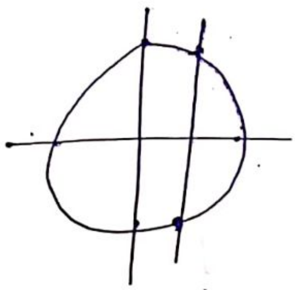
Representing a Function Numerically:

* used by engineers and Scientists.
* The graph consits only the points in the table is called scatterplot.



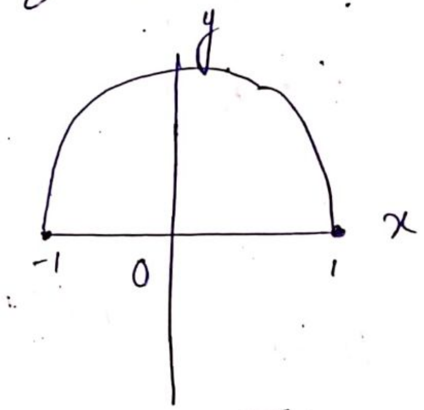
The Vertical line test:-

* A function f can have only one value $f(x)$ for each x in its domain, so no vertical line can intersect the graph of function more than once.



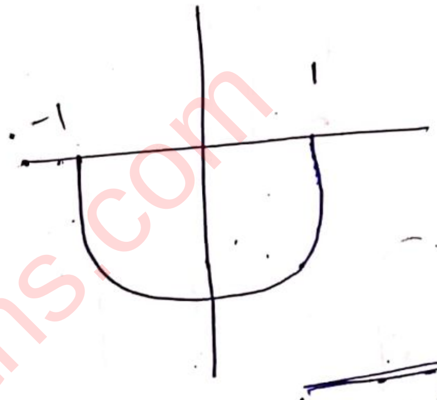
(a) $x^2+y^2=1$

The circle is not the graph of a function.



(b) $y=\sqrt{1-x^2}$

upper semicircle



(c) $y=-\sqrt{1-x^2}$

lower semicircle

Even Functions:-

A Function $y=f(x)$ is an even function of x if $f(-x)=f(x)$ for every x in the functions domain.

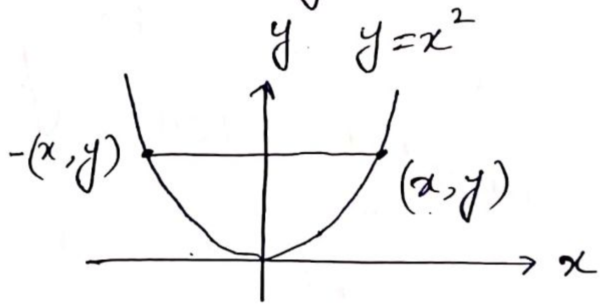
Odd Functions:-

A function $y=f(x)$ is an odd function if $f(-x)=-f(x)$ for every x in the functions domain.

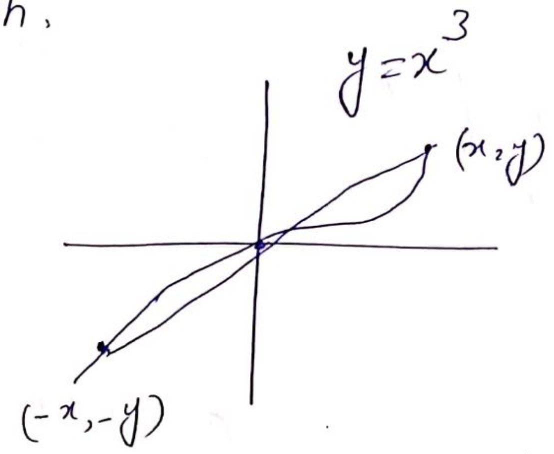
* The name even and odd come from power of x. If y is an even power of x as in $y=x^2$ or $y=x^4$, it is an even function of x because $(-x)^2 = x^2$ and $(-x)^4 = x^4$. If y is an odd power of x, as in $y=x$ or $y=x^3$, it is an odd function of x because $(-x)^1 = -x$ and $(-x)^3 = -x^3$

Symmetry:-

The graph of an even function is symmetric about the y-axis. Since $f(-x) = f(x)$, a point (x,y) lies on the graph if and only if the point $(-x,y)$ lies on the graph. A reflection across the y-axis leaves the graph unchanged



The graph of an odd function is symmetric about the origion. Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if point $(-x, -y)$ lies on the graph.



A graph is symmetric about the origion if rotation of 180° about the origion leaves graph unchanged.

Example:-

(i) $f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

So $f(x) = f(-x)$ even function, symmetry about y-axis.

(ii) $f(x) = x^2 + 1$

even function, Symmetry about y-axis.

(iii) $f(x) = x$

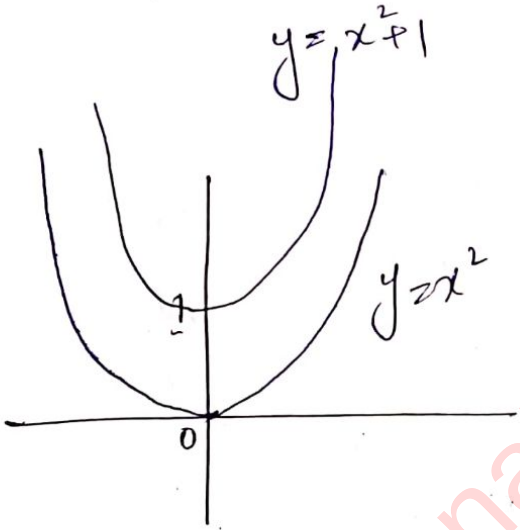
odd function and symmetry about origin.

(iv) $f(x) = x+1$

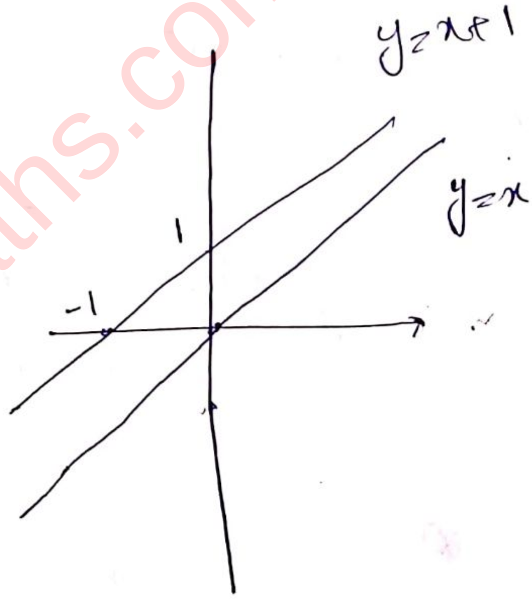
$f(-x) = -x+1$

$= -(x-1)$

Not even nor odd.



(a)

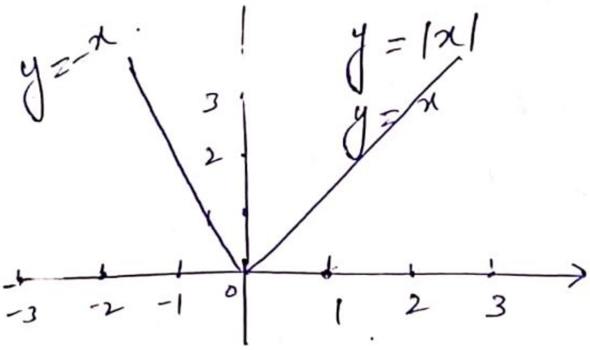


(b)

Piecewise-Defined Functions:-

Sometimes a function is described in pieces by using different formulas on different parts of its domain one example is the absolute value function

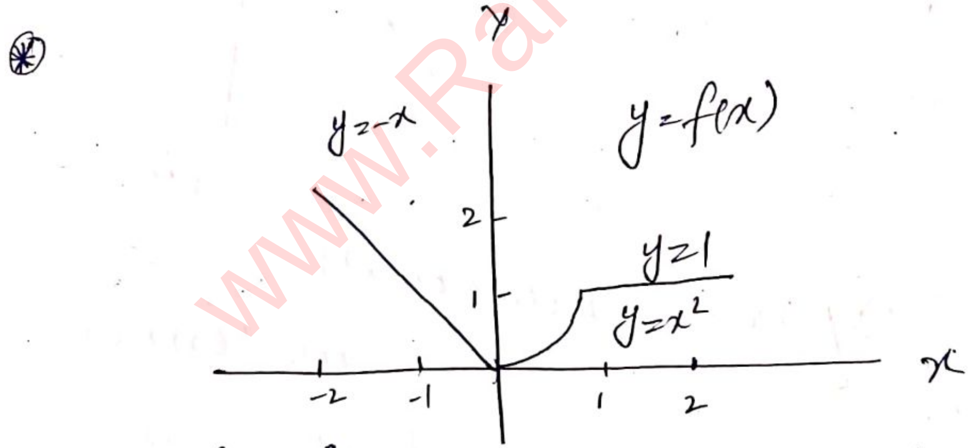
$$|x| = \begin{cases} x & x \geq 0 \quad \text{First Formula} \\ -x & x < 0 \quad \text{second formula} \end{cases}$$



Example:-

The function

$$f(x) = \begin{cases} -x & x < 0 \quad \text{1st} \\ x^2 & 0 \leq x \leq 1 \quad \text{2nd} \\ 1 & x > 1 \quad \text{3rd.} \end{cases}$$



* A function whose value at any number x is smallest integer greater than or equal to x is called least integer function or the integer ceiling

2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$ then f is said to be decreasing on I.

Everyday Life Uses of Calulus.

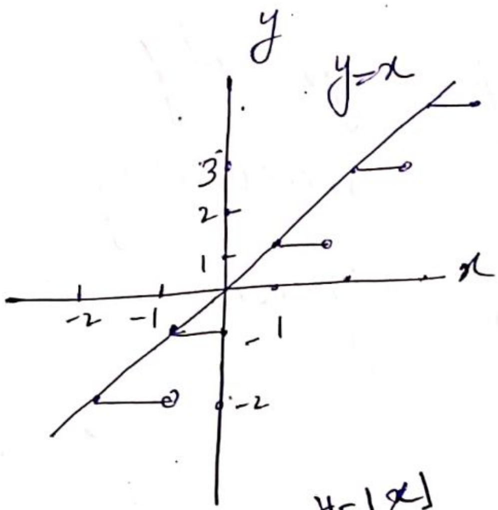
* In Physics, Integration in very much needed. For example to calculate the centre of mass, centre of gravity.
* Computer Graphics / Image processing here we also need linear Algebra and Analytic geometery.
* Scientific Computing :- Computer Algebra Systems that compute Integrals and derivatives directly, either symbolically or numerically are most Importemt examples here.

function. It is denoted by $\lceil x \rceil$

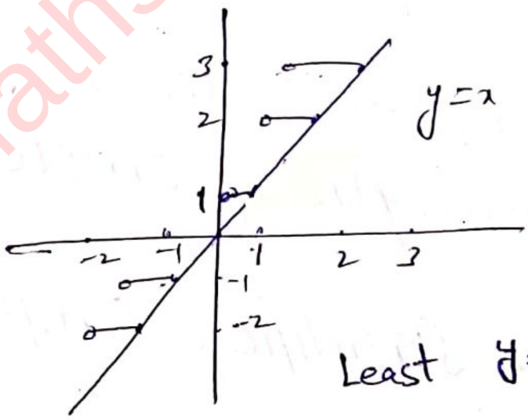
* The function whose value at any number x is greatest integer less than or equal to x is called greatest integer function or Integer floor function. It is denoted by $\lfloor x \rfloor$

$\lfloor 2.4 \rfloor = 2$.

$\lfloor 1.9 \rfloor = 1$



greatest $y = \lfloor x \rfloor$



Least $y = \lceil x \rceil$

Increasing and Decreasing

Let f be a function defined on an interval I and let x_1 and x_2 be two distinct points in I

1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be Increasing on I