2-Oct-2024.

MTH-121 Calculus - I.

Linear Equation:

→ $x+y=5$; $5x+6y=0$

$x+y=5$ —— (1)

$5x+6y=0$ —— (2)

$5x=-6y$

$\boxed{x = \frac{-6y}{5}}$ —— (3)

x value put in eq(1).

$\frac{-6y}{5}+y=5$

$\frac{-6y+5y}{5}=5$

$\frac{6y+5y}{5}=5$

Multiply 5 on b/s.

$-6y+5y=25$

$\boxed{-y=25}$

Put y value in eq(3)

$x=\frac{-6(-25)}{5}$

$\boxed{x=30}$

S.S $(30, -25)$

→ $5x_1+3x_2=5$; $6x_1-3x_2=10$

Adding b/s.

$5x_1 + \cancel{3x_2} = 5$

$6x_1 - \cancel{3x_2} = 10$

$11x_1 = 15$

$\boxed{x_1 = \frac{15}{11}}$ Put in eq(1)

$5\left(\frac{15}{11}\right)+3x_2=5$

$\frac{75}{11}+3x_2=5$

$\frac{75+33x_2}{11}=5$

$75+33x_2=55$

$33x_2=55-75$

$33x_2=-20$

$\boxed{x_2=\frac{-20}{33}}$

S.S $\left(\frac{15}{11}, \frac{-20}{33}\right)$

Quadratic Equations

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

→ $5x^2+2x+1=0.$

$a=5$, $b=2$, $c=1$

$= \frac{-2 \pm \sqrt{(2)^2-4(5)(1)}}{2(5)}$

$= \frac{-2 \pm \sqrt{4-20}}{10}$

$= \frac{-2 \pm \sqrt{-16}}{10}$

$= \frac{-2\pm 4i}{10}$

$= \frac{\cancel{2}(-1\pm 2i)}{\cancel{10}\,5}$

$\boxed{= \frac{-1\pm 2i}{5}}$

$\boxed{i^2=\sqrt{-1}}$

Factorization:

→ $6x^2+5x+1=0$

$$6x^2+5x+1=0$$
$$6x^2+3x+2x+1=0$$
$$3x(2x+1)+1(2x+1)=0$$
$$(3x+1)(2x+1)=0$$

$3x+1=0$ | $2x+1=0$

$3x=-1$ | $2x=-1$

$\boxed{x=\frac{-1}{3}}$ | $\boxed{x=\frac{-1}{2}}$

Quadratic Equations

→ $6x^2+5x+1=0$

$a=6$, $b=5$, $c=1$.

$$=\frac{-5\pm\sqrt{(5)^2-4(6)(1)}}{2(6)}$$
$$=\frac{-5\pm\sqrt{25-24}}{12}$$
$$=\frac{-5\pm\sqrt{1}}{12}$$
$$=\frac{-5\pm 1}{12}$$

$=\frac{-5+1}{12}$ | $=\frac{-5-1}{12}$

$=\frac{-4}{12}$ | $=\frac{-6}{12}$

$\boxed{=-1/3}$ | $\boxed{=-1/2}$

:Completing Square Method:

→ $6x^2+5x+1=0$.

$$x^2+\frac{5}{6}x+\frac{1}{6}=0$$
$$x^2+\frac{5}{6}x=\frac{-1}{6}$$
$$(x)^2+2(x)\left(\frac{5}{12}\right)+\left(\frac{5}{12}\right)^2=-\frac{1}{6}+\left(\frac{5}{12}\right)^2.$$
$$\left(x+\frac{5}{12}\right)^2=\frac{-1}{6}+\frac{25}{144}.$$
$$\left(x+\frac{5}{12}\right)^2=\frac{-24+25}{144}$$
$$x+\frac{5}{12}=\pm\sqrt{\frac{1}{144}}$$
$$x+\frac{5}{12}=\pm\frac{1}{12}.$$
$$x=\frac{-5}{12}\pm\frac{1}{12}$$

$x=\frac{-5}{12}-\frac{1}{12}$ | $x=\frac{-5}{12}+\frac{1}{12}$

$x=\frac{-6}{12}$ | $x=\frac{-4}{12}$

$\boxed{x=\frac{-1}{2}}$ | $\boxed{x=\frac{-1}{3}}$

4- Polynomial.

5- Division Method.

6- Circle Equation.

7- Synthetic Division.

8- Equalities.

$\rightarrow 2x^2 - 3x + 1 =$ ... ct-2024. Second Lec.

Multiplying by ...

$x^2 - \frac{3}{2}x + \ldots$

$x^2 - \frac{3}{2}x = \ldots$

$(x)^2 + 2(x)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 = -\frac{1}{2} + \left(\frac{3}{4}\right)^2.$

$\left(x + \frac{3}{4}\right)^2 = \frac{-1}{2} + \frac{9}{16}.$

$\left(x + \frac{3}{4}\right)^2 = \frac{1}{16}.$

$x + \frac{3}{4} = \pm \frac{1}{4}.$

$x + \frac{3}{4} = \frac{1}{4}$ | $x + \frac{3}{4} = -\frac{1}{4}$

$x = \frac{1}{4} - \frac{3}{4}$ | $x = -\frac{1}{4} - \frac{3}{4}$

$\boxed{x = -\frac{1}{2}}$ | $\boxed{x = -1.}$

$S.S = \left(\frac{-1}{2}, -1\right)$

-8 Polynomial 8-

$= ax^2 + bx + c$

$= ax^3 + bx^2 + cx + a'$ [Cubic polynomial.]

$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x^{n-n+1} + a_n$

OR

$p(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_0$

$\rightarrow P(x) = 5x^2 + 6x + 3.$

$Q(x) = 7x^2 + 4x + 2.$

$P(x) + Q(x) = 5x^2 + 6x + 3 + 7x^2 + 4x + 2.$

$\boxed{P(x) + Q(x) = 12x^2 + 10x + 5.}$

$P(x) - Q(x) = (5x^2 + 6x + 3) - (7x^2 + 4x + 2)$

$= 5x^2 + 6x + 3 - 7x^2 - 4x - 2$

$\boxed{P(x) - Q(x) = -2x^2 + 2x + 1}$

$P(x) \cdot Q(x) = (5x^2 + 6x + 3)(7x^2 + 4x + 2).$

$= 5x^2(7x^2 + 4x + 2) + 6x(7x^2 + 4x + 2) + 3(7x^2 + 4x + 2).$

$= 35x^4 + 20x^3 + 10x^2 + 42x^3 + 24x^2 + 12x + 21x^2 + 12x + 6.$

$= 35x^4 + 62x^3 + 34x^2 + 21x^2 + 24x + 6$

$\boxed{= 34x^4 + 62x^3 + 55x^2 + 24x + 6.}$

$\rightarrow 2x^2 - 3x + 1$ ... ct-2024. **Second Lec.**

Multiplying by (2)

$$x^2 - \frac{3}{2}x + \frac{1}{2} = 0$$

$$x^2 - \frac{3}{2}x = -\frac{1}{2}$$

$$(x)^2 + 2(x)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 = -\frac{1}{2} + \left(\frac{3}{4}\right)^2.$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{-1}{2} + \frac{9}{16}.$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{1}{16}.$$

$$x + \frac{3}{4} = \pm \frac{1}{4}.$$

$x + \frac{3}{4} = \frac{1}{4}$	$x + \frac{3}{4} = -\frac{1}{4}$
$x = \frac{1}{4} - \frac{3}{4}$	$x = -\frac{1}{4} - \frac{3}{4}$
$\boxed{x = -\frac{1}{2}}$	$\boxed{x = -1.}$

$$S.S = \left(-\frac{1}{2}, -1\right)$$

:- Polynomial :-

$$P(x) = ax^2 + bx + c$$

$$P(x) = ax^3 + bx^2 + cx + a' \quad \text{[Cubic polynomial.]}$$

$$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x^{n-n+1} + a_n$$

OR

$$p(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_0$$

$\rightarrow$ $P(x) = 5x^2 + 6x + 3.$

$Q(x) = 7x^2 + 4x + 2.$

$$P(x) + Q(x) = 5x^2 + 6x + 3 + 7x^2 + 4x + 2.$$

$$\boxed{P(x) + Q(x) = 12x^2 + 10x + 5.}$$

$$P(x) - Q(x) = (5x^2 + 6x + 3) - (7x^2 + 4x + 2)$$

$$= 5x^2 + 6x + 3 - 7x^2 - 4x - 2$$

$$\boxed{P(x) - Q(x) = -2x^2 + 2x + 1}$$

$$P(x)\,Q(x) = (5x^2 + 6x + 3)(7x^2 + 4x + 2).$$

$$= 5x^2(7x^2 + 4x + 2) + 6x(7x^2 + 4x + 2) + 3(7x^2 + 4x + 2).$$

$$= 35x^4 + 20x^3 + 10x^2 + 42x^3 + 24x^2 + 12x + 21x^2 + 12x + 6.$$

$$= 35x^4 + 62x^3 + 34x^2 + 21x^2 + 24x + 6$$

$$\boxed{= 34x^4 + 62x^3 + 55x^2 + 24x + 6.}$$

Division Method:

→ $P(x) = 2x^5 - 4x^4 + x^2 + 4x + 1.$ $\{(x-1)\}$

$$
\begin{array}{r|l}
 & 2x^4 - 2x^3 - 2x^2 - x \\
\hline
x-1 & 2x^5 - 4x^4 + x^2 + 4x + 1 \\
 & +2x^5 - 2x^4 \\
 & - \quad + \\
\hline
 & -2x^4 + 0x^2 \\
 & -2x^4 + 2x^3 \\
 & + \quad - \\
\hline
 & -2x^3 + x^2 \\
 & -2x^3 + 2x^2 \\
 & + \quad - \\
\hline
 & -x^2 + 4x \\
 & -x^2 + x \\
 & + \quad - \\
\hline
 & +3x + 1 \\
 & +3x - 3 \\
 & - \quad + \\
\hline
 & 4
\end{array}
$$

ROW:

$\frac{2x^{\cancel{5}\,4}}{\cancel{x}} = 2x^4$

$\frac{-2x^{\cancel{4}\,3}}{\cancel{x}} = -2x^3$

$\frac{-2x^3}{x} = -2x^2$

$\frac{-x^2}{x} = -x$

$\frac{3x}{x} = 3.$

→ $P(x) = 2x^5 - 4x^4 + x^2 + 4x - 3.$ ⇒ Root $x = 1$

Check the polynomial $x - 1 = 0$

~~P of 1~~ P(1) is equal to

$= 2(1)^5 - 4(1)^4 + (1)^2 + 4(1) - 3$

$= 2 - 4 + 1 + 4 - 3$

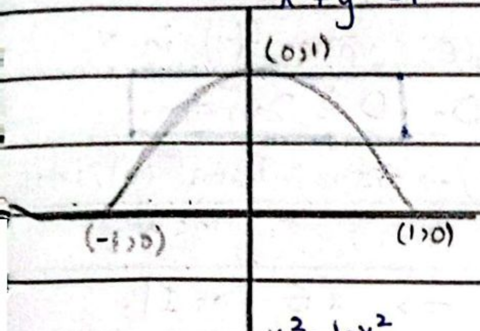
$= 3 - 3 \boxed{= 0}$

Graphic DesmosWebsite

:Circle Equations:

$x^2+y^2=r^2$

$x^2+y^2=1$



$y^2=1-x^2$

$y=\sqrt{1-x^2}$

$f(x)=\sqrt{1-x^2}$

$f(1)=0$; $f(-1)=0$.

: Synthytic Division:

→ $P(x) = 2x^5 - 4x^4 + 0x^3 + x^2 + 4x - 3$.

Root $x=1$; $x-1=0$

1	2	-4	0	1	4	-3
		2	-2	-2	-1	3
	2	-2	-2	-1	3	$\boxed{0}$

$Q(x) = 2x^4 - 2x^3 - 2x^2 - x + 3$

-3	2	-2	-2	-1	3
		-6	-24	-66	+201
	2	-8	-26	-67	$\boxed{204}$

$R(x) = 2x^3 - 8x^2 + 22x - 67$.

: Equalities:

$x > y$ → Greater than.

$y < x$ → Lessthan ⇒ $ay > ax$, $a<0$

$x \geq y$

$y \leq x$ ⇒ $ay \geq ax$, $a<0$

$|x| < -5$.

Solution Set not Possible.

→ $|x| < 5$.

$-5 < x < 5$

→ Q# $|5x+10| < 2$.

$-2 < 5x+10 < 2$

$-2-10 < 5x < 2-10$

$-12 < 5x < -8$.

→ $|x| = 0$ ⇒ $x = 0$

$|-x| = |x|$

$|xy| = |x||y|$.

Domain:
All possible inputs.
Range:
All possible outputs.
Avoid From
$\therefore$ ∞, i.
(−) sa krna or $\sqrt{-1}$ sa krna ha.

: Inequalities

$\therefore$ $\boxed{-3 \le \frac{3x-1}{2} \le 3}$

$\pm \frac{3x-4}{2} \ge 3$

$\frac{3x-4}{2} \ge 3$, $-\frac{3x-4}{2} \ge 3$.

(i) $f(x) = \frac{x^2+1}{x-1}$

(ii) $f(x) = \sqrt{x-1}$

(iii) $f(x) = \frac{1}{\sqrt{x-1}}$

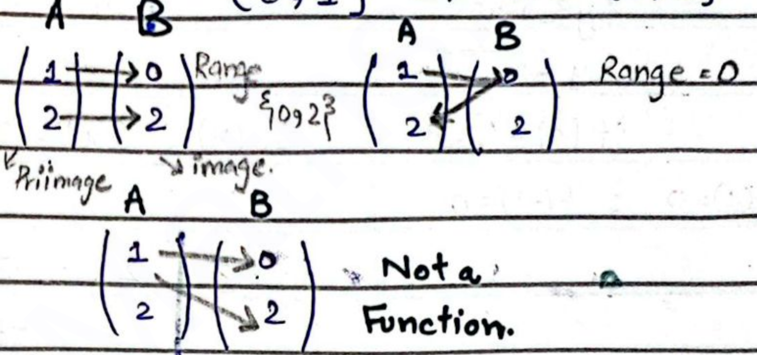
: Lec no 03: 9-Oct-2024.

: Function: Function is a rule that assign a unique value $f(x)$ in y each x in D. $\boxed{\text{D is Domain.}}$

$(-\infty, \infty) \to$ open interval (0) is not included

$[0, 1]$

$(0, 1] \Rightarrow x \in (0, 1]$.



: Onto Function:

Range = B is called on to function
{koi bi range repeat nai honi chahiay}

1-1 function
$f(x_1) \neq f(x_2)$

Bijective function
1-1 and onto function.

$*$ $f(x) = \frac{1}{x}$, $x \neq 0$.

Domain
All possible inputs
Range
All possible Outputs.

: Inequalit...

$$\left|\frac{3x-4}{2}\right| \geq 3.$$

$$\left|\frac{3x-4}{2}\right| \geq 3$$

$$\therefore \boxed{-3 \leq \frac{3x-4}{2} \leq 3}$$

$$\pm \frac{3x-4}{2} \geq 3$$

$$\frac{3x-4}{2} \geq 3, \; -\frac{3x-4}{2} \geq 3.$$

(i) $f(x) = \frac{x^2+1}{x-1}$

(ii) $f(x) = \sqrt{x-1}$

(iii) $f(x) = \frac{1}{\sqrt{x-1}}$

: Lec no 03: 9-Oct-2024.

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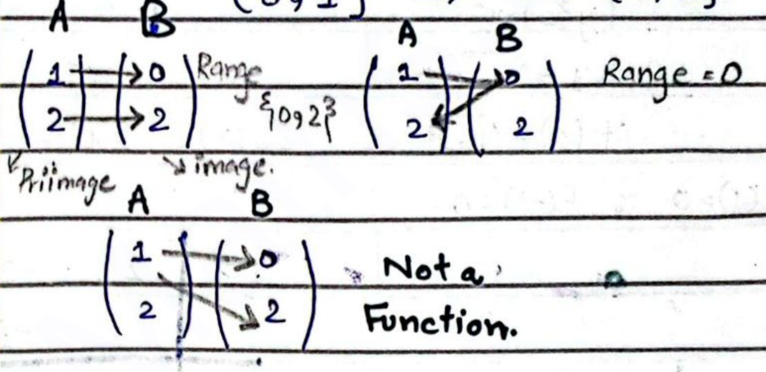


: Onto Function:

Range = B is called on to function

{koi bi range repeat nai honi chahiay}

1-1 function

$f(x_1) \neq f(x_2)$

Bijective function

1-1 and onto function.

* $f(x) = \frac{1}{x}$, $x \neq 0$.

Domain

→ All possible inputs

Range

All possible Outputs.

Lec no 04. Thursday 10/oct.

$x^2+x-1 \neq 0$

$x \neq 1, x \neq 2$

$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

$x \neq +1$

$(-\infty, 1) \cup (1, \infty)$

$\mathbb{R} - \{1\}$.

$x-1 \geq 0$

$x \geq 1$

1- $f(x) = x^2+1$.

Domain $= \mathbb{R} = (-\infty, \infty)$

$\Rightarrow x=0 \Rightarrow f(0) = (0)^2+1 \boxed{=0}$

$\Rightarrow x=1 \Rightarrow f(1) = (1)^2+(1) \boxed{=2}$

$\Rightarrow x=-1 \Rightarrow f(-1) = (-1)^2+1 = 1+1 \boxed{=2}$

Range $= [1, \infty)$.

3- $f(x) = 1-\sqrt{x}$

$x \geq 0$

Domain $= [0, \infty)$

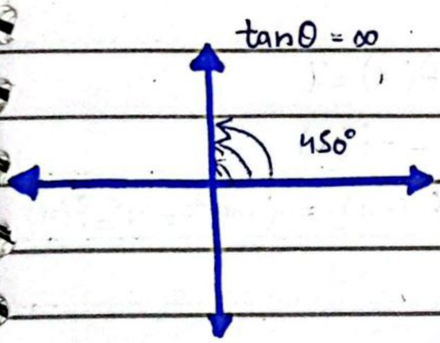
$\Rightarrow x=0 \Rightarrow f(0) = 1-\sqrt{0} \boxed{=1}$

$\Rightarrow x=1 \Rightarrow f(1) = 1-\sqrt{1} \boxed{=0}$

$\Rightarrow x=2 \Rightarrow f(2) = 1-\sqrt{2} \boxed{= -0.414.}$

Range $= (-\infty, 0]$

2- $f(x) = \tan x$.



$\tan\theta = \infty$

$x = 0 > 0 + 1$

$= 1$

$x=1 \Rightarrow 1+1$

$= 1$

$x = -1 = (-1)^2+1$

$x-1 = 2$

$(-2)^2+1 = 4+1 = 5$.

4- $f(x) = \sqrt{5x+10}$

$\sqrt{5x+10} \geq 0$

$5x+10 \geq 0$

$5x \geq -10$

$5x \geq -10$

$x \geq -10/5$

$\boxed{x \geq -2}$

Domain $= [-2, \infty)$.

Range $= [0, \infty)$

5- $f(x) = \sqrt{x^2-3x}$

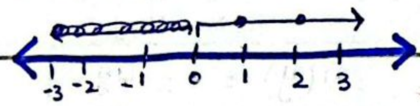
$\sqrt{x^2-3x} \geq 0$

$x^2-3x \geq 0$

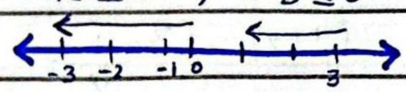
$\Rightarrow x(x-3) \geq 0$

$\Rightarrow x \geq 0 , x-3 \geq 0$

$x \geq 0 , x \geq 3$



$x \leq 0 , x-3 \leq 0$



$x \leq 0$

Domain $= (-\infty, 0] \cup [3, \infty)$.

Range $= [0, \infty)$

Range Domain ka ult ha.

$f(x) = y \rightarrow$ Dependent

$x \rightarrow$ Independent

- x ki Value independent hoti ha
- y ki Value dependent hoti ha

6- $f(x) = \frac{4}{3-x}$

$3 - x \neq 0$

$3 \neq x$

Domain $= \mathbb{R} - \{$

$= (-\infty,$

$x = 3.01 \Rightarrow$

$x = 2.99 \Rightarrow f(2.99) = 400$

Range $= R - \{0\}$

=> Inverse Functions:

$f(x) = y$

$\Rightarrow x = f^{-1}(y)$

* $f(x) = \frac{4}{3-x}$.

Let. $f(x) = y$

$x = f^{-1}(y)$

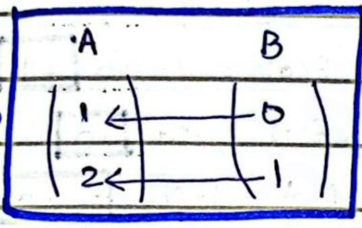
$\frac{4}{3-x} = y$

$\frac{4}{y} = (3-x)$

$x = 3 - \frac{4}{y}$

$f^{-1}(y) = -3 - \frac{4}{y}$

$\therefore f^{-1}(x) = 3 - \frac{4}{x}$



$f(x): A \longrightarrow B$

$f^{-1}(x)\ B \longrightarrow A$

Domain of $f(x)$ = Rang $f^{-1}(x)$

Range of $f(x)$ = Dom: $f^{-1}(x)$

→ Types of Functions:

* Odd function:

$f(-x) = -f(x)$

* Even function:

$f(-x) = f(x)$

x ko replace krty ha $-x$ sa to odd function ha.

or $-x$ ko $-x$ sa replace kry to even funtion ha.

$\Rightarrow f(x) = x$

$\Rightarrow f(-x) = -x = -f(x)$

$f(x) = x^2 \Rightarrow f(-x) = (-x)^2 = x^2 = f(x)$

* $f(x) = \frac{4}{3-x}$.

$f(-x) = \frac{4}{3-(-x)} = \frac{4}{3+x}$

$= -\left(\frac{4}{-(3+x)}\right)$ $\boxed{= \frac{-4}{-3-x}}$

* $f(x) = \sin x$.	$f(x) = \cos x$
$f(-x) = \sin(-x)$	$f(-x) = \cos(-x) =$
$= -\sin x$	$\cos(x) = f(x)$
$= -f(x)$	Even.
$f(x) = \cos x$	

Range Domain ka ulta ha

6- $f(x) = \frac{4}{3-x}$

$3-x \neq 0$

$3 \neq x$

Domain $= \mathbb{R} - \{3\}$

$= (-\infty, 3) \cup (3, \infty)$

$x = 3.01 \Rightarrow f(3.01) = -400$

$x = 2.99 \Rightarrow f(2.99) = 400$

Range $= \mathbb{R} - \{0\}$

=> Inverse Function:

$f(x) = y$

$\Rightarrow x = f^{-1}(y)$

* $f(x) = \frac{4}{3-x}$.

Let $f(x) = y$

$x = f^{-1}(y)$

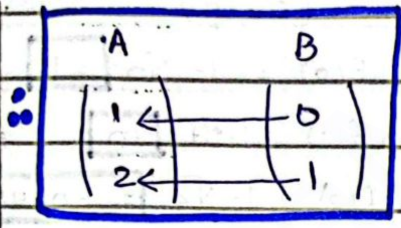
$\frac{4}{3-x} = y$

$\frac{4}{y} = (3-x)$

$x = 3 - \frac{4}{y}$

$f^{-1}(y) = -3 - \frac{4}{y}$

$f^{-1}(x) = 3 - \frac{4}{x}$



$f(x): A \longrightarrow B$

$f^{-1}(x)\ \ B \longrightarrow A$

Domain of $f(x)$ = Rang $f^{-1}(x)$

Range of $f(x)$ = Dom: $f^{-1}(x)$

=> Types of Functions:

*: **Odd function:**

$f(-x) = -f(x)$

*: **Even function:**

$f(-x) = f(x)$

x ko replace krty ha -x sa to odd function ha.
or -x ko -x sa replace kry to even funtion ha.

~~=> f(x)~~ $\Rightarrow f(x) = x$

$\Rightarrow f(-x) = -x = -f(x)$

$f(x) = x^2 \Rightarrow f(-x) = (-x)^2 = x^2 = f(x)$

* $f(x) = \frac{4}{3-x}$.

$f(-x) = \frac{4}{3-(-x)} = \frac{4}{3+x}$

$= -\left(\frac{4}{-(3+x)}\right) \boxed{= \frac{-4}{-3-x}}$

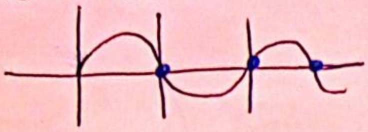
* $f(x) = \sin x$.	$f(x) = \cos x$
$f(-x) = \sin(-x)$	$f(-x) = \cos(-x) =$
$= -\sin x$	$\cos(x) = f(x)$
$= -f(x)$	→ Even.
$f(x) = \cos x$	

Polynomial:

$P(x) = x^5 + 20$

$= 0x^6 + x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 20$

Function: Agr ak Line ak sa 2da points Ko touch krti hato Ya fonction nai ha.



* $f(x) = g(x)\,h(x).$

$((-1(-1)) = 1$

odd · odd = eve

odd · Even = odc

Even · Even = Ev

Polynomial $= a_0 + a_1x + a_2x + \cdots + a_nx^n$

$P(x) = x^2 + 2x + 3$

$\Rightarrow$ Degree = 2.

Rational Function:

$$F(x) = \frac{Q(x)}{R(x)}$$

$$\Rightarrow F(x) = \frac{x^2+1}{x^2-1}$$

* Peace Function:

$$F(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

$$F(x) = \begin{cases} 2x+7, & x < -2 \\ 3x+4, & x = -2 \\ x^2+5, & x \geq -2 \end{cases}$$

Absolute Function:

$f(x) = |x|$

$f(x) = |x+1|$.

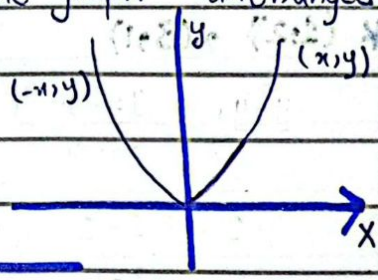
05:

iff

if and only if.

*** Symmetry:** The graph of an even Function $\{ F(-x) = f(x) \longrightarrow$ Even Function.$\}$ is Symmetric about the y-axies. Since $F(-x) = f(x)$, a point (x, y) lies on the graph if iff the point $(-x, y)$ lies on the graph. A reflection across y-axies Leaves the graph unchanged

$F(x) = x^2$.



Composite Function:

$\rightarrow F(x) = x^2+1, \quad g(x) = 2x+$

$f \circ g(x) = f(g(x)) = (2x+1)^2 + 1.$

$= (2x)^2 + (1)^2 + 2(2x)(1) + 1 = 4x^2 + 1 + 4x + 1$

$\boxed{= 4x^2 + 4x + 2}$

$\rightarrow g \circ f(x) = g(f(x))$

$= 2(x^2+1) + 1 = 2x^2 + 2 + 1 \boxed{= 2x^2 + 3.}$

$\rightarrow g \circ g = g(g(x)) = 2(2x+1) + 1$

$= 4x + 2 + 1 \boxed{= 4x+3.}$

* $f(x) = g(x)\,h(x)$.

$(-1(-1)) = 1$

odd · odd = even

odd · Even = odd

Even · Even = Even.

Polynomial $= a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

$P(x) = x^2 + 2x + 3$

$\Rightarrow$ Degree = 2.

:Rational Function:

$$F(x) = \frac{Q(x)}{R(x)}$$

$$\Rightarrow F(x) = \frac{x^2+1}{x^2-1}$$

*:Peace Function:

$$F(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

$$F(x) = \begin{cases} 2x+7, & x < -2 \\ 3x+4, & x = -2 \\ x^2+5, & x \geq -2 \end{cases}$$

: Absolute Function:

$$f(x) = |x|$$

$$f(x) = |x+1|:$$

:Lec no 05:

iff

if and only if.

*:Symmetry: The graph of an even Function $\{F(-x) = f(x) \longrightarrow$ Even Function.$\}$ Is Symmetric about the y-axies. Since $F(-x) = f(x)$, a point (x, y) lies on the graph if iff the point $(-x, y)$ lies on the graph. A reflection across y-axies Leaves the graph unchanged

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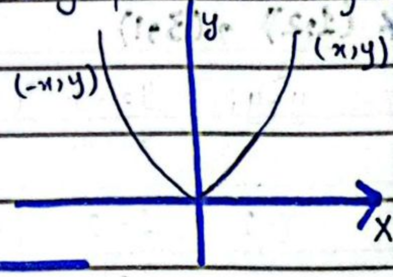


:Composite Function:

$\rightarrow F(x) = x^2+1$, $g(x) = 2x+$

$fog(x) = f(g(x)). = (2x+1)^2 + 1.$

$= (2x)^2 + (1)^2 + 2(2x)(1) + 1 = 4x^2 + 1 + 4x + 1$

$\boxed{= 4x^2 + 4x + 2}$

$\rightarrow gof(x) = g(f(x))$

$= 2(x^2+1) + 1 = 2x^2 + 2 + 1 \boxed{= 2x^2 + 3.}$

$\rightarrow gog = g(g(x)) = 2(2x+1) + 1$

$= 4x + 2 + 1 \boxed{= 4x + 3.}$

:Intercept: Intecept mai m, x hota ha.

$[m, x]$

agr m and x zero ho jay or hmary pas y ki value ajay to intercept ha

$(x_1, y_1), (x_2, y_2)$

$\rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$

$\boxed{y - y_1 = m(x - x_1).}$

Slope and intercept Form

- $y = mx + c.$
- $x = my + b.$

***:Increasing Function:** $F(x) = x^2$, If $f(a) > f(b)$

For $a > b$ => increasing Fuction.

***:Decreasing Function:** If $f(a) < f(b)$

For $a > b$ => decreasing.

* $(1,2), (3,1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}$ (x

$y - 2 = \frac{1-2}{3-1}$ (x

$y - 2 = \frac{-3}{2}(x - 1)$

$2y - 4 = -3x + 3$

$2y + 3x = -4 + 3$

$2y + 3x - 7 = 0$

$2y = -3x + 7$

$\boxed{y = \frac{-3}{2}x + \frac{7}{2}}$

* $(5,7), (2,-1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 7 = \frac{-1-7}{2-5}(x - 5)$

$y - 7 = \frac{-8}{-3}(x - 5)$

$-3(y - 7) = -8(x - 5)$

$-3y + 21 = -8x + 40$

$-3y + 8x = -21 + 40$

$-3y + 8x = 19$

$-3y = -8x + 19$

$y = -\left[\frac{-8}{3}x + \frac{19}{3}\right]$

$\boxed{y = \frac{8x}{3} - \frac{19}{3}}$

: $m = \frac{-3}{4}$ Point: $(3, -9)$

$y_1 = \frac{-3}{4}(x - x_1)$

$\frac{-3}{4}(x - 3)$

$= -3(x - 3).$

$4y + 36 = -3x + 9$, $4y + 3x = -36 + 9$

$4y + 3x = -27$

$4y = -3x - 27$

$\boxed{y = \frac{-3}{4}x - \frac{27}{4}}$

$fof(x) = f(f(x)) = (x^2+1)^2+1$

$= x^4+2x^2+1+1$

$\boxed{= x^4+2x^2+2.}$

Equation of Straight line:

$(x_1, y_1), (x_2, y_2)$

$\rightarrow y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

Slope $= m = \frac{y_2-y_1}{x_2-x_1}$

$\boxed{y-y_1 = m(x-x_1).}$

Slope and intercept Form

- $y = mx+c.$
- $x = my+b.$

$\ast\ (1,2), (3,1)$ (x_1, y_1, x_2, y_2)

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}$ (x

$y-2 = \frac{1-2}{3-1}$ (x

$y-2 = \frac{-3}{2}(x-1)$

$2y-4 = -3x+3$

$2y+3x = -4+3$

$2y+3x-7 = 0$

$2y = -3x+7$

$\boxed{y = \frac{-3}{2}x+\frac{7}{2}}$

Increasing Functions:

$F(x)=x^2$, If $f(a)>f(b)$

For $a>b \Rightarrow$ increasing Fuction.

Decreasing Functions:

If $f(a)<f(b)$

For $a>b \Rightarrow$ decreasing.

$\ast\ (5,7), (2,-1).$ (x_1, y_1, x_2, y_2)

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-7 = \frac{-1-7}{2-5}(x-5)$

$y-7 = \frac{-8}{-3}(x-5)$

$-3(y-7) = -8(x-5)$

$-3y+21 = -8x+40$

$-3y+8x = -21+40$

$-3y+8x = 19$

$-3y = -8x+19$

$y = -\left[\frac{-8}{3}x+\frac{19}{3}\right]$

$\boxed{y = \frac{8x}{3}-\frac{19}{3}}$

$m = \frac{-3}{4}$ Point: $(3,-9)$ (x_1, y_1)

$y-y_1 = \frac{-3}{4}(x-x_1)$

$= \frac{-3}{4}(x-3)$

$4(y+9) = -3(x-3).$

$4y+36 = -3x+9$, $4y+3x = -36+9$

$4y+3x = -27$

$4y = -3x-27$

$\boxed{y = \frac{-3}{4}x-\frac{27}{4}}$

To Check Lines are $\perp$ or not $\Rightarrow m_1 \cdot m_2 = -1$

$\Rightarrow m_1 = \frac{-1}{m_2}$.

Agr ans (-) mai ajay to Prependicular ha.

$m_1 = k m_2$.

$\frac{d}{dx}(x^3) = 3x^{3-1} \boxed{= 3x^2}$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

$(a^2 - b^2) = (a-b)(a+b)$.

$\rightarrow m = \frac{-1}{5}$,

$y = mx + c$

$y = \frac{-1}{5}($

$5y = -x +$

L•H•

R•H•

$x \to a^+$

System of

$5x_1 + 3x_2 = 5$

$x_1 + x_2 = 10$.

$\Rightarrow AX = B$, $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$X = A^{-1}B$.

Unit 2:

Limit and Continuity:

Let "A" ~~and B~~ be any real number and let f be a function from R to R, which is defined for all values of "x" near "a" with possible exception of point $x = a$.

The Function f is said to have Limit "ℓ" as "x" approaches to a if for every $\varepsilon > 0$, there exiests a possitive real number δ (which usually depends on ε) such that

$|F(x) - \ell| < \varepsilon$, whenever $0 < |x - a| < \delta$

In this case, we can write.

$\lim_{x \to a} f(x) = \ell$

and say that the function has limit ℓ (or f(x) coverages to ℓ as x approaches a or f(x) → l as x→ a.

First method

$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$

$\frac{(3)^3 - 27}{(3)^2 - 9}$

$\frac{27 - 27}{9 - 9} \boxed{= \frac{0}{0}}$

(0/0 Form) undeterminant form

Applying Second **L• Hôpital Rule•**

$\lim_{x \to 3} \frac{d/dx\, x^3 - d/d\,27}{d/dx\, x^2 - d/dx\, 9}$

$\lim_{x \to 3} \frac{3x^2 - 0}{2x - 0}$.

$\lim_{x \to 3} \frac{3x^{\cancel{2}}}{2\cancel{x}}$

$\lim_{x \to 3} \frac{3x}{2} = \frac{3(3)}{2}$

$\boxed{= \frac{9}{2}}$

3rd:

$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$

$\lim_{x \to 4} x^2 + 1$.

$(4)^2 + 1 = 16 + 1 = 17$

$\boxed{= 17}$

$\lim_{x \to 3} \frac{(x)^3 - (3)^3}{(x)^2 - (3)^2}$.

$\lim_{x \to 3} \frac{\cancel{(x-3)}((x)^2 + (x)(3) + (3)^2)}{\cancel{(x-3)}(x+3)}$

$\lim_{x \to 3} \frac{(x^2 + 3x + 9)}{(x+3)}$.

$= \frac{((3)^2 + 3(3) + 9)}{(3+3)}$

$= \frac{\cancel{9+9}+9}{6}$

$= 27/6 \boxed{= 9/2}$

$y = mx + c$.

$y = \frac{-1}{5}(x) + 7$.

$5y = -x + 35$, $\boxed{x = -5y + 35}$.

System of Linear Equations:

$5x_1 + 3x_2 = 5$

$x_1 + x_2 = 10$.

$\Rightarrow AX = B$, $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$X = A^{-1}B$.

Unit 2:

Limit and Continuity:

Let "A" ~~and B~~ be any real number and let f be a function from R to R, which is defined for all values of "x" near "a" with possible exception of point $x = a$.

The Function f is said to have Limit "ℓ" as "x" approaches to a if for every $\varepsilon > 0$, there exiests a possitive real number δ (which usually depends on ε) such that

$|F(x) - \ell| < \varepsilon$, whenever $0 < |x - a| < \delta$

In this case, we can write

$\lim_{x \to a} f(x) = \ell$

and say that the function has limit ℓ (or f(x) coverages to ℓ ...

L.H.L = Left hand Limits.

$\lim_{x \to a^-} f(x) = \ell_1$

R.H.L = Right hand Limits.

$\lim_{x \to a^+} f(x) = \ell_2$.

Find

$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$

First Method

$\frac{(3)^3 - 27}{(3)^2 - 9}$

$\frac{27 - 27}{9 - 9} = \boxed{\frac{0}{0}}$

(0/0 Form) undeterminant form

Applying Second L. Hôpital Rule.

$\lim_{x \to 3} \frac{d/dx\, x^3 - d/d\,27}{d/dx\, x^2 - d/dx\, 9}$

$\lim_{x \to 3} \frac{3x^2 - 0}{2x - 0}$

$\lim_{x \to 3} \frac{3x^{\cancel{2}}}{2\cancel{x}}$

$\lim_{x \to 3} \frac{3x}{2} = \frac{3(3)}{2}$

$\boxed{= \frac{9}{2}}$

3rd:

$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$

$\lim_{x \to 3} (x^2 + 1)$.

$(4)^2 + 1 = 16 + 1 = 17$

$\boxed{= 17}$

$\lim_{x \to 3} \frac{(x)^3 - (3)^3}{(x)^2 - (3)^2}$.

$\lim_{x \to 3} \frac{\cancel{(x-3)}\,((x)^2 + (x)(3) + (3)^2)}{\cancel{(x-3)}\,(x+3)}$

$\lim_{x \to 3} \frac{(x^2 + 3x + 9)}{(x+3)}$.

$= \frac{((3)^2 + 3(3) + 9)}{(3+3)}$

$= \frac{9 + 9 + 9}{6}$

$= 27/6$ $\boxed{= 9/2}$

$$\to f(x) = \begin{cases} 5x, & \text{if } x<2 \\ 6x-2 & \text{if } x=2 \\ x^2+6 & \text{if } x>2 \end{cases}$$

L.H.L:

$$\lim_{x\to 2^-} f(x)$$

$$\lim_{x\to 2^-} 5(x)$$

$$5(2) \; \boxed{=10}$$

R.H.L: $\lim_{x\to 2^+} f(x)$

$$\lim_{x\to 2^+} (6x-2)$$

$$= 6(2)-2 \; \boxed{=10.}$$

~~L.H.L Lim~~

$$\lim_{x\to 2^+} x^2+6$$

$$(2)^2+6$$

$$4+6 \; \boxed{=10}$$

$\to$ Find L.H.L and R.H.L.

$$\lim_{x\to 3^0} \frac{x^3-27}{x^2-9}$$

L.H.L.

$$\lim_{x\to 3^-} \frac{x^3-27}{x^2-9}$$

$$\lim_{x\to 3-0} \frac{(x-h)^3-27}{(x-h^2)-9}$$

$$\lim_{h\to 0} \frac{(3-h)^3-27}{(3-h)^2-9}$$

$$= \frac{d/dx(x^3) - d/dx(27)}{d/dx(x^2) - d/dx(9)}$$

$$= \frac{3x^2}{2x} = \frac{3x}{2} = \frac{9}{2}$$

$h) + (3)^2$

$h \to 0$ $\quad 6-h$

$$\frac{(3-0)^2+3(3-0)+9}{6-0}$$

$$= \frac{9+9+9}{6} = \frac{27}{6} \; \boxed{=\frac{9}{2}}$$

:Lec#07: 24-Oct-2024

Theorems on Limits

Limit of Constant Function:

If $f(x)=c$ $f(x)$ all $x \in R$, where c is a fixed real number. then $\lim_{x\to a} f(x) = c$

Identity Function:

$$\lim_{x\to a} f(x) = a.$$

Sum (Difference):-

$\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) =$

$$\lim_{x\to a} (f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = l \pm m$$

$\rightarrow f(x) = \begin{cases} 5x, & \text{if } x<2 \\ 6x-2 & \text{if } x=2 \\ x^2+6 & \text{if } x>2 \end{cases}$

L.H.L:

$\lim_{x\to 2^-} f(x)$

$\lim_{x\to 2^-} 5(x)$

$5(2)$ $\boxed{=10}$

R.H.L: $\lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^+} (6x-2)$

$= 6(2)-2$ $\boxed{=10.}$

~~L.H.L Lim~~

$\lim_{x\to 2^+} x^2+6$

$(2)^2+6$

$4+6$ $\boxed{=10}$

→Find L.H.L and R.H.L.

$\lim_{x\to 3^0} \frac{x^3-27}{x^2-9}$

L.H.L.

$\lim_{x\to 3^-} \frac{x^3-27}{x^2-9}$

$\lim_{x\to 3-0} \frac{(x-h)^3-27}{(x-h^2)-9}$

$\lim_{h\to 0} \frac{(3-h)^3-27}{(3-h)^2-9}$

$\lim_{h\to 0} \frac{(3-h)^3-(3)^3}{(3-h)^2-(3)^2}$

$\lim_{h\to 0} \frac{(3h-3)\,(3-h)^2+3(3-h)+(3)^2}{(3h-3)\,(3h+3)}$

$\lim_{h\to 0} \frac{(3-h)^2+3(3+h)+9}{6-h}$

$\frac{(3-0)^2+3(3-0)+9}{6-0}$

$= \frac{9+9+9}{6} = \frac{27}{6}$ $\boxed{=\frac{9}{2}}$

:Lec#07: 24-Oct-2024

:Theorems on Limits:

:Limit of Constant Function:

If $f(x)=c$ f(x) all $x \in R$, where c is a fixed real number. then $\lim_{x\to a} f(x)=c$

:Identity Function:

$\lim_{x\to a} f(x) = a.$

Sum (Difference):-

$\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$

$\lim_{x\to a} (f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm$

$\lim_{x\to a} g(x) = l \pm m$

Product:

$$\lim_{x\to a} (f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$
$$= lm$$

Quotient:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{l}{m}.$$

Inequalities:

If $f(x) \le g(x)$

$$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$$

$$l \le m.$$

Sandwich Theorm: Suppose that f, g and h are functions defined on $0<|x-a|<K$. If $f(x)\le g(x)\le h(x)$

If $\lim_{x\to a} f(x) = l = \lim_{x\to a} h(x)$.

$$\Rightarrow l \le \lim_{x\to a} g(x) \le l.$$

$$\Rightarrow \lim_{x\to a} g(x) = l$$

→: Formulas:

1- $\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1.$

2- $\displaystyle\lim_{\theta\to 0} \frac{a^x - 1}{x} = \ln a$

3- $\displaystyle\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = 0$

4- $\displaystyle\lim_{x\to a} \frac{x^n - a^n}{x-a} = n a^{n-1}$

Q1: $\displaystyle\lim_{x\to 1} \frac{x^3-1}{x-1}.$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$$

$$\lim_{x\to 1} x^2+x+1$$

$$= (1)^2+1+1$$

$$\boxed{=3}.$$

Q2: $\displaystyle\lim_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}.$

$$\lim_{x\to 0} \frac{1/\sin x - \cos x/\sin x}{x}$$

$$\lim_{x\to 0} \frac{\cancel{\sin}1-\cos x}{\sin x} \Big/ x$$

$$\lim_{x\to 0} \frac{1-\cos x}{x\sin x}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x\sin x} \times \frac{1+\cos x}{1+\cos x}.$$

$$\lim_{x\to 0} \frac{(1)^2-(\cos x)^2}{x\sin x(1+\cos x)}$$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x\sin(1+\cos x)}$$

$$\lim_{x\to 0} \frac{\sin^{\cancel{2}} x}{x\cancel{\sin x}(1+\cos x)} ; \lim_{x\to 0} \frac{\sin x}{x(1+\cos x)}$$

$$\lim_{x\to 0} \frac{\sin x}{x}^{=1} \cdot \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$\frac{1}{1+\cos(0)} = \frac{1}{1+1} \boxed{= \frac{1}{2}}.$$

$\pi = 180^\circ$ $\left(\frac{0}{0} \text{ form}\right).$

Q $\lim_{x \to \pi} \frac{\tan(\sin x)}{\sin x}$

Lets $\sin x = \theta$

when $x \to \pi$ then $\theta \to 0$

$= \lim_{\theta \to 0} \frac{\tan\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\theta}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\cancel{\theta}} \to 1 \cdot \lim_{\theta \to 0} \frac{1}{\cos\theta}$

$= 1 \cdot \frac{1}{\cos(0)} = \frac{1}{1}$

$\boxed{= 1.}$

Q: $\lim_{x \to -2} \frac{x^2 + 2x - 8}{x^2 - 4}.$

~~Lets~~ $\lim_{x \to -2-0} \frac{(x-h)^2 + 2(x-h) - 8}{(x-h)^2 - 4}$

$\lim_{h \to 0} \frac{(-2-h)^2 + 2(-2-h) - 8}{(-2-h)^2 - 4}$

$\frac{(-2-0)^2 + 2(-2-0) - 8}{(-2-0)^2 - 4}$

$\frac{(\cancel{4-4}) - 8}{4 - 4} = \frac{-8}{0}$

$\boxed{= \infty}$

Appling L'Hopital Rule.

:Limit of Polynomials:

Limit of polynomials can be found by substitution. If $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots a_0.$

then

$\lim_{x \to c} P(x) = P(c) = a_n c^n + a_{n-1} c^{n-1} + \cdots + a_0$

:Lec no 8:

30-Oct-2024.

:Continuty: A function f is said to be continous at point "a" if.

(i) f(a) is defined.

(ii) Limit exiest.

(iii) L.H.L = R.H.L = f(a).

Q: Discuss the Continuty of f defind by

$$f(x) = \begin{cases} \frac{x-4}{\sqrt{x}-2} & \text{if } x \geq 0 \text{ and } x \neq 4 \\ 4 & \text{if } x = 4 \end{cases}$$

at $x = 4.$

(i) $f(4) = 4$

(ii) $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{x-4}{\sqrt{x} - 2}.$

$\left\{ \left(\frac{0}{0} \text{ form}\right) \right\}$

$\because a^2 - b^2 = (a+b)(a-b)$.

LoHoL

$= \lim_{x\to 4^-} \frac{(\sqrt{x})^2 - (2)^2}{\sqrt{x} - 2}$

$= \lim_{x\to 4^-} \frac{(\cancel{\sqrt{x}-2})(\sqrt{x}+2)}{(\cancel{\sqrt{x}-2})}$

$\lim_{x\to 4^-} \sqrt{x} + 2$

$= \sqrt{4} + 2$

$= 2+2 \boxed{=4}$

RoHoL

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}$

$= \frac{x-4}{x-2} \boxed{=4.}$

$\Rightarrow f(4) =$ LoHoL = RoHoR

So, f(x) is Continuos.

Q: $f(x) = \begin{cases} 3 & \text{if } x \le -2 \\ -1/2\, x^2 & \text{if } -2 < x < 2 \\ 3 & \text{if } 2 \le x. \end{cases}$

at $x = -2$.

LoHoL: $\lim_{x\to -2^-} f(x) = \lim_{x\to 2^-} (3) \boxed{=3}$

RoHoL: $\lim_{x\to -2^+} (-1/2x^2) = -1/2(-2)^2 = \frac{1}{2}(4) \boxed{=-2}$

at $x=2$

LoHoL $= \lim_{x\to 2^-} f(x) = \lim_{x\to 2} (3) \boxed{=3}$

RoHoL $= \lim_{x\to 2^+} f(x) = \frac{-1}{2}x^2 = \frac{-1}{2}(2)^2 \boxed{=-2}$

Limits are not Continuous

Q: $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x}+1} & , x \ne 0 \\ 0 & , x = 0 \end{cases}$

at $x=0$ (i) $f(0) = 0$

(ii) LoHoL: $\lim_{x\to 0^-} \frac{e^{1/x} - 1}{e^{1/x}+1}$

$= \frac{e^{-1/0} - 1}{e^{1/0}+1} = \frac{e^{-\infty} - 1}{e^{-\infty}+1}$

$= \frac{1/e^{\infty} - 1}{1/e^{\infty} + 1} = \frac{1/\infty - 1}{1/\infty + 1} = \frac{0-1}{0+1} \boxed{=-1}$

RoHoL: $\lim_{x\to 0^+} \frac{e^{1/x} - 1}{e^{1/x}+1}$

$\frac{e^{1/0} - 1}{e^{1/0}+1} = \frac{e^{\infty} - 1}{e^{\infty}+1}$

$= \frac{\infty - 1}{\infty + 1} = \frac{\infty}{\infty}$

$\lim_{x\to 0^+} \frac{1 - 1/e^{1/x}}{1 + 1/e^{1/x}}$

$= \frac{1 - 1/e^{\infty}}{1 + 1/e^{\infty}}$

$= \frac{1 - 1/\infty}{1 + 1/\infty} = \frac{1-0}{1+0} \boxed{=1.}$

Limit are not exiest.

Q $\lim_{x\to\infty} \frac{4x^2 + 5x + 7}{3x^2 + 2x + 4}$

$= \lim_{x\to\infty} \frac{\frac{4x^2}{x^2} + \frac{5x}{x^2} + \frac{7}{x^2}}{\frac{3x^2}{x^2} + \frac{2x}{x^2} + \frac{4}{x^2}}$

$= \lim_{x\to\infty} \frac{4 + 5/x + 7/x^2}{3 + 2/x + 4/x^2}$

$= \cancel{\lim_{x\to\infty}} \frac{4 + 5/\infty + 7/(\infty)^2}{3 + 2/\infty + 4/(\infty)^2} = \frac{4+0+0}{3+0+0} \boxed{= \frac{4}{3}}$

Q: $\lim_{x\to\infty} \frac{x^2 + x - 1}{5x^3 + x^2 - 2}$

$\lim_{x\to\infty} \frac{x^2/x^3 + x/x^3 - 1/x^3}{5x^3/x^3 + x^2/x^3 - 2/x^3}$

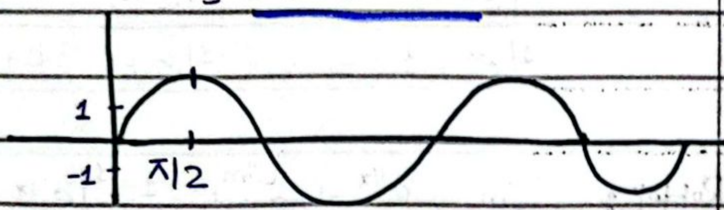
Desmos graphic Calculator

$$\lim_{x\to\infty} \frac{1/x - 1/x^2 - 1/x^3}{5 - 1/x - 2/x^3}$$

$$= \frac{1/\infty - 1/\infty - 1/\infty}{5 - 1/\infty - 2/\infty}$$

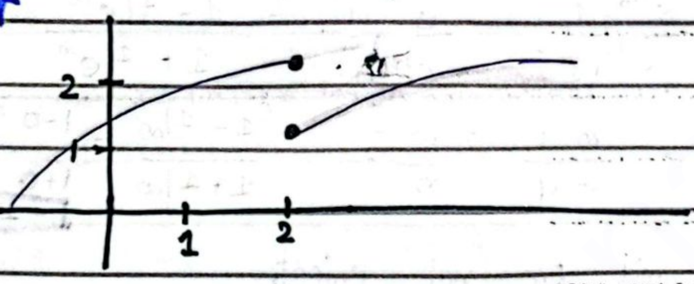
$$= \frac{0 - 0 - 0}{5 - 0 - 0} \quad \boxed{= 0}$$

*



LoHoL $= 1 =$ RoHoL $= f(\pi/2)$ {Continous}.

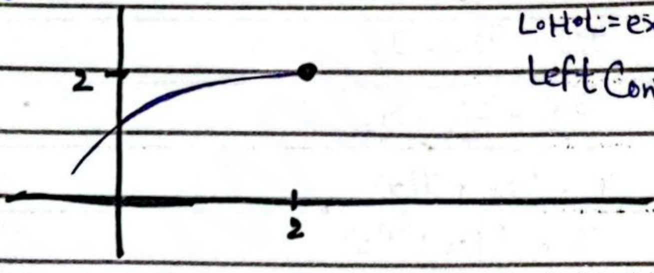
*



LoHoL $= 2 = f(2)$

RoHoL $= 1$.

*

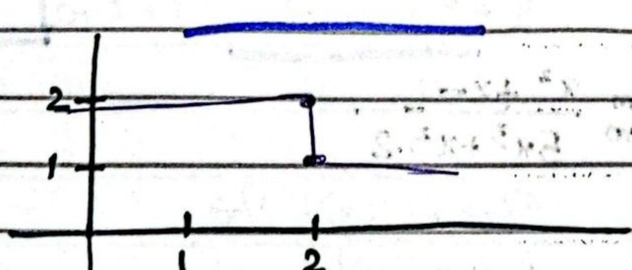


LoHoL = exiest

Left Continous

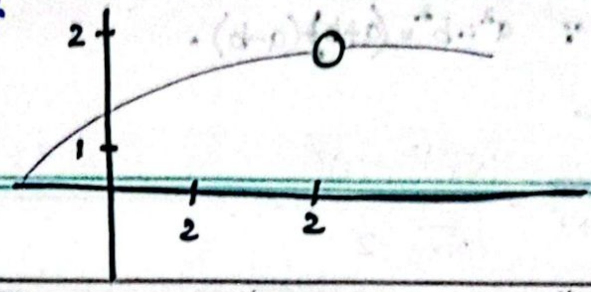
It is dis-Continous.

*



This is Dis Continous

*

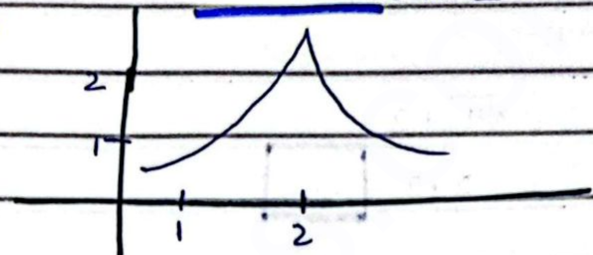


This is Dis-Continous.

$f(2) = 1$

LoHoL $= 2 =$ RoHoL.

*



This is Continous.

Lec no 9: 31-Oct-2024.

Q: $g(x) = \begin{cases} ax+2b, & \text{if } x \le 0 \\ x^2+3a-b, & \text{if } 0 < x \le 2 \\ 3x-5, & \text{if } 2 < x. \end{cases}$

at $x=0$

LoHoL: $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} (ax+2b)$

$= a(0) + 2b \quad \boxed{= 2b}$

RoHoL: $\lim_{x\to 0^+} g(x) = \lim_{x\to 0} x^2+3a-b$

$= (0)^2 + 3a - b \quad \boxed{= 3a-b.}$

$\Rightarrow 2b = 3a - b$

$2b + b = 3a$

$3b = 3a$

$\boxed{b = a} \longrightarrow (1)$

At $x=2$.

L.H.L: $g(2) = (2)^2 + 3a - b$

$\boxed{= 4+3a-b}$

R.H.L: $\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} 3x - 5$

$3(2)-5 = 6-5 \boxed{= 1.}$

$\Rightarrow 4+3a-b = 1$ —— ②

Using Eq (1)

$3a - a = 1-4$

$2a = -3$

$\boxed{a = -3/2}$

$\boxed{b = -3/2}$

Q: $\lim_{x \to \infty} \left(5 + \frac{1}{x}\right)$.

$\left(5 + \frac{1}{\infty}\right) = 5+0 \boxed{= 5.}$

Q: $\lim_{x \to -\infty} \frac{11x+2}{2x^3-1}$.

$\lim_{x \to -\infty} \frac{11x/x^3 + 2/x^3}{2x^3/x^3 - 1/x^3}$

$\lim_{x \to -\infty} \frac{11/x^2 + 2/x^3}{2 - 1/x^3}$

$\frac{11/(-\infty)^2 + 2/(-\infty)^3}{2 - 1/(-\infty)^3}$

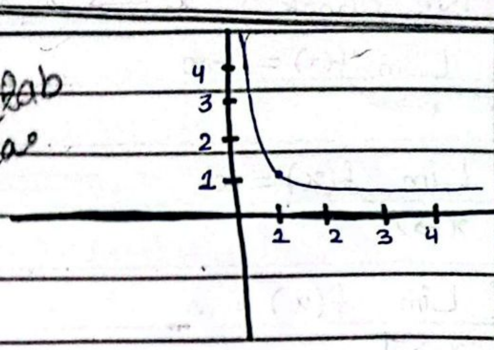
$\frac{11/\infty + 2/\infty}{2 + 1/\infty}$

$\frac{0-0}{2+0} \boxed{= 0}$

Q: Find the horizontal asymptotes of the graph of.

$$f(x) = \frac{x^3 - 2}{|x|^3 + 1}.$$

Asymptotes is kamatlab kya hai



$f(x) = \frac{1}{x}$

$x = 0.5 \Rightarrow f(0.5) = 1/0.5 = 2$

$x = 0.4 \Rightarrow f(0.4) = 1/0.4 = 2.5.$

$x = 1,\ f(1) = 1,\ (1,1)$

$x = 2 \to f(2) = \frac{1}{2} = 0.5.$

When $x > 0$, $\lim_{x \to \infty}$

$f(x) = \lim_{x \to \infty} \frac{x^3-2}{|x|^3+1}$

$= \lim_{x \to \infty} \frac{x^3-2}{x^3+1} = \lim_{x \to \infty} \frac{x^3/x^3 - 2/x^3}{x^3/x^3 + 1/x^3}$

$\lim_{x \to \infty} \frac{1 - 2/x^3}{1 + 1/x^3}$

$= \frac{1 - 2/\infty}{1 + 1/\infty} = \frac{1-0}{1+0} = 1.$

When

$\lim_{x \to \infty} \frac{x^3-2}{|x|^3+1} = \lim_{x \to \infty} \frac{x^3-2}{-x^3+1}.$

$\lim_{x \to -\infty} \frac{1 - 2/x^3}{-1 + 1/x^3} = \frac{1-0}{1+0} = -1$

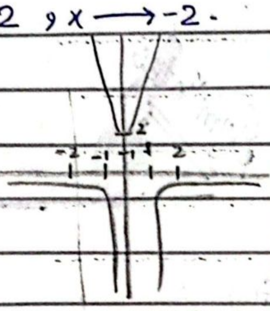
at $y = 1$ and $y = -1$.

Q: $f(x) = \dfrac{-8}{x^2-4}$

Find vertical asymptotes.

We Check $x \to 2$, $x \to -2$.

$\lim\limits_{x\to 2^+} f(x) = -\infty$



$\lim\limits_{x\to 2^-} f(x) = \infty$

$\lim\limits_{x\to 2^+} f(x) = \infty$

$\lim\limits_{x\to -2^-} f(x) = -\infty$.

Q $\lim\limits_{x\to\infty} \dfrac{\sin x}{x} = 0$

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} \boxed{= 1.}$

Q: $\lim\limits_{x\to\infty} \dfrac{x+\sin x}{2x+7-5\sin x}$.

$= \lim\limits_{x\to\infty} \dfrac{x/x + \sin x/x}{2x/x + 7/x - 5\sin x/x}$

$= \lim\limits_{x\to\infty} \dfrac{1+\sin x/x}{2+7/x-5\sin x/x}$

$\dfrac{1 + \lim\limits_{x\to\infty} \sin x/x}{2 + \lim\limits_{x\to\infty} 7/x + 5\lim\limits_{x\to\infty} \sin x/x}$

$= \dfrac{1+0}{2+0-0}$ $\boxed{= \dfrac{1}{2}}$.

Vertical mai denominator ko zero rakhty ha, Aur joh x ki value nikalti ha voh limit mai put krty ha.

Horizantal mai ∞ put krty ha } symtotes.

X———————X

Lec no 10: 6-Nov-2024.

Derivatives:

→ Find the slope of $f(x) = x^2+1$, $(2, 5)$.

Differentiate w.r.t 'x'

$f'(x) = 2x+0$

$\boxed{f'(x) = 2x}$

$m = f'(2) = 2(2) \boxed{= 4.}$

→ $f(x) = x - 2x^2$.

$f'(x) = 1-2(2x)$

$\boxed{f'(x) = 1-4x}$.

→ Formulas:

→ $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$ [Power rule].

→ $f(x) = g(x)\cdot h(x) \Rightarrow f'(x) = g(x)\cdot h'(x) + g'(x)\,h(x)$.

(Product Rule).

→ $f(x) = \dfrac{g(x)}{h(x)} \Rightarrow f'(x) = \dfrac{h(x)\cdot g'(x) - (g)(x)\,h'(x)}{(h(x))^2}$

(Quotient Rule.)

Prime kind value x

→ $f(x) = \frac{x}{x-2}$. $x(3)$.

$f(x) = \frac{x}{x-2}$

Diff w.r.t 'x'

$f'(x) = \frac{(x-2)\,d/dx(x) - (x)\,d/dx(x-2)}{(x-2)^2}$

$= \frac{(x-2)(1) - (x)(1-0)}{(x-2)^2}$

$= \frac{(x-2) - (x)}{(x-2)^2}$

$= \frac{x-2-x}{(x-2)^2}$

$\boxed{f(x) = \frac{-2}{(x-2)^2}}$

$\cancel{f(x)}\ m = \frac{-2}{(3-2)^2}$

$= \frac{-2}{1}$

$\boxed{m = -2}$

→ $f(x) = \sqrt{x+1}$, $x = 8$

$f(x) = (x+1)^{1/2}$

$f'(x) = \frac{1}{2}(x+1)^{1/2-1}$

$f'(x) = \frac{1}{2}(x+1)^{-1/2}$

$f'(x) = \frac{1}{2\sqrt{x+1}}$

$m = \frac{1}{2\sqrt{8+1}} = \frac{1}{2\sqrt{9}} = \frac{1}{2(3)}$

$\boxed{m = \frac{1}{6}}$

→ $y = \frac{x-1}{x+1}$, $x = 2$

Diff w.r.t to "x".

$f'(y) = \frac{(x+1)\,d/dx(x-1) - (x-1)\,d/dx(x+1)}{(x+1)^2}$

$= \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

$= \frac{(x+1) - (x-1)}{(x+1)^2} = \frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}$

$\boxed{f'(y) = \frac{2}{(x+1)^2}}$

→ $m = \frac{2}{(2+1)^2} = \frac{2}{(3)^2}$ $\boxed{= \frac{2}{9}}$

→ $f(x) = 4 - x^2$, $f'(-3)$, $f'(0)$, $f'(1)$.

$f'(x) = 0 - 2x$

$\boxed{f'(x) = -2x}$

$f'(3) = -2(3)$ $\boxed{= -6}$

$f'(0) = -2(0)$ $\boxed{= 0}$

$f'(1) = -2(1)$ $\boxed{= -2}$

* $f(x) = \frac{1-x}{x}$, $f'(-1)$, $f'(1)$, $f'(\sqrt{2})$

$f'(x) = \frac{(x)\,d/dx(1-x) - (1-x)\,d/dx(x)}{(x)^2}$

$f'(x) = \frac{x(-1) - (1-x)(1)}{(x)^2}$

3.1 (11, 22)

3.2 (1, ... 22) 17/18 Not include

$$f'(x) = \frac{(x)(-1) - (1-x)(1)}{(x)^2}$$

$$f'(x) = \frac{-\cancel{x} - 1 + \cancel{x}}{(x)^2}$$

$$\boxed{f'(x) = \frac{-1}{x^2}}$$

$$f(-1) = \frac{-1}{(-1)^2} \boxed{= -1}$$

$$f(1) = \frac{-1}{1} \boxed{= -1}$$

$$f(\sqrt{2}) = \frac{-1}{\sqrt{2}} \boxed{= \frac{-1}{2}}$$

$y = \frac{x+3}{1-x}$, $x = -2$.

$$y = \frac{x+3}{1-x}.$$

$$f'(y) = \frac{(1-x)d/dx(x+3) - (x+3)\,d/dx(1-x)}{(1-x)^2}$$

$$f'(y) = \frac{(1-x)(1) - (x+3)(-1)}{(1-x)^2}$$

11: 8-Nov-2024.

Ex 3.2.

$$\lim_{x \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$= \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}. \text{ (Formula)}$$

Steps.
(i) Replace x by x+h.
(ii) Subtract f(x).
(iii) Divided by h.
(iv) Apply limit $h \to 0$

Different by definition.
By first Principle.
By ab intialy method.

Ya teno ak method ka name ya question ay to phr (x+h) lgana ha.

∴ Example

$$f(x) = x$$

Step 1: $f(x+h) \boxed{= x+h.}$

Step 2: $f(x+h) - f(x)$

$= (x+h-x).$

$\boxed{= h}$

$$f'(x) = \frac{(x)(-1) - (1-x)(1)}{(x)^2}$$

$$f'(x) = \frac{\cancel{-x} - 1 + \cancel{x}}{(x)^2}$$

$$\boxed{f'(x) = \frac{-1}{x^2}}$$

$$f(-1) = \frac{-1}{(-1)^2} \boxed{= -1}$$

$$f(1) = \frac{-1}{1} \boxed{= -1}$$

$$f(\sqrt{2}) = \frac{-1}{\sqrt{2}} \boxed{= \frac{-1}{2}}$$

$$y = \frac{x+3}{1-x}, \quad x = -2.$$

$$y = \frac{x+3}{1-x}.$$

$$f'(y) = \frac{(1-x)d/dx(x+3) - (x+3)\, d/dx(1-x)}{(1-x)^2}$$

$$f'(y) = \frac{(1-x)(1) - (x+3)(-1)}{(1-x)^2}$$

: Lec no 11 : 8-Nov-2024.

Ex 3.2.

:Derivatives:

$$* \; f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$= \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}. \text{ (Formula)}$$

Steps.
(i) Replace x by $x+h$.
(ii) Subtract $f(x)$.
(iii) Divided by h.
(iv) Apply limit $h \to 0$

- Different by definition.
- By frst Principle.
- By ab intialy method.

Ya teno ak method ka name ya question ay to Phr $(x+h)$ lgana ha.

∴ Example

$$f(x) = x$$

Step 1: $f(x+h)$ $\boxed{= x+h.}$

Step 2: $f(x+h) - f(x)$

$$= (x+h-x).$$

$$\boxed{= h}$$

Step3:

$$\frac{f(x+h)-f(x)}{h} = \frac{h}{h} \boxed{=1.}$$

Step4:

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} 1$$

$$\boxed{= 1} \quad \boxed{f'(x) = 1.}$$

* **Differenciate by difinition**

$f(x) = x^2.$

$f(x) = x^2$

Step 1: $f(x+h) = (x+h)^2.$

$$\boxed{f(x+h) = x^2+h^2+2xh}$$

Step 2:

$= f(x+h) - f(x)$

$= \cancel{x^2}+h^2+2xh - \cancel{x^2}$

$$\boxed{= h^2+2xh.}$$

Step3:

$$= \frac{f(x+h)-f(x)}{h}$$

$$= \frac{h^2+2xh}{h} = \frac{\cancel{h}(h+2x)}{\cancel{h}}$$

$$\boxed{= h+2x}$$

Step 4:

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (h+2x)$$

$$= 0 + 2x \quad \boxed{= 2x}$$

$$\boxed{f'(x) = 2x.}$$

* $f(x) = c.$

$f(x) = c.$

Step1:

$$\boxed{f(x+h) = c.}$$

Step2:

$= f(x+h) - f(x)$

$= c - c \quad \boxed{= 0}$

Step3:

$$= \frac{f(x+h)-f(x)}{h}$$

$$= \frac{0}{h} \quad \boxed{= 0}$$

Step4:

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (0) \quad \boxed{= 0}$$

$$\boxed{f'(x) = 0}$$

Formulas:

***: Differentiates:**

(i) Derivative of a Constant function:

$$f(x) = c \implies \frac{df}{dx} = \frac{d}{dx}(c) \boxed{= 0}$$

(ii) Power Rule:

$$f(x) = x^n \implies \frac{df}{dx} = \frac{d}{dx}(x^n) \boxed{= nx^{n-1}}$$

(iii) Derivative Constantmultiple rule:

$$f(x) = cx \implies \frac{df}{dx} = \frac{d}{dx}(cx)$$

$$= c\frac{dx}{dx} \quad \boxed{= c}$$

iv- **Derivative Sum Rule.** $u+v$.

$$\Rightarrow \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}.$$

v- **Derivative Sub Rule** $u-v$

$$\Rightarrow \frac{d}{dx}(u-v) = \frac{du}{dv} - \frac{dv}{dx}.$$

vi- **Product Rule:**

$$\Rightarrow \frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{du}{dx}.$$

v- **Quotient Rule:** $\frac{u}{v}$

$$\Rightarrow \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du/dx - u\left(dv/dx\right)}{v^2}.$$

Second and Higher order Derivatives:

$$y = f(x)$$

1st $\Rightarrow$ $y' = f(x)$.

2nd $\Rightarrow$ $y'' = f''(x)$.

3rd $\Rightarrow$ $y''' = f'''(x)$.

Q1: $y = -x^2+3$.

Diff w.r.t. x.

$y' = -2x^{2-1}+0$

$\boxed{y' = -2x}$

Again diff w.r.t. x

$\boxed{y'' = -2}$

Q7: $w = 3z^{-2} - \frac{1}{z}$

$W = 3z^{-2} - z^{-1}$

Diff w.r.t. z.

$\frac{dw}{dz} = 3(-2)z^{-2-1} - (-1)z^{-1-1}$.

$\frac{dw}{dz} = -6z^{-3} + z^{-2}$

$f(w)' = -6z^{-3} + z^{-2}$.

Diff w.r.t. to z.

$f(w)'' = -6(-3)z^{-4} - 2z^{-3}$.

$= 18z^{-4} - 2z^{-3}$

$= \frac{18}{z^4} - \frac{2}{z^3}$.

$\boxed{= \frac{18-2z}{z^4}}$

Q:12 $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$

$$\gamma = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}.$$

$\Rightarrow \gamma = 12\theta^{-1} - 4\theta^{-3} + \theta^{-4}.$

Diff w.r.t. "γ".

$f(\gamma)' = 12(-1)\theta^{-2} - 4(-3)\theta^{-4} + (-4)\theta^{-5}.$

$f(\gamma)' = -12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}$

Diff w.r.t. "γ"

$f(\gamma)'' = -12(-2)\theta^{-3} + 12(-4)\theta^{-5} - 4(-5)\theta^{-6}.$

$= 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

$$\boxed{f(\gamma)'' = \frac{24}{\theta^3} - \frac{48}{\theta^5} + \frac{20}{\theta^6}}.$$

Q: $y = (3-x^2)(x^3-x+1).$

Product Formula:

$d(y) = (3-(x)^2)\frac{d}{dx}(x^3-x+1) + (x^3-x+1)\frac{d}{dx}(3-x^2).$

$d(y) = (3-x^2)[3(x)^2-1+0] + (x^3-x+1)(0-2x).$

$= (3-x^2)[3x^2-1] + (x^3-x+1)(-2x)$

$= 3(3x^2-1) - x^2(3x^2-1) + -2x(x^3-x+1).$

$= 9x^2 - 3 - 3x^4 + x^2 + (-2x^4 + 2x^2 - 2x)$

$= 9x^2 - 3 - 3x^4 + x^2 - 2x^4 + 2x^2 + 2x$

$= -2x^4 - 3x^4 + 9x^2 + 2x^2 + x^2 + 2x - 3$

$$\boxed{= -5x^4 + 12x^2 + 2x - 3.}$$

~~Again Diff w.r.t. x.~~

Ex 3.2.

Q (1 - 28).

$y = 3x^3 - 3x + 3 - (x^5 - x^3 + x^2)$

$= 3x^3 - 3x + 3 - x^5 + x^3 - x^2.$

$= -x^5 + 4x^3 - x^2 - 3x + 3.$

Diff w.r.t to x.

$y' = -5x^4 + 4(3)x^2 - 2x - 3$

$$\boxed{y' = -5x^2 + 12x^2 - 2x - 3}$$

Q17: $y = \frac{2x+5}{3x-2}.$

~~$dy = (3x-2)d/dx(2x+5) - (3x-5)d/dx(2x+5$~~

$$\frac{dy}{dx} = \frac{(3x-2)d/dx(2x+5) - (2x+5)d/dx(3x-2)}{(3x-2)^2}$$

$$\frac{dy}{dx} = \frac{(3x-2)(2(1)) - (2x+5)(3(1))}{(3x-2)^2}$$

$$= \frac{(3x-2)2 - (2x+5)(3)}{(3x-2)^2}$$

$$= \frac{6x-4-[6x+15]}{(3x-2)^2}$$

$$= \frac{\cancel{6x} - 4 - \cancel{6x} - 15}{(3x-2)^2}$$

$\gamma = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$.

$\Rightarrow \gamma = 12\theta^{-1} - 4\theta^{-3} + \theta^{-4}$.

Diff w.r.t. "γ".

$f(\gamma)' = 12(-1)\theta^{-2} - 4(-3)\theta^{-4} + (-4)\theta^{-5}$.

$f(\gamma)' = -12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}$

Diff w.r.t. "γ"

$f(\gamma)'' = -12(-2)\theta^{-3} + 12(-4)\theta^{-5} - 4(-5)\theta^{-6}$.

$= 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

$$\boxed{f(\gamma)'' = \frac{24}{\theta^3} - \frac{48}{\theta^5} + \frac{20}{\theta^6}}$$

Q: $y = (3-x^2)(x^3-x+1)$.

Product Formula:

$d(y) = (3-(x)^2)\frac{d}{dx}(x^3-x+1) + (x^3-x+1)\frac{d}{dx}(3-x^2)$.

$d(y) = (3-x^2)[3(x)^2-1+0] + (x^3-x+1)(0-2x)$.

$= (3-x^2)[3x^2-1] + (x^3-x+1)(-2x)$

$= 3(3x^2-1) - x^2(3x^2-1) + -2x(x^3-x+1)$.

$= 9x^2 - 3 - 3x^4 + x^2 + (-2x^4 + 2x^2 - 2x)$

$= 9x^2 - 3 - 3x^4 + x^2 - 2x^4 + 2x^2 + 2x$

$= -2x^4 - 3x^4 + 9x^2 + 2x^2 + x^2 + 2x - 3$

$$\boxed{= -5x^4 + 12x^2 + 2x - 3}$$

~~Again Diff w.r.t.~~ x.

$-5(4)x$

Now We Multiply.

$\Rightarrow y = (3-x^2)(x^3-x+1)$.

$y = 3(x^3-x+1) - x^2(x^3-x+1)$

$y = 3x^3 - 3x + 3 - [x^5 - x^3 + x^2]$

$= 3x^3 - 3x + 3 - x^5 + x^3 - x^2$.

$= -x^5 + 4x^3 - x^2 - 3x + 3$.

Diff w.r.t to x.

$y' = -5x^4 + 4(3)x^2 - 2x - 3$

$$\boxed{y' = -5x^2 + 12x^2 - 2x - 3}$$

Q17: $y = \frac{2x+5}{3x-2}$.

~~$dy = (3x-2)\,d/dx(2x+5) - (3x-5)\,d/dx(2x+5$~~

$\frac{dy}{dx} = \frac{(3x-2)\,d/dx(2x+5) - (2x+5)\,d/dx(3x-2)}{(3x-2)^2}$

$\frac{dy}{dx} = \frac{(3x-2)(2(1)) - (2x+5)(3(1))}{(3x-2)^2}$

$= \frac{(3x-2)2 - (2x+5)(3)}{(3x-2)^2}$

$= \frac{6x-4 - [6x+15]}{(3x-2)^2}$

$= \frac{\cancel{6x} - 4 - \cancel{6x} - 15}{(3x-2)^2}$

$$= \frac{-4-15}{(3x-2)^2}$$

$$\boxed{y' = \frac{-19}{(3x-2)^2}}$$

Lec 12:

:4-Dec-2024:

→: Derivative Rate of Change:

:Formula:

$$V(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

Average Velocity (AV)

$$= \frac{\Delta s}{\Delta t} = \frac{S(t_2) - S(t_1)}{t_2 - t_1}$$

$$, \; t_1 \leq t \leq t_2.$$

Exercise 3.4

Q1: $S(t) = t^2 - 3t + 2, \quad 0 \leq t \leq 2.$

$S(0) = (0)^2 - 3(0) + 2$

$= 2$

Average velocity $\frac{S(2)-S(0)}{2-0}$

$S(2) = (2)^2 - 3(2) + 2$

$= 4 - 6 + 2$

$= 0$

$\Delta S = S(2) - S(0)$

$= 0 - 2$

$= -2m$

$\Delta t = 2 - 0 = 2s$

$$V_{av} = \frac{\Delta s}{\Delta t} = \frac{-2}{2} = -1 \; m/s$$

Q2: $S(t) = t^2 - 3t + 2, \quad 0 \leq t \leq 2.$

Diff w.r.t t.

$$V = \frac{ds}{dt} = 2t - 3.$$

$$\text{Speed} = \left|\frac{ds}{dt}\right|_{t=0} = |2(0) - 3|$$

Displacement Velocity Speed

$$\text{Speed} = |-3| \; \boxed{= 3 \; ms^{-1}}$$

$$\text{Speed} = \left|\frac{ds}{dt}\right|_{t=2}$$

$$= |2(2) - 3| = |4 - 3|$$

$$\boxed{= 1 ms^{-1}.}$$

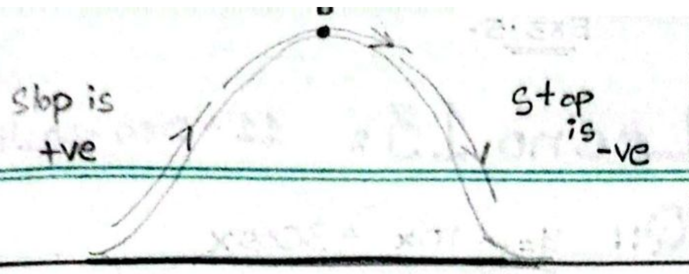
(Double Derivative

acceleration $a = \frac{dV}{dt} = 2 \; ms^{-2}$

$a(0) = 2 \; ms^{-2} = a(2).$

* If the Velocity is Positive then the direction of motion of a body is forward.

* If ~~Veloti~~ Velocity is Negative then the direction of motion of a body is backWord.



Forward Method ✓	Backword Method. ✓
$2t-3=0$	$v(1)=2(1)-3$
$2t=3$	$\boxed{=1\ m/s}$
$\boxed{t=3/2}$	$v(1.8)=2(1.8)-3$
	$=3.6-3$
	$\boxed{=0.6\ m/s.}$

Q3 $s(t)=6t^2-t^2$, $0\le t\le 6$

$s(0)=6(0)-(0)^2$

$=0$

$s(6)=6(6)-(6)^2$

$=36-36=0$

$\Delta s = s(6)-s(6)\ \boxed{=6}$

~~$\Delta t = 6-0$~~

$\boxed{\cancel{\Delta t = 6}}$

$\Delta t = t_2 - t_1$

$=6-0\ \boxed{=6}$

$V_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{0}{6}\ \boxed{=0}$

Speed:

$s(t)=6t-t^2 \quad 0\le t\le 6.$

$v\ \dfrac{ds}{dt} = 6t-t^2$

$\text{Speed} = \left|\dfrac{ds}{dt}\right|$

$=|6(6)-(6)^2|$

$=|36-36|=|0|\ \boxed{=0.}$

Diff w.r.t. t.

$a=\dfrac{d^2s}{dt^2}$

$a = 6t - t^2$

$\boxed{=6-2t.}$

~~$a = 6t$~~

$a = 6-2t$

$a = 0-2(1)$

$\boxed{a=-2\ m/s^{-2}.}$

$a(0)=6-2t.$

$=0-2(1)$

$\boxed{=-2\ m/s^{-2}}$

$a(0)=6t-2t = 6-2(0)\ \boxed{=6}.$

$a(6) = 6-2(6) = 6-12\ \boxed{=-6}.$

Q4. $s(t)=\dfrac{t^4}{4}-t^3+t^2$, $0\le t\le 2.$

(a) $s(0)=0m$

$s(3)=\dfrac{9}{4}m$

$\Delta s = \dfrac{9}{4}m$

$\Delta t = 3$

$V_{av} = \dfrac{9/4}{3}\ \boxed{=\dfrac{3}{4}\ ms^{-1}.}$

(b) $\frac{ds}{dt} = t^3 - 3t^2 + 2t.$

$$\text{Speed} = \left|\frac{ds}{dt}\right|_{t=0} = 0\,ms^{-1}.$$

$$\text{Speed} = \left|\frac{ds}{dt}\right|_{t=3} = 6\,ms^{-1}.$$

$a = 3t^2 - 6t + 2$

$a(0) = 2ms^{-2}.$

$a(3) = 11ms^{-2}.$

(c) $\frac{ds}{dt} = 0$

$t^3 - 3t^2 + 2t = 0$

$t(t^2 - 3t + 2) \geq 0$

$\Rightarrow \boxed{t = 0}$, $t^3 - 3t + 2 = 0$

$t(t-2) - 1(t-2) = 0$

$(t-2)(t-1) = 0$

$t - 2 = 0$, $t - 1 = 0$

$\boxed{t = 2}$; $\boxed{t = 1}$

Body Change its direction at $t = 0, 1, 2.$

Ex 3.5.

Lec no 13: 11-Dec-2024.

Q1: $y = -10x + 3\cos x$

Differenciate w.r.t "x".

$\frac{dy}{dx} = -10(1) + 3(-\sin x)$

$\boxed{= -10 - 3\sin x.}$

Q9: $y = (\sec x + \tan x)(\sec x - \tan x).$

$y = \sec^2 x - \tan^2 x$

$a^2 - b^2 = (a-b)(a+b)$

$\sin^2 x + \cos^2 x = 1$

$\frac{\sin^2 x}{\cos^2 x} + \cancel{\frac{\cos^2 x}{\cos^2 x}} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

$y = 1$

Differen w.r.t. x.

$\frac{dy}{dx} = 0$

Q10: $y = (\sin x + \cos x)\sec x.$

$y = (\sin x + \cos x)\left(\frac{1}{\cos x}\right)$

$y = \frac{\sin x + \cos x}{\cos x}$

$y = \frac{\sin x}{\cos x} + \frac{\cancel{\cos x}}{\cancel{\cos x}}$

$y = \tan^2 x + 1$

Diff w.r.t "x"

$\frac{dy}{dx} = \frac{d}{dx}(\tan^2 x) + 1$

$\boxed{\frac{dy}{dx} = \sec^2 x}$

Q18: $y = \frac{4}{\cos x} + \frac{1}{\tan x}$.

$y = 4(\sec x) + (\cot x)$

$y = 4\sec x + \cot x$.

Diff w.r.t "x".

$\frac{dy}{dx} = 4\sec x\tan x - \operatorname{cosec}^2 x$.

Q21: $S = \frac{1 + \csc t}{1 - \csc t}$.

$\csc t = \operatorname{cosec}$

$s = \frac{1 + \operatorname{cosec} t}{1 - \operatorname{cosec} t}$

$s = \frac{1 + \operatorname{cosec} t}{1 - \operatorname{cosec} x} \times \frac{1 + \operatorname{cosec} t}{1 + \operatorname{cosec} t}$

$s = \frac{1 + \operatorname{cosec}^2 t + 2\operatorname{cosec} t}{1 - \operatorname{cosec}^2 t}$.

$s = \frac{1 + \operatorname{cosec}^2 t + 2\operatorname{cosec} t}{1 - \operatorname{cosec}^2 t}$

$\cos^2 x + \sin^2 x = 1$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$1 - \operatorname{cosec}^2 x = -\cot^2 x$.

$= \tan^2(t)\ (1 + \operatorname{cosec}^2 t + 2\operatorname{cosec} t)$.

$= -\tan^2 t - \frac{\sin^2 t}{\cos^2 t} \cdot \frac{1}{\sin^2 t} - 2\tan^2 t\operatorname{cosec} t$

~~Diff~~ $= -\tan^2 t - \sec^2 t - 2\tan^2 t\operatorname{cosec} t$

$\frac{ds}{dt} = -2\tan^{2-1} t \cdot \sec^2 t - 2\sec^{2-1} t\ \sec(t)\tan(t)$.

$= -2\tan(t)\sec^2(t) - 2\sec^2 t\tan t + 2\tan^2(t)\operatorname{cosec} t\left(\frac{1}{\tan t}\right)$.

$= -2\left[\tan^2(t)(-\operatorname{cosec} t\cot(t)) + \operatorname{cosec}(t)\cdot 2\tan^2(t)\sec^2(t)\right]$.

$= -2\tan(t)\sec^2(t) - 2\sec^2(t)\tan(t) + 2(\tan^2(t))\operatorname{cosec} t\left(\frac{1}{\tan t}\right) - 4\operatorname{cosec} t\ \tan t\ \sec^2 t$.

$= -4\tan t\sec^2 t + 2\tan t\operatorname{cosec} t - 4\left(\frac{1}{\sin t}\right)\left(\frac{\sin t}{\cos t}\right)\cdot\left(\frac{1}{\cos^2 t}\right)$.

$= -4\left(\frac{\sin t}{\cos t}\right)\left(\frac{1}{\cos^2 t}\right) + 2\left(\frac{\sin t}{\cos t}\right)\left(\frac{1}{\sin t}\right) - 4\left(\frac{1}{\cos^3 t}\right)$.

$$\boxed{= -4\left(\frac{\sin t}{\cos^3 t}\right) + 2\sec t - 4\sec^3 t}$$

Q4. $y = \sqrt{x}\sec x + 3$.

$y = \sqrt{x}\sec x + 3$.

$\frac{dy}{dx} = \left[\sqrt{x}\frac{d}{dx}(\sec x) + \sec x\frac{d}{dx}(\sqrt{x})\right] + 0$

$\frac{dy}{dx} = \sqrt{x}(\tan x) + \sec x\left(\frac{1}{2}\right)(x)^{-1/2}$.

$\frac{dy}{dx} = \sqrt{x}(\tan x \sec x) + \frac{\sec x}{2\sqrt{x}}$

$= \sqrt{x}(\tan x + \sec x) + \frac{\sec x}{2\sqrt{x}}$.

$$= \frac{\cancel{2(\sqrt{x})(\sqrt{x})(\tan x + \sec x) + \sec x}}{\cancel{2\sqrt{x}}}$$

$$= \frac{\cancel{2x\ (\tan x + \sec x) + \sec x}}{\cancel{2\sqrt{x}}}$$

$$= \sqrt{x}\ (\tan x \sec x) + \frac{\sec x}{2\sqrt{x}}.$$

$$= \frac{2(\sqrt{x})^2\ (\tan x \sec x) + \sec x}{2\sqrt{x}}$$

$$\boxed{= \frac{2x\ (\tan x \sec x) + \sec x}{2\sqrt{x}}}$$

Q29. $p = \dfrac{\sin q + \cos q}{\cos q}$.

Diff w.r.t 'x'

$$\frac{dp}{dq} = \frac{\cos q\ d/dx((\sin q) + \cos q) - (\sin q + \cos q)\, d/dx(\cos q)}{(\cos q)^2}$$

$$\frac{dP}{dq} = \frac{\cos q\ (\cos q - \sin q) - (\sin q + \cos q)(-\sin q)}{\cos^2 q}$$

$$= \frac{\cos^2 q - \sin q \cos q + \cos q \sin q + \sin^2 q}{\cos^2 q}$$

$$= \frac{\cos^2 q + \sin^2 q}{\cos^2 q}$$

$$= \frac{1}{\cos^2 q}$$

$$\boxed{= \sec^2 q.}$$

Q32: $p = \dfrac{3q + \tan q}{q \sec q}$.

Differenciate w.r.t "x".

$$\frac{dP}{dq} = \frac{(q \sec q)\, d/dq\, (3q + \tan q) + (3q + \tan q)\, d/dq\, (q \sec q)}{(q \sec q)^2}$$

$$\frac{dP}{dq} = \frac{(q \sec q)(3 + \sec^2 q) - (3q + \tan q)\ (q \sec q \tan q + \sec q \cdot 1)}{(q \sec q)^2}$$

$$= \frac{3q \sec q + q \sec^3 q - 3q^2 \sec q \tan q - 3q \sec q - q \sec q \tan^2 q - \tan q \sec q}{(q \sec q)^2}$$

$$= \frac{\sec q\,[q \sec^2 q - 3q^2 \tan q - q \tan^2 q - \tan q]}{q^2 \sec^2 q}$$

$$= \frac{q \sec^2 q - 3q^2 \tan q - q \tan^2 q - \tan q}{q^2 \tan q}.$$

Q: 34 (b) $y = 9 \cos x$

$$\frac{dy}{dx} = -9 \sin x$$

$$\frac{d^2y}{dx^2} = -9 \cos x$$

$$\frac{d^3y}{dx^2} = 9 \sin x$$

$$\frac{d^4y}{dx^4} = 9 \cos x$$

$\vdots$

$$\frac{d^{99}y}{dx^{99}} = 9 \sin x$$

Q 50

Trigonometric Limits:

Q 50

$$\lim_{x \to -\pi/6} \sqrt{1 - \cos \pi (\operatorname{cosec} x)}.$$

$$= \sqrt{1 + \cos\left(\pi\left(\operatorname{cosec}\left(-\frac{\pi}{6}\right)\right)\right)}$$

$$= \sqrt{1 + \cos(-2\pi)}.$$

$$= \sqrt{1 - \cos 2\pi}$$

$$= \sqrt{1 + \cos(360)^\circ}$$

$$= \sqrt{1+1}$$

$$\boxed{= \sqrt{2}}$$

$\frac{\pi}{6} \times \frac{180}{\pi} = 30^\circ$

$\operatorname{cosec}(-30^\circ) = -2$

$1/\sin(-30)$

$= 1/(-1/2)$

$\boxed{= -2.}$

Formulas:-

$\rightarrow \frac{d}{dx}(\sin x) = \cos x.$

$\rightarrow \frac{d}{dx}(\cos x) = -\sin x.$

$\rightarrow \frac{d}{dx}(\tan x) = \sec^2 x.$

$\rightarrow \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$\rightarrow \frac{d}{dx}(\sec x) = \sec x \tan x.$

$\rightarrow \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$1 + \cot^2 x = \sec^2 x$

$\cot^2 x = \sec^2 x - 1.$

$\rightarrow \cos^2 x + \sin^2 x = 1$

$\rightarrow \sin^2 x = 1 - \cos^2 x$

$\rightarrow \cos^2 x = 1 - \sin^2 x.$

$\rightarrow 1 + \tan^2 x = \sec^2 x.$

$\rightarrow \tan^2 x = \sec^2 x - 1$

Lec no 14:

12-Dec-2024.

Ex 3.6

Chain Rule:

If $f(u) = y$ and $u = g(x)$

then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}.$

$y \rightarrow u \rightarrow x$ agr y w.r.t "x" then we apply Chain Rule.

Ex 3.6.

Q1: $y = 6u - 9$, $u = (1/2)x^4.$

$\rightarrow \frac{dy}{du} = 6\frac{d}{du}(u) - \frac{d}{du}(9)$

$\frac{dy}{du} = 6(1)$, $\boxed{\frac{dy}{du} = 6}$

$\rightarrow \frac{du}{dx} = \frac{1}{2}\frac{d}{dx}(x^4)$

$= \frac{4}{2}x^3$ $\boxed{= 2x^3}$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$= 6 \times 2x^3$

$\boxed{\frac{dy}{dx} = 12x^3.}$

Q3: $y = \sin u \;,\; u = 3x+1.$

$\rightarrow y = \sin u$

$\frac{dy}{du} = \frac{d}{du}(\sin u)$

$\boxed{\frac{dy}{du} = \cos u}$

$\rightarrow u = 3x+1$

$\frac{du}{dx} = 3(1)$

$\boxed{\frac{du}{dx} = 3}$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \cos u \; (3)$

$\boxed{= 3\cos u.}$ $= 3\cos(3x+1)$

$\{1 \text{ to } 8\}$

Q9: $y = (2x+1)^5.$

Let: $u = (2x+1)^5$

$\rightarrow y = u^5$

Diff y w.r.t "u"

$\rightarrow \boxed{\frac{dy}{du} = 5u^4.}$

Diff u w.r.t to "x"

$\rightarrow \boxed{\frac{du}{dx} = 2.}$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 5u^5 \; (2)$

$\boxed{= 10u^5}$

$\boxed{\frac{dy}{dx} = 10(2x+1)^5.}$

Q13: $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4.$

Let:

$\rightarrow u = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)$

$y = (u)^4.$

Diff w.r.t. "u"

$\rightarrow \frac{dy}{du} = 4u^3$

Diff w.r.t "x"

$\rightarrow \frac{du}{dx} = \frac{2}{8}x + 1 (-1)x^{-2}$

$= \frac{x}{4} + 1 + x^{-2}$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 4u^3\left(\frac{x}{4} + 1 + x^{-2}\right)$$

$$\boxed{\frac{dy}{dx} = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3\left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)}$$

Q18: $y = 5\cos^{-4}x.$

Let:

$\rightarrow u = \cos x$

$y = 5u^{-4}.$

Diff. w.r.t 'u'

$$\rightarrow \frac{dy}{dx} = 5(-4)u^{-4-1}$$

$$\boxed{\frac{dy}{du} = -20\,u^{-5}}$$

Differenciate w.r.t x.

$$\rightarrow \boxed{\frac{du}{dx} = -\sin x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= -20u^{-5}(-\sin x)$$

$$\boxed{\frac{dy}{dx} = 20\cos^{-5}x \sin x}$$

Q21.

$$S = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$$

Diff w.r.t to "t".

$$\rightarrow \frac{ds}{dt} = \frac{d}{dt}\left(\frac{4}{3\pi}\sin 3t\right) + \frac{d}{dt}\left(\frac{4}{5\pi}\cos 5t\right).$$

$$\Rightarrow \frac{ds}{dt} = \frac{4}{3\pi}\cos 3t \frac{d}{dt}(3t) + \frac{4}{5\pi}(-\sin 5t)\frac{d}{dt}(5t)$$

$$\frac{ds}{dt} = \frac{4}{3\pi}(\cos 3t)(3) + \frac{4}{5\pi}(\sin 5t)(5)$$

$$\frac{ds}{dt} = \frac{4}{\cancel{3}\pi}(\cancel{3})(\cos 3t) + \frac{4}{\cancel{5}\pi}(\cancel{5})(-\sin 5t)$$

$$\frac{ds}{dt} = \frac{4}{\pi}(\cos 3t) + \frac{4}{\pi}(-\sin 5t)$$

$$\boxed{\frac{ds}{dt} = \frac{4}{\pi}(\cos 3t - \sin 5t)}$$

Q25. $y = x^2\sin^4 x + x\cos^{-2}x.$

Diff w.r.t "x".

$$\frac{dy}{dx} = \frac{d}{dx}(x^2\sin^4 x) + \frac{d}{dx}(x\cos^{-2}x)$$

$$\frac{dy}{dx} = \left[x^2\frac{d}{dx}(\sin^4 x) + \sin^4 x\frac{d}{dx}(x^2)\right] + \left[x\frac{d}{dx}(\cos^{-2}x) + \cos^{-2}x\frac{d}{dx}(x)\right].$$

$= x^2 \{ 4(\cos x)(\sin^3 x) \} + \sin^4 x (2x) +$

$x [-2(\cos^{-3} x)(-\sin x)] + \cos^{-2}(x)(1)$

$= x^2 [4\cos x \sin^3 x + 2x\sin^4 x$

$- 2x \sin x \cos^{-3} x + \cos^{-2} x.$

$= 4x^2 \sin^3 x \cos x + 2x \sin^4 x + 2x \sin x$

$\cos^{-3} x + \cos^{-2} x.$

Q36: $g(t) = \left(\frac{1+\sin 3t}{3-2t}\right)^{-1}$

$g(t) = \frac{3-2t}{1+\sin 3t}.$

Q40. $q = \cot\left(\frac{\sin t}{t}\right)$

$\frac{dq}{dt} = \frac{d}{dt}\left(\cot\left(\frac{\sin t}{t}\right)\right).$

$\frac{dq}{dt} = -\operatorname{Cosec}^2 x \left(\frac{\sin t}{t}\right) \frac{d}{dt}\left(\frac{\sin t}{t}\right).$

$\frac{dq}{dt} = -\operatorname{Cosec}^2 x \left(\frac{\sin t}{t}\right) \cdot \left(\frac{(t)\frac{d}{dt}(\sin t) + (\sin t)(t)}{(t)^2}\right)$

$\frac{dq}{dt} = -\operatorname{Cosec}^2 x \left(\frac{\sin t}{t}\right) \left(\frac{+\cos t - \sin t}{t^2}\right)$

Q42: $y = \sec^2 \pi t.$

$\frac{d}{dx} = \frac{d}{dx} \sec \pi t$

$\frac{d}{dx} = \sec \pi t \tan \pi t$

$\frac{dy}{dt} = \frac{d}{dt}\left(\sec^2 \pi t\right).$

$= 2\sec \pi t (\sec \pi t) \tan \pi t \cdot \pi.$

$= 2\pi \sec^2 \pi t \tan \pi t.$

Q58 $y = \sqrt{3t + \sqrt{2t\sqrt{1-t}}}.$

Differenciat w.r.t to "x"

Formulas:

→ $d/dx\,(\cos x) = -\sin x.$

→ $d/dx\,(\sin x) = \cos x.$

→ $d/dx\,(\tan x) = \sec^2 x.$

→ $d/dx\,(\sec x) = \sec x \tan x.$

→ $d/dx\,(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x.$

→ $d/dx\,(\cot x) = -\operatorname{cosec}^2 x$

→ $\sin x = 1/\operatorname{cosec} x.$

→ $\operatorname{cosec} x = 1/\sin x.$

→ $\cos x = 1/\sec x.$

→ $\sec x = 1/\cos x.$

→ $\tan x = \sin x/\cos x.$

→ $\cot x = \cos x/\sin x.$

Lec no 15. 18 - Dec-2024

:Ex 3.6:

Qno 62: $y = 9\tan\left(\frac{x}{3}\right)$.

Diff w.r.t to "x".

$\frac{dy}{dx} = 9\frac{d}{dx}\left(\tan(x/3)\right)$.

$= 9\left(\sec^2x(x/3)\,d/dx(x/3)\right)$.

$= 9\left(\sec^2x\,(x/3)\,(1/3)\right)$

$dy/dx = 3\sec^2x\left(\frac{x}{3}\right)$.

Again Diff w.r.t to "x"

$y'' = 3(2)\sec x(x/3)\,d/dx\left(\sec x/3\right)$

$= 6\sec x(x/3)\sec(x/3)\tan(x/3)\,d/dx(x/3)$

$= 6\left(\sec^2(x/3)\tan(x/3)\left(1/(3)\right)\right)$.

$= 2\sec^2(x/3)\tan\left(\frac{x}{3}\right)$.

Qno64. $y = x^2(x^3-1)^5$.

$y = x^2(x^3-1)^5$.

$dy/dx = \left(d/dx\left[x^2(x^3-1)^5\right]\right.$.

$= x^2\frac{d}{dx}(x^3-1)^5 + (x^3-1)^5\frac{d}{dx}(x^2)$.

$= x^2 \, (5)(x^3-1)^4 \, d/dx\,(x^3-1) + (x^3-1)^5 (2x).$

$= 5x^2 (x^3-1)^4 (3x^2) + (2x)(x^3-1)^5.$

$= 15x^4 (x^3-1)^4 + 2x(x^3-1)^5.$

Again w.r.t to "x".

- $y'' = d/dx\,[15x^4(x^3-1)^4 + 2x(x^3-1)^5].$

$y'' = [15x^4 \, d/dx\,(x^3-1)^4 + (x^3-1)^4 \, d/dx\,(15x^4)] +$

$[2x \, d/dx\,(x^3-1)^5 + (x^3-1)^5 \, d/dx\,(2x)].$

$y'' = [15x^4 (4(x^3-1)^3 \, d/dx(x^3-1)) + (x^3-1)(15x^3)].$

$+ [(2x(5(x^3-1)^4 \, d/dx(x^3-1)) + (x^3-1)^5 (2)].$

$y'' = [15x^4 (4(x^3-1)^3 (3x^2)) + (15x^3(x^3-1)^4)].$

$+ [10x(x^3-1)^4 (3x) + 2(x^3-1)^5].$

$= \cancel{180x^6(x^3-1)^3} + \cancel{15x^6 - 15x^3} + \cancel{30x^2(x^3-1)^4} +$

$2\cancel{(x^3-1)^5}.$

Composite Function.

Find the value of $(f \circ g)'$ at the given "x".

Q67. $F(u) = u^5 + 1$, $u = g(x) = \sqrt{x} = x^{1/2}$, $x = 1$.

Put u value in $F(u)$.

$F(u) = (x^{1/2})^5 + 1.$

$= x^{5/2} + 1.$

Diff w.r.t "x".

$\frac{d(f(u))}{dx} = \frac{5}{2}(x)^{5/2-1} + 0$

$$\boxed{= \frac{5}{2} x^{3/2}}$$

$\frac{df(u(1))}{dx} = \frac{5}{2}(1)^{3/2}$ $\boxed{= \frac{5}{2}}.$

Lecno 10. 19-Dec-2024.

:Implicit Functions.

$x^3 + y^3 - 9xy = 0 \longrightarrow$ Implicit funtion.

$y = x^3 + 1 \longrightarrow$ Explicit function

:Exercise 3.7: Use implicit differenciation dy/dx

1- $x^2y + xy^2 = 6$.

$d/dx(x^2y) + d/dx(xy^2) = d/dx(6)$.

$[x^2\, d/dx(y) + (y)\, d/dx(x^2)] + [x\, d/dx(y^2) + y^2\, d/dx(x)] = 0$

$[x^2\, dy/dx + y\,(2x)] + [(x)2y\, dy/dx + y^2(1)] = 0$

$[x^2\, dy/dx + 2xy + 2xy\, dy/dx + y^2] = 0$

$y^2 + 2xy + x^2\, dy/dx + 2xy\, dy/dx = 0$

$y^2 + 2xy + [dy/dx\,(x^2 + 2xy)] = 0$

$[dy/dx\,(x^2 + 2xy)] = -y^2 - 2xy$

$$dy/dx = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

$$\boxed{\frac{dy}{dx} = \frac{-\cancel{y}(x - y^2 - 2y)}{\cancel{x}(x + 2y)}}$$

Q2: $x^3 + y^3 = 18xy.$

$3x^3 + 3y^2 dy/dx = 18\,[x\,dy/dx + y]$

$3x^3 + 3y^2\,dy/dx = 18x\,dy/dx + 18y.$

$3x^3 - 18y = 18x\,dy/dx - 3y^2\,dy/dx$

$3x^3 - 18y = dy/dx\,(18x - 3y^2.$

$$\frac{dy}{dx} = \frac{\cancel{3}(x^3 - 6y)}{\cancel{3}(6x - y^2)}$$

$$\boxed{y = \frac{x^3 - 6y}{6x - y^2}.}$$

Q8: $x^3 = \dfrac{2x - y}{x + 3y}.$

$d/dx(x^3) = d/dx\,(2x - y/x + 3y).$

$$3x^2 = \frac{(x+3y)\,d/dx(2x-y) - (2x-y)\,d/dx(x+3y)}{(x+3y)^2}$$

$$3x^2 = \frac{(x+3y)\,(2(1) - dy/dx) - (2x-y)(1 + 3dy/dx}{(x+3y)^2}$$

$$3x^2 = \frac{(2 - dy/dx)(x+3y) - (2x-y)(1 + 3dy/dx)}{(x+3y)^2}$$

$$3x^2 = \frac{[2x + 6y - x\,dy/dx - 3y\,dy/dx] - [2x + 6x\,dy/dx - y - 3y\,dy/dx]}{(x+3y)^2}$$

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$$3x^2 = \frac{\cancel{2x} + 6y - x\frac{dy}{dx} - 3y\frac{dy}{dx} - \cancel{2x} + 6x\frac{dy}{dx} - y - 3y\frac{dy}{dx}}{(x+3y)^2}$$

$$3x^2 = \frac{6y - y - \frac{dy}{dx} - 3y\frac{dy}{dx} + 6\frac{dy}{dx} - 3y\frac{dy}{dx}}{(x+3y)^2}$$

$$3x^2 = \frac{5y - dy/dx\left[x + 3y - 6 + 3y\right]}{(x+3y)^2}$$

$$3x^2\left(x^2 + 3y^2 + 2(x)(3y)\right) = 5y - \frac{dy}{dx}\left[x + 6y - 6\right]$$

$$3x^4 + 9y^4 + 18x^3y = 5y - \frac{dy}{dx}\left[x + 6y - 6\right].$$

$$3x^4 + 9y^4 + 18x^3y - 5y = -\frac{dy}{dx}$$

On Product Rule.

Q10: $xy = \cot xy$.

$d/dx\,(xy) = d/dx\,(\cot(xy))$

$\left[x\,dy/dx + y\,d/d(1)\right] = -\operatorname{Cosec}^2 xy \left[\frac{d}{dx}\,xy\right]$.

$\left[x\,dy/dx + y\right] = -\operatorname{Cosec}^2(xy)\left[x\,dy/dx + y(1)\right]$.

$x\,dy/dx + y = -\operatorname{Cosec}^2(xy)\left[\frac{x\,dy}{dx} + y\right]$.

$x\frac{dy}{dx} + y = -x\operatorname{Cosec}^2(xy)\,dy/dx - y\operatorname{Cosec}^2(xy)$

$\frac{x\,dy}{dx} + x\operatorname{Cosec}^2(xy)\frac{dy}{dx} = -y - y\operatorname{Cosec}^2(xy)$

$dy/dx\left(x + x\operatorname{Cosec}^2(xy)\right) = -y - y\operatorname{Cosec}^2(xy)$

$$\frac{dy}{dx} = \frac{-y\,\cancel{(1 + \operatorname{Cosec}^2 xy)}}{x\,\cancel{(1 + \operatorname{Cosec}^2 xy)}}$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x}}$$

Q16: $r - 2(\theta)^{1/2} = \frac{3}{2}(\theta)^{2/3} + \frac{4}{3}(\theta)^{3/2}$.

$\frac{dr}{d\theta} - 2 \cdot \frac{1}{2}\left(\theta^{1/2-1} \cdot (1)\right) = \frac{3}{2} \cdot \frac{2}{3} \theta^{2/3-1}(1) + \frac{4}{3} \cdot \frac{3}{4} \theta^{3/4-1}(1).$

$\frac{dr}{d\theta} = \theta^{-1/3} + \theta^{-1/4} + \theta^{-1/2} = 1/\theta^{1/3} + 1/\theta^{1/4} + 1/\theta^{1/2}$

$\boxed{dr/d\theta = \frac{1}{\theta^{1/3}} + \frac{1}{\theta^{1/4}} + \frac{1}{\theta^{1/2}}}$.

Q17: $\sin(\theta r) = 1/2$.

$\cos(r\theta)\, d/d\theta\, (r\theta) = 0$

$r(1) + \theta(dr/d\theta) = 0/\cos(r\theta) = 0$

$\theta\,(dr/d\theta) = -r$

$\boxed{dr/d\theta = -r/\theta.}$

→ : Second Derivative:

Q#19. $x^2 + y^2 = 1.$

$2x + 2y\,dy/dx = 0$

$2y\,dy/dx = 0 - 2x$

$\boxed{dy/dx = 0 - 2x/2y.}$ 1st Derivative

$dy/dx = -\cancel{2}x/\cancel{2}y$

$y_1 = \frac{dy}{dx} = y'$

$y\,(dy/dx) = -x.$

$y\left(\frac{d^2y}{dx}\right) + dy/dx \cdot dy/dx = -1.$

$y\left(\frac{d^2y}{dy}\right) + (dy/dx)^2 = -1.$

$y\,(d^2y/dx^2) = -1 - (dy/dx)^2.$

$d^2y/dx^2 = \dfrac{-1 - \frac{x^2}{y^2}}{y}.$

$= \dfrac{-y^2 - x^2/y^2}{y}$

$= \dfrac{-y^2 - x^2}{y^3}$

$= \dfrac{-(y^2 + x^2)}{y^3}$ $\boxed{= \dfrac{-1}{y^3}.}$

Q20: $x^{2/3} + y^{2/3} = 1.$

$2/3\,(x)^{2/3-1} + 2/3(y)^{2/3-1} = 0$

$2/3\,[x^{-1/3} + y^{-1/3}\, dy/dx] = 0$

$x^{-1/3} + y^{-1/3}\dfrac{dy}{dx} = 0$ 1st Derivative.

$1/y^{1/3}\,(dy/dx) = -x^{1/3}.$

$\boxed{dy/dx = -y^{1/3}/x^{1/3}.}$

27---.

Q28: $xy + y^2 = 1$, $(0, -1)$

$[x\,dy/dx + y\cdot 1] + 2y\,dy/dx = 0$

$x\,dy/dx + y + 2y\,dy/dx = 0$

$y + dy/dx\,(x+2y) = 0$

$dy/dx\,(x+2y) = -y$

$\dfrac{dy}{dx}\Big|_{(0,-1)} = \dfrac{-y}{x+2y}$

$= \dfrac{-(-1)}{0+(2)(-1)}.$

$\boxed{= (1/-2)}$

dy/dx

Again Derivative.

$(x+2y)\,d^2y/dx^2 + (1+2(dy/dx))\,dy/dx = -dy/dx.$

$(x+2y)\,d^2y/dx^2 + (1+2(-1/2))(-1/2) = +1/2.$

Using $dy/dx\,\Big|_{(0,-1)} = -1/2$ and $x = 0,\ y = -1.$

$(0+2(-1))\dfrac{d^2y}{dx^2} + (1+2(-1/2))(-1/2) = -1/2.$

$$-2\frac{d^2y}{dx^2} + 0 = 1/2$$

$$\boxed{d^2y/dx^2 = -1/4.}$$

Q30. $(x^2+y^2)^2 = (x-y)^2$ at $(1,0)$ and $(1,-1)$.

$$d/dx\,(x^2+y^2)^2 = d/dx\,(x-y)^2.$$

$$2(x^2+y^2)\;d/dx\,(x^2+y^2) = 2(x-y)\;d/dx\,(x-y).$$

$$\cancel{2}(x^2+y^2)\;(2x+2y\,dy/dx) = \cancel{2}(x-y)\;(1-dy/dx).$$

$$(x^2+y^2)\;(2x+2y\,dy/dx)\,(x-y)\,(1-dy/dx) \text{ —— } ①$$

Using $(1,0)$ in Eq (1).

$$(1+0)\,(2(1)+0) = (1-0)\,(1-dy/dx).$$

$$2 = 1-dy/dx.\Big|_{(1,0)}.$$

$$dy/dx\Big|_{(1,0)} = 1-2 = -1.$$

Using $(1,-1)$ in Eq (1).

$$(1+1)\left(2-2\,dy/dx\Big|_{(1,-1)}\right) = (1+1)\left(1-dy/dx\Big|_{(1,-1)}\right)$$

$$2-1 = 2\,dy/dx\Big|_{(1,-1)} - dy/dx\Big|_{(1,-1)}.$$

$$1 = \frac{dy}{dx}\Big|_{(1,-1)}$$

Qno: Slope, Tangent and normal to the curve.

31. $x^2+xy-y^2=1$, $(2,3)$

Put $x=2$, $y=3$.

L.H.S: x^2+xy-y^2 R.H.S: 1.

$(2)^2+(2)(3)-(3)^2$ L.H.S = R.H.S.

$4+6-9$

$=1$

$\Rightarrow y-y_1=m(x-x_1)$, $(2,3)$.

Diff w.r.t. x.

$2x^{2-1}+y(1)+x\,dy/dx-2y^{2-1}dy/dx=0$

$2x+y+x\frac{dy}{dx}-2y\frac{dy}{dx}=0$

Put $(2,3)$.

$2(2)+3+2m-2(3)m=0$

$7-4m=0$, $7=4m$.

$\boxed{m=7/4}$

(a) Eq of tangent line

For $m=7/4$ and Point $(2,3)$.

$y-3=7/4(x-2)$.

$y = 7/4\,x - 7/4(2) + 3$

$\boxed{y = 7/4\,x - 1/2.}$

(b) Eq of normal line: For $m = -4/7$, $(2,3)$.

$y - y_1 = m(x - x_1).$

$y - 3 = -4/7\,(x-2).$

$y = -4/7\,x + 8/7 + 3.$

$\boxed{y = -4/7\,x + 29/7.}$

Q40: $x^2\cos^2 y - \sin y = 0$, $(0, \pi)$

Put $x = 0$, $y = \pi.$ $\pi = 180°$

L.H.S $x^2\cos^2 y - \sin y$

$(0)^2\cos^2(180) - \sin(180°)$

$(0)(-1) - 0$

$\boxed{0}$ L.H.S = R.H.S

$y - y_1 = m(x - x_1)$, $(0, \pi).$

$x^2\cos^2 y = 0$

Diff w.r.t. x.

$\frac{d}{dx}(x^2\cos^2 y) - \sin y = 0$

$$\left[x^2(2\cos y(-\sin y)\,dy/dx + 2x\cos^2 y\right] - \cos y\, dy/dx = 0$$

$$-2x^2\cos y \sin y\, dy/dx + 2x\cos^2 y - \cos y\, dy/dx = 0$$

$$(-2x^2\cos y \sin y - \cos y)\, dy/dx = -2x\cos^2 y.$$

$$\frac{dy}{dx} = \frac{-2x\cos^2 y}{-\cos y(2x^2\sin y + 1)}$$

$$\boxed{\frac{dy}{dx} = \frac{2x\cos y}{2x^2\sin y + 1}}.$$

Put $(0,\pi)$.

$$\left.\frac{dy}{dx}\right|_{(0,\pi)} = m = \frac{2(0)\cos\pi}{2(0)^2\sin\pi + 1} = \frac{0}{0+1} = 0.$$

$$y - y_1 = \cancel{m}^{0}(x - x_1)$$

$$y - \pi = 0$$

$$\boxed{y = \pi.}$$

$m = -1/0$, $(0,\pi)$.

$$y - y_1 = m(x - x_1)$$

$$y - \pi = -1/0\,(x - 0)$$

$$0 = -x + 0$$

$$\boxed{x = 0.}$$

$-4/7$, $(2,3)$.

$(0,\bar{\lambda})$

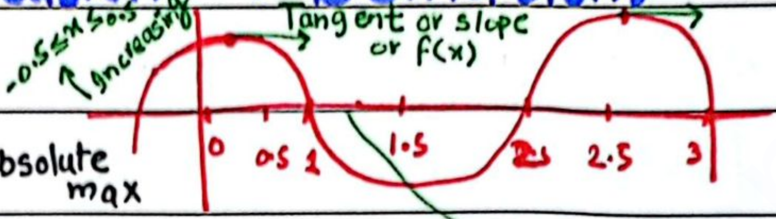
$=180°$

$=0$

Chap no 4: :Lec no 18: :26-Dec-2024:

→ :Application of Derivation:

-: **Extreme Value** :-



$m \le F(x) \le M$, Absolute max

For $x \in [a, b]$.

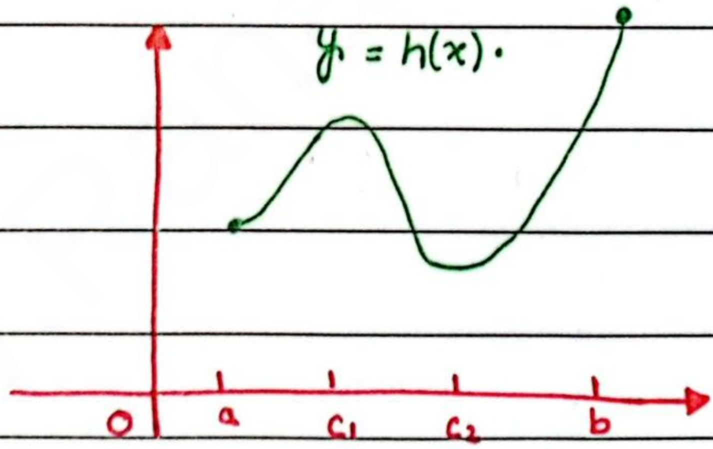
→ Absolute minimum.

Decreasing $0.5 \le x \le 1.5$.

: Ex 4.1 :

{Agr function increassing Sa ~~decrea~~ decreasing (local maximum) hoga} {Agr function decreasing Sa increasing hoto (local minimum) hoga}

Q1:



local maximum.

{7-20} Skip:

921-58P

Q no 21: $F(x) = \frac{2}{3}x - 5 , -2 \leq x \leq 3.$

$F(x) = \frac{2}{3}x - 5$

$F'(x) = \frac{2}{3}.$

$F'(x) = 0$

$2/3 \neq 0$ There is no Critical Point.

Agr function (0) ho aur (∞) is a critical Point hoga.

Q24: $F(x) = 4 - x^3 , -2 \leq x \leq 1.$

$F'(x) = 0 - 3x^2$

$F'(x) = -3x^2 , F'(x) = 0$

$-3x^2 = 0$

$\boxed{x = 0}$

(i) $F'(x) < 0 \rightarrow (a, c_1)$

$\Rightarrow$ Funct is decreassing

(ii) $F'(x) > 0 \rightarrow (c, b)$

$\Rightarrow$ Function is increassing.

$\Rightarrow (-2, 0) , (0, 1).$

(Ham na (-2,0) ~~ka~~ bich mai sa value leni ha)

$\Rightarrow (2,0).$

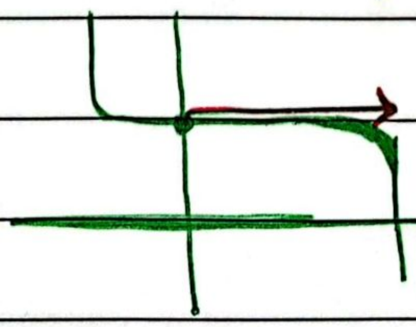
$F'(-1) = -3(-1)^2 = -3(1) = -3 \leq 0$

Function is decreassing in $(-2, 0)$

$\Rightarrow (0, 1)$

$F'(0.5) = -3(0.5)^2 < 0 .$

Function is decreassing in $(0, 1)$.



$F(x) = 4 - x^3$

$F(-2) = 4 - (-2)^3 = 4 + 8 \boxed{= 12}$

$F(1) = 4 - (1)^3 \boxed{= 3}$

F(x) has absolute maximum at $x = -2$ and absolute minimum at $x = 1$.

Q27. $F(x) = (x)^{1/3}, \; -1 \le x \le 1$.

$F'(x) = 1/3 (x)^{1/3 - 1} = 1/3 \, x^{-2/3}$

$= \frac{1}{3} \cdot \frac{1}{x^{2/3}}$.

$F'(x) = 0$.

$1/3x^{2/3} = 0$

$1/x^{2/3} = 0$

$x^{2/3} = 1/0 = \infty$

$\boxed{x = \infty}$

$F(-1) = -1$

$F(8) = (8)^{1/3} = ((2)^3)^{1/3} = 2$

Lec no 19: 27-12-2024.

Ex 4.1.

Qno35. $F(t) = 2 - |t|, \quad -1 \le t \le 3.$

$F(t) = 2 - |t|.$

$F(t) = |t|$

Let: $= \sqrt{t^2}$.

$F'(t) = 0 - \frac{d}{dt}(\sqrt{t^2}).$

$= -\frac{1}{\cancel{2}}(t^2)^{1/2-1} \cdot \cancel{2}(t)^{2-1}$

$= -(t^2)^{-1/2} \cdot t$

$= -t/(t^2)^{1/2}$ $\boxed{= -t/|t|.}$

$F'(t) = -\frac{t}{|t|}.$

Put $F'(t) = 0$

$t/|t| = 0$, $\boxed{t = 0.}$

$\rightarrow (-1, 0), (0, 3).$

$\Rightarrow (-1, 0)$

$F'(-0.5) = -(-0.5)/|(-0.5)| = 0.5/0.5$ $\boxed{= 1 > 0.}$

$\Rightarrow$ in $(-1, 0)$ Function is increassing.

$\Rightarrow (0, 3).$

$F'(1) = -1/|1|$ $\boxed{= -1 < 0.}$

$\Rightarrow$ in $(0,3)$ Function in decreassing.

$F(-1) = 2 - |-1| = 2 - 1$ $\boxed{= 1.}$

$F(0) = 2 - |0|$ $\boxed{= 2.}$

$F(3) = 2 - |3| = 2 - 3$ $\boxed{= -1}$

* at $t = 0$, absolute max and at $t = 3$ absolute min.

Q33: $g(x) = \operatorname{cosec} x$, $\pi/3 \le \theta \le 2\pi/3$.

$g'(x) = -\operatorname{cosec} x \cot x.$

Put $g'(x) = 0$.

$-\operatorname{cosec} x \cot x = 0$. $\Rightarrow$ $\operatorname{cosec} x \cot x = 0$

$\Rightarrow$ $\operatorname{cosec} x = 0$, $\Rightarrow \cot x = 0$.

$\frac{1}{\sin x} = 0$, $\frac{\sin x}{1} = \frac{1}{0} = \infty$ (Reciprocal).

$\boxed{\sin x = \infty}$

$\Rightarrow$ $\cot x = 0$

$\frac{1}{\tan x} = 0$, $\frac{\tan x}{1} = \frac{1}{0} = \infty$ (Reciprocal).

$\tan x = \frac{1}{0} = \infty$, $x = \tan^{-1}(\infty)$

$\boxed{x = \pi/2.}$

$\Rightarrow$ $(\pi/3, \pi/2)$, $(\pi/2, 2\pi/3)$.

$g'(80^\circ) = -\operatorname{cosec}(80^\circ)\cot(80^\circ)$

$= \frac{-1}{\sin 80^\circ \tan 80^\circ} < 0$

Function is dec in $(\pi/3, \pi/2)$.

$\Rightarrow \quad g'(100°) = -\operatorname{cosec}(100°) \cdot \operatorname{Cot}(100°) > 0$

Function is increasing in $(\pi/2, 2\pi/3)$.

$g(\pi/3) = \operatorname{cosec}(\pi/3) = \dfrac{1}{\sin 60°} \boxed{= \dfrac{2}{\sqrt{3}}} = 1.1.$

$g(\pi/2) = \operatorname{cosec}(\pi/2)$

$\boxed{= 1.}$

$g\left(\dfrac{2\pi}{3}\right) = \operatorname{cosec}\left(\dfrac{2\pi}{3}\right) \boxed{= \dfrac{2}{\sqrt{3}}.} = 1.1.$

absolute min $(\pi/2)$.

Q:37. $F(x) = x^{4/3}, \quad -1 \leq x \leq 8.$

$F'(x) = 4/3\, x^{1/3}.$

$F'(x) = 0, \quad 4/3\, x^{1/3} = 0.$

$4x^{1/3} = 0$

$x^{1/3} = 0$

$\Rightarrow (-1, 0).$

$F'(-0.5) = (-0.5)^{1/3} = -0.79.$

Function is decreassing at $(-1, 0)$.

$\Rightarrow (0, 8).$

$F'(7) = (7)^{1/3} = 1.91.$

Function is increassing at $(0, 8)$.

=> Function is $(-1, 8)$.

$F(-1) = x^{4/3}$.

$= (-1)^{4/3}$ $\boxed{= 1.}$

$F(0) = (0)^{4/3}$

$\boxed{= 0}$

$F(8) = (8)^{4/3}$ $\boxed{= 16.}$

Q: Finding Critical Function.

46 - $F(x) = \dfrac{x^2}{x-2}$.

$$f'(x) = \frac{(x-2)(2x) - (x^2)(1)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2}{(x-2)^2} \quad , \quad = \frac{x^2 - 4x}{(x-2)^2}$$

$(x-2)^2 = 0$, $\dfrac{x^2 - 4x}{(x-2)^2} = 0$

$x - 2 = 0$,

$\boxed{x = 2}$ $x(x-4) = 0$

, $\boxed{x = 0}$, $\boxed{x = 4}$

Critical Points $(2, 0, 4)$.

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So. $y = x - 3x^{2/3}$.

$y' = 1 - \cancel{3}\left(\frac{2}{\cancel{3}}\right)(x)^{2/3-1}$

$= 1 - 2x^{-1/3}$.

$= 1 - \frac{2}{x^{1/3}}$.

+ve >0 increasing

-ve <0 Decreasing.

Q# $y = \dfrac{x+1}{x^2+2x+2}$ [ExtremValues].

$$y' = \frac{(x^2+2x+2)\, d/dx(x+1) - (x+1)\, d/dx(x^2+2x+2)}{(x^2+2x+2)^2}$$

$$y' = \frac{(x^2+2x+2)(1) - (x+1)(2x+2(1))}{(x^2+2x+2)^2}$$

$$y' = \frac{(x^2+2x+2) - (x+1)(2x+2)}{(x^2+2x+2)^2}$$

$$y' = \frac{(x^2+2x+2) - (2x^2+2x+2x+2)}{(x^2+2x+2)^2}$$

$$= \frac{x^2+2x+2-2x^2-2x-2x-2}{(x^2+2x+2)^2}$$

$$= \frac{x^2+2x+\cancel{2}-2x^2-4x-\cancel{2}}{(x^2+2x+2)^2}$$

$$= \frac{x^2-2x}{(x^2+2x+2)^2}.$$

$$(x^2+2x+2)^2 = 0$$

$$x^2+2x+2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2ac}$$

$a = 1,\ b = 2$
$c = 2.$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)(2)}$$

$$= \frac{-2 \pm \sqrt{4-8}}{4} \qquad \boxed{= \frac{-2 \pm \sqrt{-4}}{4}.}$$

$\Rightarrow x^2 - 2x = 0$

$x(x-2) = 0$

$\boxed{x = 0},\ \boxed{x = 2.}$

Critical Point $(0,2)$.

:Lec no 20: 1-Jan-2025. Wednesday

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TRADE AND INVESTMENT FORUM 2020
BUILDING A PROSPEROUS FUTURE

* -: EXERCISE NO 4.2 :-

→ Find the value of c that Satisfy the equation.

$$\frac{F(b)-F(a)}{b-a} = f'(c).$$

(1-6).

1- $F(x) = x^2 + 2x + 1 \quad , [\overset{\nearrow a}{0}, \overset{\to b}{1}].$

$F(x) = -1$

$f(1) = 1+2-1 = 2$

$F'(x) = 2x + 2 \quad \Rightarrow F'(c) = 2c + 2.$

$$2c + 2 = \frac{2-(-1)}{1-0} = \frac{2+1}{1} \boxed{= 3.}$$

→ $2C = 3-2 \quad , \boxed{C = \frac{1}{2}.}$

4- $f(x) = \sqrt{x-1} \quad , [1, 3].$

$F(1) = 0.$

$F(3) = \sqrt{2}$

$$F'(x) = \frac{1}{2\sqrt{x-1}} \quad , \quad F'(c) = \frac{1}{2\sqrt{c-1}}$$

$$\frac{1}{2\sqrt{c-1}} = \frac{\sqrt{2}-0}{3-1}$$

$$\frac{1}{2\sqrt{c-1}} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{c-1}} = \sqrt{2}, \quad \frac{1}{\sqrt{2}} = \sqrt{c-1}.$$

$$\frac{1}{2} = c-1, \quad \frac{1}{2}+1 = c$$

$$\boxed{\frac{3}{2} = c.}$$

Ex 4.3: .Analyzing From Derivatives.

On (a,b).

$F'(x) > 0 \longrightarrow$ Increassing.

$F'(x) < 0 \longrightarrow$ Decreassing.

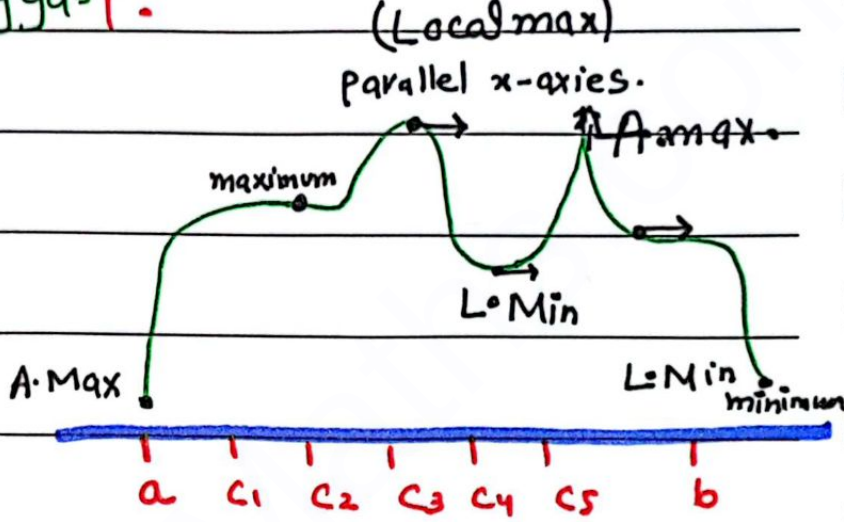
First derivative test.

There is a local.

(i) minimum b/w decreassing to increassing.

(ii) maximum b/w increassing to decreassing.

(iii) positive sa positive ho, Negative sa negative ho, Agr increasing sa decreasing hoto Local maximum ayga.



(a, c_1), (c_1, c_2), (c_2, c_3)

Critical point has Frst derivative $= 0$ or undefined value or infinity.

Q2: $f'(x) = \dfrac{x^2(x-1)}{(x+2)}$, $x \neq -2$.

$x + 2 = 0 \Rightarrow x = -2$

$$\frac{x^2(x-1)}{(x+2)} = 0.$$

$x^2 = 0$, $x - 1 = 0$

$\boxed{x = 0}$, $\boxed{x = 1}$

$(-\infty, -2)$, $(-2, 0)$, $(0, 1)$, $(1, \infty)$.

(15, 16, 17, 18) Graph Skiped.

$$F'(-3) = \frac{(-3)^2(-3-1)}{-3+2} = \frac{9(-4)}{-1} > 0 \rightarrow \text{Inc.}$$

$$F'(-1) = \frac{(-1)^2(-1-1)}{-1+2} = \frac{1(-2)}{1} = < 0 \rightarrow \text{Dec.}$$

$$f'(0.5) = \frac{(0.5)^2(0.5-1)}{0.5+2} = \frac{0.25(-0.5)}{2.5} < 0 \rightarrow \text{Dec.}$$

$$F'(2) = \frac{(2)^2(2-1)}{2+2} = \frac{4(1)}{4} = 1 > 0 \rightarrow \text{Inc.}$$

Local max at $x = -2$.
Local min at $x = 1$.

Point of Inflection: Jha per Second derivative zero ho ya not defined / indefined ho point of inflection hoga.

→ Canvave Down / up: Second Derivative Positive ho kisi bi point mai (Canvave up) hoga. And Second Derivative Negative hoga to (Canvave down) hoga.

$f'' > 0 \longrightarrow I \longrightarrow$ Cancave up

$f'' < 0 \longrightarrow I \longrightarrow$ Canvave Down.

→:Second Devivative test for local extreme Value:-

When $F'(c) = 0$.

i) If $f''(c) > 0$ then Local minimum at $x = c$.

ii) If $f''(c) < 0$ then Local maximum at $x = c$.

iii) If Point $F''(c) = 0$ ah jata ha to test fail hojata ha.

→ $F' < 0$ on I then Function is increassing.

→ $F' > 0$ on I then function is decreassing.

→: EXERCISE 4.4: (1 − 70)

1- $y = x^3/3 - x^2/2 - 2x + \frac{1}{3}$.

$y' = 3/3\,(x)^2 - \frac{2}{2}(x) - 2(1) = x^2 - x - 2$.

$\boxed{y'' = 2x - 1.}$

$y' = x^2 - x - 2 = 0$, $x^2 - 2x + x - 2 = 0$, $(x - 2)(x + 1) = 0$

$\boxed{x = 2}$, $\boxed{x = -1}$, → $y''(2) = 2(2) - 1 = 3 > 0$.

Find critical Point.

Local minimum at $x = 2$.

$y''(-1) = 2(-1) - 1 = -3 < 0$ Local max at $x = -1$.

Find inflection Points.

→ $y'' = 0 \Rightarrow 2x - 1 = 0$, $x = 1/2$ (increasing Point)

$(-\infty, 1/2)$, $(1/2, \infty)$.

$y'' = 0 = -1 < 0$

Cancave Down in $(-\infty, 1/2)$, $y''(1) = 2(1) - 1 = 1 > 0$ Cancave up in $(1/2, \infty)$

2- $y = x\left(x/2 - 5\right)^4$.

$y' = x \cdot 4\left(\frac{x}{2} - 5\right)^3 \cdot 1/2 + 1\left(x/2 - 5\right)^4$.

$y' = \left(x/2 - 5\right)^3 \left[2x + x/2 - 5\right]$ Common Liaha.

$y' = \left(x/2 - 5\right)^3 \left[\cancel{5x} \quad 5x - 10/2\right]$.

$y' = 5/2 \left[x/2 - 5\right]^3 (x-2)$

$y'' = 5/2\left[\left(x/2 - 5\right)^3 (1) + (x-2) \cdot 3 \cdot \left(x/2 - 5\right)^2 1/2\right]$.

$y'' = 5/2\left[\left(x/2 - 5\right)^2 \left(x/2 - 5 + 3/2\,(x-2)\right]\right.$.

$y'' = 5/2\left[x/2 - 5 + 3x/2 - 6/2\right]$.

$= 5/2\left[x - 10 + 3x - 6/2\right]\left(x/2 - 5\right)^2$

$= 5/4\left(x/2 - 5\right)^2 (4x - 16)$.

$= 5/\cancel{4} \cdot \cancel{4}\left(x/2 - 5\right)^2 (x - 4)$

$y' = 0$, $\Rightarrow 5/2\left(x/2 - 5\right)^3 (x-2) = 0$

$\left(x/2 - 5\right)^3 = 0$, $x - 2 = 0$

$x/2 = 5$, $x = 2$

$x = 10$, $x = 2$

$y''(10) = 5\left(10/2 - 5\right)^2 (10 - 4)$.

$= 0$ Test Fails.

$y'(9) = 5/2\left(9/2 - 5\right)^7 (9 - 2) < 0$ Decreasing

$y'(11) = 5/2\left(11/2 - 5\right)(11 - 2) > 0$ increasing.

$\Rightarrow$ Local mimum at $x=10$

$y''_{(x)} \Rightarrow 5(x/2-5)^2(x-4) < 0$

$y''(2) = 5(2/2-5)^2(2-4) < 0.$

$\Rightarrow$ Local max at $x=2$.

$\Rightarrow y''(x)=0$ (Point of inflection).

$5(x/2-5)^2(x-4)=0$

$x=10, \; x=4.$

$\Rightarrow y''(3) = 5(3/2-5)^3 \times (3-4) < 0$ (Negative

$\Rightarrow$ Cancave Down in $(-\infty,4)$.

Cancave Up. in $(4-\infty) > 0$ (Positive).

Ex 4.1 Question no 55.

$\rightarrow y = \begin{cases} 4-2x, & x \le 1 \\ x+1, & x > 1 \end{cases}$

For $x=1$

$y = 4-2x$

$y' = -2$

For $x>1$

$y = x+1$

$y' = 1$

$\Rightarrow$ No Critical Points.

When $x=1 \Rightarrow y(1) = 4(2)(1) = 4-2 = 2.$

$y(0) = 4$

$y(1) = 2$

$\boxed{y=2}.$

$y(2) = 2+1 \boxed{= 3}$

Absolute minima at $x=1$.

$$\rightarrow \quad y = \begin{cases} -1/4\, x^2 - 1/2\, x + 15/4 & , x \leq 1 \\ x^3 - 6x + 8x & , x > 1 \end{cases}$$

For $x \leq 1$, $\quad y = -1/4\, x^2 - 1/2\, x + 15/4$

$y' = -1/2\, x - 1/2.$

$y' = 0 \Rightarrow -1/2\,(x+1) = 0, \quad \boxed{x = -1}.$

For $x > 1 \Rightarrow y = x^3 - 6x^2 + 8x$

$y' = 0 \Rightarrow 3x^2 - 12x + 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$x = \frac{12 \pm \sqrt{144 - 12 \times 8}}{2(3)}$$

$\boxed{x = 3.15}$

$(-\infty, -1)$ → In, $(-1, 1)$ → Dec

$(1, 3.15)$ → Dec, $(3.15, \infty)$ → Inc.