Exercise 3.6

Sr. No	$y = f(x)$	$\frac{dy}{dx}$
1.	$y = \sinh x$ ✓	$\cosh x$
2.	$y = \cosh x$	$\sinh x$
3.	$y = \tanh x$	$\text{sech}^2 x$
4.	$y = \coth x$	$-\text{csch}^2 x$
5.	$y = \text{csch}\, x$	$-\text{csch}\, x\ .\coth x$
6.	$y = \text{sech}\, x$	$-\text{sech}\, x\ .\tanh x$
7.	$y = \sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}, -\infty < x < \infty$
8.	$y = \cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}},\ x > 1$
9.	$y = \tanh^{-1} x$	$\frac{1}{1-x^2}, \lvert x\rvert < 1$
10.	$y = \coth^{-1} x$	$\frac{1}{1-x^2}, \lvert x\rvert > 1$
11.	$y = \text{csch}^{-1} x$	$-\frac{1}{x\sqrt{1+x^2}}, x > 0$
12.	$y = \text{sech}^{-1} x$	$-\frac{1}{x\sqrt{1-x^2}}, 0 < x < 1$

Exercise # 3.6

Q1 Differentiate the following w.r.t. 'x'.

(i) Let $y = \sinh[\ln(x+3)]$

$$\frac{dy}{dx} = \frac{d}{dx}\sinh[\ln(x+3)]$$

$$= \cosh[\ln(x+3)] \cdot \frac{d}{dx}\ln(x+3)$$

$$= \cosh[\ln(x+3)] \cdot \frac{1}{x+3} \cdot \frac{d}{dx}(x+3)$$

$$= \cosh(\ln(x+3)) \cdot \frac{1}{x+3} \cdot 1$$

$$= \frac{\cosh[\ln(x+3)]}{x+3}.$$

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(ii) Let $y = \sinh(e^{3x})$

$(\sinh x)' = \cosh x$

$$\frac{dy}{dx} = \frac{d}{dx}\sinh(e^{3x})$$

$$= \cosh(e^{3x}) \cdot \frac{d}{dx}(e^{3x})$$

$(e^x)' = e^x$

$$= \cosh(e^{3x}) \cdot e^{3x}\frac{d}{dx}(3x)$$

$$= \cosh(e^{3x}) \cdot e^{3x} \cdot 3$$

$$= 3e^{3x}\cosh(e^{3x}).$$

(iii) Let $y = \cosh(2x^2 + 3x)$

$$\frac{dy}{dx} = \frac{d}{dx}\cosh(2x^2+3x)$$

$\frac{d}{dx}(\cosh x) = \sinh x$

$$= \sinh(2x^2+3x) \cdot \frac{d}{dx}(2x^2+3x)$$

$$= \sinh(2x^2+3x) \cdot (4x+3)$$

$$= (4x+3)\sinh(2x^2+3x).$$

(iv) Let $y = \dfrac{\tanh\sqrt{x}}{\sqrt{\cosh x}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\tanh\sqrt{x}}{\sqrt{\cosh x}}\right)$$

$$= \frac{\sqrt{\cosh x}\cdot\frac{d}{dx}\left(\tanh\sqrt{x}\right) - \tanh\sqrt{x}\,\frac{d}{dx}\left((\cosh x)^{1/2}\right)}{\left(\sqrt{\cosh x}\right)^2}$$

$(\tanh x)' = \operatorname{sech}^2 x$

$$= \frac{1}{\cosh x}\left[\sqrt{\cosh x}\ \operatorname{sech}^2\sqrt{x}\,\frac{d}{dx}\left(x^{1/2}\right) - \tanh\sqrt{x}\cdot\frac{1}{2}(\cosh x)^{\frac{1}{2}-1}\frac{d}{dx}(\cosh x)\right]$$

$$= \frac{1}{\cosh x}\left[\sqrt{\cosh x}\cdot\operatorname{sech}^2\sqrt{x}\cdot\frac{1}{2}\frac{1}{\sqrt{x}} - \tanh\sqrt{x}\cdot\frac{1}{2}\,\frac{1}{\sqrt{\cosh x}}\cdot\sinh x\right]$$

$$= \frac{1}{\cosh x}\left[\frac{\sqrt{\cosh x}\cdot\operatorname{sech}^2\sqrt{x}}{2\sqrt{x}} - \frac{\tanh\sqrt{x}\cdot\sinh x}{2\sqrt{\cosh x}}\right]$$

$$= \frac{1}{\cosh x}\left[\frac{\sqrt{\cosh x}\,\operatorname{sech}^2\sqrt{x}\,\sqrt{\cosh x} - \tanh\sqrt{x}\cdot\sinh x\,\sqrt{x}}{2\sqrt{x}\,\sqrt{\cosh x}}\right]$$

$$= \frac{\cosh x\cdot\operatorname{sech}^2\sqrt{x} - \sqrt{x}\,\sinh x\,\tanh\sqrt{x}}{2\sqrt{x}\,(\cosh x)^{3/2}}$$

(v) Let $y = \tan\left(e^{\sinh^{-1}x}\right)$

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(e^{\sinh^{-1}x}\right)$$

$$= \sec^2\left(e^{\sinh^{-1}x}\right)\cdot\frac{d}{dx}e^{\sinh^{-1}x}$$

$(e^x)' = e^x$

$$= \sec^2\left(e^{\sinh^{-1}x}\right)\cdot e^{\sinh^{-1}x}\cdot\frac{d}{dx}\left(\sinh^{-1}x\right)$$

$(\sinh^{-1}x)' = \dfrac{1}{\sqrt{x^2+1}}$

$$= \sec^2\left(e^{\sinh^{-1}x}\right)\cdot e^{\sinh^{-1}x}\cdot\frac{1}{\sqrt{x^2+1}}$$

$$= \frac{e^{\sinh^{-1}x}}{\sqrt{x^2+1}}\cdot\sec^2\left(e^{\sinh^{-1}x}\right).$$

(vi) Let $y = \dfrac{\sinh^{-1}x}{\operatorname{sech}^{-1}x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sinh^{-1}x}{\operatorname{sech}^{-1}x}\right)$$

$$= \frac{(\operatorname{sech}^{-1}x)\frac{d}{dx}(\sinh^{-1}x) - (\sinh^{-1}x)\frac{d}{dx}(\operatorname{sech}^{-1}x)}{(\operatorname{sech}^{-1}x)^2}$$

$$(\sinh^{-1}x)' = \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{1}{(\operatorname{sech}^{-1}x)^2}\left[(\operatorname{sech}^{-1}x)\cdot\frac{1}{\sqrt{x^2+1}} - \sinh^{-1}x\cdot\left(\frac{-1}{x\sqrt{1-x^2}}\right)\right]$$

$$(\operatorname{sech}^{-1}x)' = \frac{-1}{x\sqrt{1-x^2}}$$

$$= \frac{1}{(\operatorname{sech}^{-1}x)^2}\left[\frac{\operatorname{sech}^{-1}x}{\sqrt{1+x^2}} + \frac{\sinh^{-1}x}{x\sqrt{1-x^2}}\right]$$

$$= \frac{1}{(\operatorname{sech}^{-1}x)^2}\left[\frac{\operatorname{sech}^{-1}x\cdot x\sqrt{1-x^2} + \sinh^{-1}x\cdot\sqrt{1+x^2}}{x\sqrt{1+x^2}\sqrt{1-x^2}}\right]$$

$$= \frac{x\sqrt{1-x^2}\,\operatorname{sech}^{-1}x + \sqrt{1+x^2}\,\sinh^{-1}x}{x\sqrt{1-x^4}\,(\operatorname{sech}^{-1}x)^2}$$

(vii) Let $y = \cosh x \cdot \coth x^2$

$$\frac{dy}{dx} = \frac{d}{dx}(\cosh x\cdot\coth x^2)$$

$$= (\cosh x)\frac{d}{dx}(\coth x^2) + \coth x^2\cdot\frac{d}{dx}(\cosh x)$$

$$(\coth x)' = -\operatorname{cosech}^2 x$$

$$= \cosh x\cdot\left(-\operatorname{cosech}^2 x^2\right)\cdot\frac{d}{dx}(x^2) + \coth x^2\cdot\sinh x$$

$$= -2x\cosh x\operatorname{cosech}^2 x^2 + \coth x^2\cdot\sinh x.$$

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(viii) Let $y = \sinh x \cdot \tanh x^2$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sinh x \cdot \tanh x^2\right)$$

$$= (\sinh x)\frac{d}{dx}(\tanh x^2) + \tanh x^2 \cdot \frac{d}{dx}(\sinh x)$$

$$= (\sinh x)(\operatorname{sech}^2 x^2)\frac{d}{dx}(x^2) + \tanh x^2 \cdot \cosh x$$

$$= 2x \sinh x \operatorname{sech}^2 x^2 + \tanh x^2 \cosh x.$$

(ix) Let $y = \ln[\tanh(x^2+2x+1)]$

$(\ln f)' = \frac{1}{f} \cdot f'$

$$\frac{dy}{dx} = \frac{d}{dx}\ln[\tanh(x^2+2x+1)]$$

$$\frac{dy}{dx} = \frac{1}{\tanh(x^2+2x+1)} \cdot \frac{d}{dx}\left(\tanh(x^2+2x+1)\right)$$

$$= \coth(x^2+2x+1)\ \operatorname{sech}^2(x^2+2x+1)\frac{d}{dx}(x^2+2x+1)$$

$$= \frac{\cancel{\cosh(x^2+2x+1)}}{\sinh(x^2+2x+1)} \cdot \frac{1}{\cosh^{\cancel{2}}(x^2+2x+1)} \cdot (2x+2)$$

$$= 2(x+1)\operatorname{cosech}(x^2+2x+1)\operatorname{sech}(x^2+2x+1).$$

② Find $\frac{dy}{dx}$ for the following functions.

(i) $y = x\cosh^{-1}x - \sqrt{x^2-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x\cosh^{-1}x\right) - \frac{d}{dx}\left((x^2-1)^{1/2}\right)$$

$$= x\frac{d}{dx}(\cosh^{-1}x) + \cosh^{-1}x\,\frac{d}{dx}(x) - \frac{1}{2}(x^2-1)^{\frac{1}{2}-1}\frac{d}{dx}(x^2-1)$$

$$= x\frac{1}{\sqrt{x^2-1}} + \cosh^{-1}x - \frac{1}{\cancel{2}}(x^2-1)^{-1/2}(\cancel{2}x)$$

$$= \cancel{\frac{x}{\sqrt{x^2-1}}} + \cosh^{-1}x - \cancel{\frac{x}{\sqrt{x^2-1}}}$$

$$= \cosh^{-1}x.$$

(ii) $y = x\tanh^{-1}(3x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x\tanh^{-1}(3x)\right)$$

$$= x\frac{d}{dx}\left(\tanh^{-1}(3x)\right) + \tanh^{-1}(3x)\cdot\frac{d}{dx}(x)$$

$$= x\cdot\frac{1}{1-(3x)^2}\cdot\frac{d}{dx}(3x) + \tanh^{-1}(3x)\cdot 1$$

$$= \frac{3x}{1-9x^2} + \tanh^{-1}(3x)$$

$$= \frac{3x + (1-9x^2)\tanh^{-1}(3x)}{1-9x^2}$$

(iii) $\ln(\cosh^{-1}x) + \sinh^{-1}y = C$

$$\frac{d}{dx}\left[\ln(\cosh^{-1}x)\right] + \frac{d}{dx}(\sinh^{-1}y) = \frac{d}{dx}(C)$$

$$\frac{1}{\cosh^{-1}x}\cdot\frac{d}{dx}(\cosh^{-1}x) + \frac{1}{\sqrt{y^2+1}}\cdot\frac{dy}{dx} = 0$$

$$\frac{1}{\cosh^{-1}x}\cdot\frac{1}{\sqrt{x^2-1}} + \frac{1}{\sqrt{y^2+1}}\,\frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{y^2+1}}\,\frac{dy}{dx} = -\frac{1}{\sqrt{x^2-1}\,\cosh^{-1}x}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y^2+1}}{\sqrt{x^2-1}\,\cosh^{-1}x}.$$

(iv) $y = \ln(1-x^2) + 2x\tanh^{-1}x$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(1-x^2)\right] + 2\frac{d}{dx}\left[\underline{x}\,\underline{\tanh^{-1}x}\right]$$

$$= \frac{1}{1-x^2}\cdot\frac{d}{dx}(1-x^2) + 2\left[x\frac{d}{dx}(\tanh^{-1}x) + \tanh^{-1}x\cdot\frac{d}{dx}(x)\right]$$

$$= \frac{1}{1-x^2}(-2x) + 2\left[x\cdot\frac{1}{1-x^2} + \tanh^{-1}x\right]$$

$$= \cancel{\frac{-2x}{1-x^2}} + \cancel{\frac{2x}{1-x^2}} + 2\tanh^{-1}x$$

$$= 2\tanh^{-1}x.$$

$\cancel{\tan^{-1}}(\cancel{\tan} -)$

(v) $y = \tanh^{-1}(\tan x^3)$

$(\tanh^{-1} x)' = \frac{1}{1-x^2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\tanh^{-1}(\tan x^3)\right]$$

$$= \frac{1}{1-(\tan x^3)^2} \cdot \frac{d}{dx}(\tan x^3)$$

$$= \frac{1}{1-\tan^2 x^3} \cdot \sec^2 x^3 \cdot \frac{d}{dx}(x^3)$$

$$= \frac{1}{1-\tan^2 x^3} \cdot \sec^2 x^3 \cdot 3x^2$$

$$= \frac{3x^2 \sec^2 x^3}{1-\tan^2 x^3}.$$

(vi) $y = x \operatorname{sech}^{-1}(\sqrt{x})$

$(\operatorname{sech}^{-1} x)' = \frac{-1}{x\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x \operatorname{sech}^{-1}(\sqrt{x})\right)$$

$$= x \frac{d}{dx}\left(\operatorname{sech}^{-1}\sqrt{x}\right) + \operatorname{sech}^{-1}\sqrt{x} \cdot \frac{d}{dx}(x)$$

$$= x \cdot \frac{-1}{\sqrt{x}\sqrt{1-(\sqrt{x})^2}} \cdot \frac{d}{dx}\left(x^{1/2}\right) + \operatorname{sech}^{-1}\sqrt{x}$$

$$= \frac{-x}{\sqrt{x}\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} + \operatorname{sech}^{-1}\sqrt{x}$$

$$= \frac{-1}{2\sqrt{1-x}} + \operatorname{sech}^{-1}\sqrt{x}$$

$$= \frac{-1 + 2\sqrt{1-x}\operatorname{sech}^{-1}\sqrt{x}}{2\sqrt{1-x}}.$$