Important Formulas for Exercise 3.5

$\log_a x$

$a = 10$, common

$a = e$, natural

$\log_e x = \ln x.$

1- $\frac{d}{dx}(a^x) = a^x \ln a$, $a > 0$, $a \neq 1$

2- $\frac{d}{dx}(e^x) = e^x$

3- $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

4- $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$$\left(a^{f(x)}\right)' = a^{f(x)} \ln a \cdot f'(x).$$

$$\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x).$$

$$y = (f(x))^{g(x)}$$

$$\ln y = \ln (f(x))^{g(x)} = g(x) \cdot \ln (f(x)).$$

1) $\ln(ab) = \ln a + \ln b$

2) $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

3) $\ln a^m = m \ln a$

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Exercise 3.5

1. Differentiate the following w.r.t. 'x'.

(i) Let $y = x^2 + 2^x$

$\boxed{(a^x)' = a^x \ln a}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + 2^x\right)$$
$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(2^x)$$
$$= 2x + 2^x \ln 2.$$

(ii) Let $y = 4^x + 5^x$

$$\frac{dy}{dx} = \frac{d}{dx}\left(4^x + 5^x\right)$$
$$= \frac{d}{dx}(4^x) + \frac{d}{dx}(5^x)$$
$$= 4^x \ln 4 + 5^x \ln 5.$$

(iii) Let $y = e^{\tan x + \cot x}$

$\boxed{\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} \cdot f'(x)}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan x + \cot x}\right)$$
$$= e^{\tan x + \cot x} \cdot \frac{d}{dx}(\tan x + \cot x)$$
$$= e^{\tan x + \cot x} \cdot \left(\sec^2 x - \operatorname{cosec}^2 x\right)$$
$$= \left(\sec^2 x - \operatorname{cosec}^2 x\right) e^{\tan x + \cot x}.$$

(iv) Let $y = e^{\tan x^2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan x^2}\right)$$
$$= e^{\tan x^2} \cdot \frac{d}{dx}(\tan x^2)$$
$$= e^{\tan x^2} \cdot \sec^2 x^2 \cdot \frac{d}{dx}(x^2)$$
$$= e^{\tan x^2} \cdot \sec^2 x^2 \cdot 2x$$
$$= 2x\, e^{\tan x^2} \cdot \sec^2 x^2.$$

(v) Let

$$y = e^{2\ln(2x+1)}$$

$$y = e^{\ln(2x+1)^2}$$

$$\boxed{\ln a^m = m \ln a}$$

$$y = (2x+1)^2$$

$$y = 4x^2 + 1 + 4x$$

$$\frac{dy}{dx} = 8x + 0 + 4$$

$$= 4(2x+1).$$

(vi) Let

$$y = \log_{10} x$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log_{10} x\right)$$

$$\boxed{\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a}}$$

$$= \frac{1}{x \ln 10}$$

(vii) Let

$$y = \frac{e^x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)e^x - e^x(2x)}{(x^2+1)^2}$$

$$= \frac{e^x(x^2+1-2x)}{(x^2+1)^2}$$

$$= \frac{e^x(x-1)^2}{(x^2+1)^2} \quad \text{Ans.}$$

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(viii) Let $y = x^2 + 2^x + a^{2x}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + 2^x + a^{2x}\right)$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(2^x) + \frac{d}{dx}(a^{2x})$$

$\boxed{(a^x)' = a^x \ln a}$

$$= 2x + 2^x \ln 2 + a^{2x} \cdot \ln a \cdot \frac{d}{dx}(2x)$$

$$= 2x + 2^x \ln 2 + 2a^{2x} \ln a.$$

(ix) Let $y = (\ln x)^x$

$$\ln y = \ln (\ln x)^x$$

$$\ln y = x \ln (\ln x)$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}\left[\underline{x}\ \underline{\ln(\ln x)}\right]$$

$\boxed{[\ln f]' = \frac{1}{f} \cdot f'}$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \frac{d}{dx}[\ln(\ln x)] + \ln(\ln x) \frac{d}{dx}(x)$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{\ln x} \cdot \frac{d}{dx}(\ln x) + \ln(\ln x) \cdot 1$$

$$\frac{1}{y} \frac{dy}{dx} = \cancel{x} \cdot \frac{1}{\ln x} \cdot \frac{1}{\cancel{x}} + \ln(\ln x)$$

$$\frac{dy}{dx} = y\left[\frac{1}{\ln x} + \ln(\ln x)\right]$$

$$\frac{dy}{dx} = (\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x)\right]$$

(x) Let $y = \ln\left(\sqrt{e^{3x}+e^{-3x}}\right)$

$$y = \ln\left(e^{3x}+e^{-3x}\right)^{1/2}$$

$$y = \frac{1}{2}\ln\left(e^{3x}+e^{-3x}\right)$$

$$\boxed{(\ln f)' = \frac{1}{f}\cdot f'}$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left[\ln\left(e^{3x}+e^{-3x}\right)\right]$$

$$= \frac{1}{2}\,\frac{1}{e^{3x}+e^{-3x}}\cdot\frac{d}{dx}\left(e^{3x}+e^{-3x}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{e^{3x}+e^{-3x}}\cdot\left(3e^{3x}-3e^{-3x}\right)$$

$$= \frac{3}{2}\cdot\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$$

$$\boxed{\tanh x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}}$$

$$= \frac{3}{2}\cdot\tanh 3x.$$

(xi) Let $y = \ln\left(\sin(\ln x)\right)$

$$\boxed{(\ln f)' = \frac{1}{f}\cdot f'}$$

$$\boxed{(\sin f)' = (\cos f)\cdot f'}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln\left(\sin(\ln x)\right)\right]$$

$$= \frac{1}{\sin(\ln x)}\cdot\frac{d}{dx}\left(\sin(\ln x)\right)$$

$$= \frac{1}{\sin(\ln x)}\cdot\cos(\ln x)\cdot\frac{d}{dx}(\ln x)$$

$$= \frac{\cos(\ln x)}{\sin(\ln x)}\cdot\frac{1}{x} = \frac{\cot(\ln x)}{x}$$

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(xii) Let $y = \ln\left[\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right]$

$$\boxed{\frac{d}{dx}(\ln f) = \frac{1}{f}\cdot\frac{d}{dx}f}$$

$$\frac{dy}{dx} = \frac{d}{dx}\ln\left[\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\frac{d}{dx}\left[\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\sec^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\cdot\frac{d}{dx}\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

$$= \frac{\cancel{\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}}{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\frac{1}{\cos^{\cancel{2}}\left(\frac{x}{2}+\frac{\pi}{4}\right)}\cdot\frac{1}{2}$$

$$\boxed{2\sin\theta\cos\theta = \sin 2\theta}$$

$$= \frac{1}{2\sin\left(\underline{\frac{x}{2}+\frac{\pi}{4}}\right)\cos\left(\underline{\frac{x}{2}+\frac{\pi}{4}}\right)}$$

$$= \frac{1}{\sin 2\left(\frac{x}{2}+\frac{\pi}{4}\right)} = \frac{1}{\sin\left(x+\frac{\pi}{2}\right)}$$

$$= \frac{1}{\cos x} = \sec x$$

Ans

② Use logarithmic differentiation to find $\frac{dy}{dx}$ if

(i) $y = \sqrt{\frac{x^2-1}{x^2+1}}$

$$\ln y = \ln \left(\frac{x^2-1}{x^2+1}\right)^{1/2}$$

$$\ln y = \frac{1}{2} \ln \left(\frac{x^2-1}{x^2+1}\right)$$

$$\ln y = \frac{1}{2}\left[\ln (x^2-1) - \ln (x^2+1)\right]$$

$$\frac{d}{dx} \ln y = \frac{1}{2} \frac{d}{dx}\left[\ln (x^2-1) - \ln (x^2+1)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x^2-1}(2x) - \frac{1}{x^2+1}(2x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}}\left[\frac{1}{x^2-1} - \frac{1}{x^2+1}\right]$$

$$\frac{dy}{dx} = x\,y\left[\frac{\cancel{x^2}+1 - \cancel{x^2}+1}{(x^2-1)(x^2+1)}\right]$$

$$\frac{dy}{dx} = x\left(\frac{x^2-1}{x^2+1}\right)^{1/2}\left(\frac{2}{(x^2-1)(x^2+1)}\right)$$

$$= x\,\frac{(x^2-1)^{1/2}}{(x^2+1)^{1/2}} \cdot \frac{2}{(x^2-1)(x^2+1)}$$

$$= \frac{2x}{(x^2+1)^{\frac{1}{2}+1}(x^2-1)^{1-\frac{1}{2}}}$$

$a^{3/2} = a \cdot a^{1/2}$

$$= \frac{2x}{(x^2+1)^{3/2}(x^2-1)^{1/2}}$$

$$= \frac{2x}{(x^2+1)(x^2+1)^{1/2}(x^2-1)^{1/2}} = \frac{2x}{(x^2+1)\sqrt{x^4-1}}$$

Ans

(ii) $y = x^3\sqrt{x}$

$$\ln y = \ln\left(x^3\sqrt{x}\right)$$

$$\ln y = \ln x^3 + \ln x^{1/2}$$

$$\ln y = \ln x^3 + \frac{1}{2}\ln x$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\ln x^3 + \frac{1}{2}\frac{d}{dx}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cancel{x^3}}\cdot 3\cancel{x^2} + \frac{1}{2}\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = y\left[\frac{3}{x} + \frac{1}{2x}\right] = \cancel{x^3}^{\,2}\sqrt{x}\left[\frac{6+1}{2\cancel{x}}\right]$$

$$= \frac{7}{2}x^2\sqrt{x} = \frac{7}{2}x^{2+\frac{1}{2}} = \frac{7}{2}x^{5/2}$$

(iii) $y = x\,e^{\cos x}$

$$\ln y = \ln\left(x\,e^{\cos x}\right)$$

$$\ln y = \ln x + \ln e^{\cos x}$$

$$\ln y = \ln x + \cos x\,(\ln e)$$

$$\ln y = \ln x + \cos x$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}(\ln x) + \frac{d}{dx}(\cos x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} - \sin x$$

$$\frac{dy}{dx} = y\left[\frac{1}{x} - \sin x\right]$$

$$= x e^{\cos x}\left[\frac{1}{x} - \sin x\right]$$

$$= \cancel{x}\, e^{\cos x}\cdot\frac{1}{\cancel{x}} - x e^{\cos x}\sin x$$

$$= (1 - x\sin x)\,e^{\cos x}$$

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(iv)

$$y = e^{-2x}(x^2+2x+1)$$

$$\ln y = \ln\left[e^{-2x}(x^2+2x+1)\right]$$

$$\ln y = \ln e^{-2x} + \ln(x^2+2x+1)$$

$$\ln y = -2x(\ln e) + \ln(x^2+2x+1)$$

$$\ln y = -2x + \ln(x^2+2x+1)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(-2x) + \frac{d}{dx}\left(\ln(x^2+2x+1)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = -2 + \frac{1}{x^2+2x+1}(2x+2)$$

$$\frac{dy}{dx} = e^{-2x}(x^2+2x+1)\left[-2 + \frac{2x+2}{x^2+2x+1}\right]$$

$$= e^{-2x}\cancel{(x^2+2x+1)}\left[\frac{-2x^2-4x\cancel{-2}+2x\cancel{+2}}{\cancel{x^2+2x+1}}\right]$$

$$= e^{-2x}(-2x^2-2x)$$

$$= -2xe^{-2x}(x+1)$$

(v)

$$y = \ln\left(\frac{e^x}{1+e^x}\right)$$

$$y = \ln e^x - \ln(1+e^x)$$

$$y = x(\ln e) - \ln(1+e^x)$$

$$y = x - \ln(1+e^x)$$

$$\frac{dy}{dx} = \frac{d}{dx}(x) - \frac{d}{dx}\left(\ln(1+e^x)\right)$$

$$= 1 - \frac{1}{1+e^x}\frac{d}{dx}(1+e^x)$$

$$= 1 - \frac{e^x}{1+e^x} = \frac{1+\cancel{e^x}-\cancel{e^x}}{1+e^x}$$

$$= \frac{1}{1+e^x}$$

(vi) $y = \sqrt{\dfrac{1+e^x}{1-e^x}}$

$$\ln y = \ln\left(\frac{1+e^x}{1-e^x}\right)^{1/2}$$

$$\ln y = \frac{1}{2}\ln\left(\frac{1+e^x}{1-e^x}\right)$$

$$\ln y = \frac{1}{2}\left[\ln(1+e^x) - \ln(1-e^x)\right]$$

$$\frac{d}{dx}\ln y = \frac{1}{2}\left[\frac{d}{dx}\ln(1+e^x) - \frac{d}{dx}\ln(1-e^x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1+e^x}.e^x - \frac{1}{1-e^x}(-e^x)\right]$$

$$\frac{dy}{dx} = y \cdot \frac{e^x}{2}\left[\frac{1}{1+e^x} + \frac{1}{1-e^x}\right]$$

$$\frac{dy}{dx} = \left(\frac{1+e^x}{1-e^x}\right)^{1/2} \cdot \frac{e^x}{2}\left[\frac{1-\cancel{e^x}+1+\cancel{e^x}}{(1+e^x)(1-e^x)}\right]$$

$$\frac{dy}{dx} = \frac{(1+e^x)^{1/2}}{(1-e^x)^{1/2}} \cdot \frac{e^x}{\cancel{2}} \cdot \frac{\cancel{2}}{(1+e^x)(1-e^x)}$$

$$= \frac{e^x}{(1-e^x)^{\frac{1}{2}+1} \cdot (1+e^x)^{1-1/2}}$$

$$= \frac{e^x}{(1-e^x)^{3/2} \cdot (1+e^x)^{1/2}}$$

$$= \frac{e^x}{(1-e^x)(1-e^x)^{1/2}(1+e^x)^{1/2}} = \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$$

③ Find $\frac{dy}{dx}$ if

(i) $$y = \frac{1-x^2}{\sqrt{1+x^2}}$$

$$\ln y = \ln\left(\frac{1-x^2}{\sqrt{1+x^2}}\right) = \ln(1-x^2) - \ln(1+x^2)^{\frac{1}{2}}$$

$$\ln y = \ln(1-x^2) - \frac{1}{2}\ln(1+x^2)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\ln(1-x^2) - \frac{1}{2}\frac{d}{dx}\ln(1+x^2)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1-x^2}(-2x) - \frac{1}{\cancel{2}}\cdot\frac{1}{1+x^2}(\cancel{2}x)$$

$$\frac{dy}{dx} = y\left[\frac{-2x}{1-x^2} - \frac{x}{1+x^2}\right]$$

$$= \frac{\cancel{1-x^2}}{\sqrt{1+x^2}}\left[\frac{-2x(1+x^2) - x(1-x^2)}{\cancel{(1-x^2)}(1+x^2)}\right]$$

$$= \frac{-2x - 2x^3 - x + x^3}{(1+x^2)^{\frac{1}{2}+1}}$$

$$= \frac{-x^3 - 3x}{(1+x^2)^{3/2}}$$

(ii) $$y = \sqrt{\frac{1-x}{1+x}}$$

$$\ln y = \ln\left(\frac{1-x}{1+x}\right)^{1/2}$$

$$\ln y = \frac{1}{2}\left[\ln(1-x) - \ln(1+x)\right]$$

$$\frac{d}{dx}\ln y = \frac{1}{2}\left[\frac{d}{dx}\ln(1-x) - \frac{d}{dx}\ln(1+x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1-x}(-1) - \frac{1}{1+x}(1)\right]$$

$$\frac{dy}{dx} = y\cdot\frac{1}{2}\left[\frac{-1-\cancel{x}-1+\cancel{x}}{(1-x)(1+x)}\right]$$

$$\frac{dy}{dx} = \frac{(1-x)^{1/2}}{(1+x)^{1/2}}\cdot\frac{1}{\cancel{2}}\left[\frac{-\cancel{2}}{(1-x)(1+x)}\right]$$

$$= \frac{-1}{(1+x)^{1/2+1}(1-x)^{1-1/2}} = \frac{-1}{(1+x)^{3/2}(1-x)^{1/2}}.$$

(4) Find $\frac{dy}{dx}$ if

(i) $y = x^{\sin x}$

$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \cdot \ln x$$

$$\frac{d}{dx} \ln y = \frac{d}{dx}\left(\underline{\sin x} \cdot \underline{\ln x}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{d}{dx}(\ln x) + \ln x \cdot \frac{d}{dx}(\sin x)$$

$$\frac{dy}{dx} = y\left[\sin x \cdot \frac{1}{x} + \ln x \cdot \cos x\right]$$

$$= x^{\sin x}\left[\frac{\sin x}{x} + \cos x \cdot \ln x\right]$$

(ii) $y = (\sin^{-1} x)^{\ln x}$

$$\ln y = \ln (\sin^{-1} x)^{\ln x}$$

$$\ln y = \ln x \cdot \ln(\sin^{-1} x)$$

$$\frac{d}{dx} \ln y = \frac{d}{dx}\left[\underline{\ln x} \cdot \underline{\ln(\sin^{-1} x)}\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x \cdot \frac{d}{dx}\left(\ln(\sin^{-1} x)\right) + \ln(\sin^{-1} x) \cdot \frac{d}{dx}(\ln x)$$

$$\frac{dy}{dx} = y\left[\ln x \cdot \frac{1}{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} + \ln \sin^{-1} x \cdot \frac{1}{x}\right]$$

$$\frac{dy}{dx} = (\sin^{-1} x)^{\ln x}\left[\frac{\ln x}{\sqrt{1-x^2}\,\sin^{-1} x} + \frac{\ln(\sin^{-1} x)}{x}\right]$$

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(iii)

$$y = (\tan^{-1}x)^{\sin x + \cos x}$$

$$\ln y = \ln (\tan^{-1}x)^{\sin x + \cos x}$$

$$\ln y = (\sin x + \cos x) \ln (\tan^{-1}x)$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} \left[\underline{(\sin x + \cos x)}\ \underline{\ln (\tan^{-1}x)} \right]$$

$$\frac{1}{y} \frac{dy}{dx} = (\sin x + \cos x) \frac{d}{dx} \left(\ln (\tan^{-1}x) \right) + \ln (\tan^{-1}x) \cdot \frac{d}{dx} (\sin x + \cos x)$$

$$\frac{dy}{dx} = y \left[(\sin x + \cos x) \frac{1}{\tan^{-1}x} \cdot \frac{1}{1+x^2} + \ln (\tan^{-1}x) (\cos x - \sin x) \right]$$

$$\frac{dy}{dx} = (\tan^{-1}x)^{\sin x + \cos x} \left[\frac{\sin x + \cos x}{(1+x^2) \tan^{-1}x} + (\cos x - \sin x) \ln (\tan^{-1}x) \right]$$

(iv)

$$y = (\ln x)^{\cos x}$$

$$\ln y = \ln (\ln x)^{\cos x}$$

$$\ln y = \cos x \cdot \ln (\ln x)$$

$$\frac{d}{dx} (\ln y) = \frac{d}{dx} \left(\underline{\cos x} \cdot \underline{\ln (\ln x)} \right)$$

$$\frac{1}{y} \frac{dy}{dx} = \cos x \cdot \frac{d}{dx} \left(\ln (\ln x) \right) + \ln (\ln x) \cdot \frac{d}{dx} (\cos x)$$

$$\frac{dy}{dx} = y \left[\cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln (\ln x) \cdot (-\sin x) \right]$$

$$= (\ln x)^{\cos x} \left[\frac{\cos x}{x \ln x} - \sin x \cdot \ln (\ln x) \right]$$

(v) $y = x^x$

$$\ln y = \ln x^x = x \ln x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\underline{x}\,\underline{\ln x})$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\ln x) + \ln x \cdot \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = y\left[\cancel{x}\frac{1}{\cancel{x}} + \ln x \cdot 1\right]$$

$$\frac{dy}{dx} = x^x(1 + \ln x).$$

(vi) $y = \ln\left(\frac{\sqrt{x^2+1} - x}{\sqrt{x^2+1} + x}\right)$

$$y = \ln\left(\sqrt{x^2+1} - x\right) - \ln\left(\sqrt{x^2+1} + x\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln\left(\sqrt{x^2+1} - x\right)\right] - \frac{d}{dx}\left[\ln\left(\sqrt{x^2+1} + x\right)\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1} - x}\frac{d}{dx}\left((x^2+1)^{\frac{1}{2}} - x\right) - \frac{1}{\sqrt{x^2+1} + x}\cdot\frac{d}{dx}\left((x^2+1)^{\frac{1}{2}} + x\right)$$

$$= \frac{1}{\sqrt{x^2+1} - x}\left[\frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x) - 1\right] - \frac{1}{\sqrt{x^2+1} + x}\left[\frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x) + 1\right]$$

$$= \frac{1}{\sqrt{x^2+1} - x}\left[\frac{x}{\sqrt{x^2+1}} - 1\right] - \frac{1}{\sqrt{x^2+1} + x}\left[\frac{x}{\sqrt{x^2+1}} + 1\right]$$

$$= \frac{-1}{\cancel{\sqrt{x^2+1} - x}}\left(\frac{\cancel{-x + \sqrt{x^2+1}}}{\sqrt{x^2+1}}\right) - \frac{1}{\cancel{\sqrt{x^2+1} + x}}\left(\frac{\cancel{x + \sqrt{x^2+1}}}{\sqrt{x^2+1}}\right)$$

$$= \frac{-1}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{-2}{\sqrt{x^2+1}}$$

(5) Find $\frac{dy}{dx}$, when :

(i) $x^y \cdot y^x = 1$

$$y^x = \frac{1}{x^y} = x^{-y}$$

$$\ln y^x = \ln x^{-y}$$

$$x \ln y = -y \ln x$$

$$\frac{d}{dx}(x \ln y) = -\frac{d}{dx}(y \ln x)$$

$$x \frac{d}{dx}(\ln y) + \ln y \frac{d}{dx}(x) = -\left[y \frac{d}{dx}(\ln x) + \ln x \cdot \frac{dy}{dx} \right]$$

$$x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y = -y \frac{1}{x} - \ln x \frac{dy}{dx}$$

$$\frac{x}{y} \frac{dy}{dx} + \ln x \frac{dy}{dx} = -\frac{y}{x} - \ln y$$

$$\frac{dy}{dx}\left(\frac{x}{y} + \ln x \right) = -\left(\frac{y}{x} + \ln y \right)$$

$$\frac{dy}{dx}\left(\frac{x + y \ln x}{y} \right) = -\left(\frac{y + x \ln y}{x} \right)$$

$$\frac{dy}{dx} = -\frac{y}{x} \frac{(y + x \ln y)}{(x + y \ln x)}.$$

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(ii) $\ln(xy) = x^2 + y^2$

$$\ln x + \ln y = x^2 + y^2$$

$$\frac{d}{dx}\left(\ln x + \ln y\right) = \frac{d}{dx}\left(x^2 + y^2\right)$$

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 2x + 2y\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - \frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - 2y\right) = 2x - \frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{1-2y^2}{y}\right) = \frac{2x^2-1}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}\frac{(2x^2-1)}{(1-2y^2)} = \frac{-y(2x^2-1)}{x(2y^2-1)}$$

(iii) $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\cos^2 x}}\frac{d}{dx}(\cos x) - \frac{1}{\sqrt{1-\sin^2 x}} \cdot \frac{d}{dx}(\sin x)$$

$$= \frac{-\sin x}{\sqrt{\sin^2 x}} - \frac{\cos x}{\sqrt{\cos^2 x}}$$

$$= \frac{-\cancel{\sin x}^{\,1}}{\cancel{\sin x}} - \frac{{}^{1}\cancel{\cos x}}{\cancel{\cos x}}$$

$$= -1 - 1 = -2.$$

(iv) $y = x^y$

$$\ln y = \ln x^y = y \ln x$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}(\underline{y}\,\underline{\ln x})$$

$$\frac{1}{y}\frac{dy}{dx} = y\cdot\frac{1}{x} + \ln x\,.\,\frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} - \ln x\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \ln x\right) = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1-y\ln x}{y}\right) = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y^2}{x(1-y\ln x)}.$$

(v) $y = \cos x\,.\,\ln(\sin^{-1}x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\underline{\cos x}\,.\,\underline{\ln(\sin^{-1}x)}\right)$$

$$= \cos x\,.\,\frac{1}{\sin^{-1}x}\,.\,\frac{1}{\sqrt{1-x^2}} + \ln(\sin^{-1}x)\,.\,(-\sin x)$$

$$= \frac{\cos x}{\sqrt{1-x^2}\,\sin^{-1}x} - \sin x\ln(\sin^{-1}x).$$

For videos, visit YouTube

Suppose Math.

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(vi) $x^n\cdot y^n = a^n$

$$(xy)^n = a^n$$

$$xy = a$$

$$y = \frac{a}{x}$$

$$\frac{dy}{dx} = a\frac{d}{dx}(x^{-1}) = a(-x^{-2}) = -\frac{a}{x^2}.$$