Important Formulas for Exercise 3.3

$$f'(x) = \frac{df}{dx}$$

1- $$\left(c f(x)\right)' = c\left(f(x)\right)'$$

2- $$\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$$

3- $$\left(f(x)\, g(x)\right)' = f(x)\, g'(x) + g(x)\, f'(x)$$ (Product Rule).

4- $$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)\, f'(x) - f(x)\, g'(x)}{[g(x)]^2}$$ (Quotient Rule).

5- $$(x^n)' = n x^{n-1} \quad , \quad (x)' = 1 \quad , \quad (c)' = 0$$

6- $$[f(x)^n]' = n\, f(x)^{n-1} \cdot f'(x)$$

(Power Rule)

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Suppose Math.

0332-6297570

AKHTAR ABBAS.

Exercise 3.3

1. Differentiate the following w.r.t. 'x',

(i) $5x^5$

Let $y = 5x^5$

$$y' = (5x^5)'$$

$$y' = 5(x^5)'$$

$$= 5(5x^{5-1})$$

$$= 25x^4$$

(ii) $\frac{7}{9}x^9$

Let $y = \frac{7}{9}x^9$

$$y' = \left(\frac{7}{9}x^9\right)'$$

$$y' = \frac{7}{9}(x^9)' = \frac{7}{\cancel{9}}(\cancel{9}x^8)$$

$$= 7x^8$$

(iii) $-25x^{-3/5}$

Let $y = -25x^{-3/5}$

$$y' = \left(-25x^{-3/5}\right)'$$

$$= -25\left(x^{-3/5}\right)'$$

$(x^n)' = nx^{n-1}$

$$= -\cancel{25}^{\,5}\left(-\frac{3}{\cancel{5}}x^{-\frac{3}{5}-1}\right)$$

$$= 15x^{-8/5}$$

(iv) $124\sqrt{x}$

Let $y = 124x^{1/2}$

$$y' = (124x^{1/2})' = 124(x^{1/2})'$$

$$y' = \cancel{124}^{\,62}\left(\frac{1}{\cancel{2}}x^{\frac{1}{2}-1}\right) = 62x^{-1/2} = \frac{62}{\sqrt{x}}$$

(v) $\frac{1}{22}x^{22}$

Let $y = \frac{1}{22}x^{22}$

$$y' = \left(\frac{1}{22}x^{22}\right)' = \frac{1}{22}(x^{22})' = \frac{1}{\cancel{22}}(\cancel{22}x^{21})$$

$$y' = x^{21}$$

(vi) x^{-100}

Let $y = x^{-100}$

$$y' = (x^{-100})' = -100x^{-101}$$

(vii) $15\sqrt[3]{x}$

Let $y = 15x^{1/3}$

$$y' = (15x^{1/3})' = 15(x^{1/3})'$$

$$y' = \cancel{15}^{\,5}\left(\frac{1}{\cancel{3}}x^{\frac{1}{3}-1}\right) = 5x^{-2/3}$$

(viii) $16\sqrt[4]{x^{3/4}}$

Let $y = 16\left(x^{3/4}\right)^{\frac{1}{4}}$

$$y = 16x^{\frac{3}{16}}$$

$$y' = \left(16x^{\frac{3}{16}}\right)'$$

$$y' = 16\left(x^{\frac{3}{16}}\right)' = \cancel{16}\left(\frac{3}{\cancel{16}}x^{\frac{3}{16}-1}\right)$$

$$y' = 3x^{\frac{-13}{16}} = 3\left(x^{\frac{-13}{4}}\right)^{1/4}$$

$$y' = 3\sqrt[4]{x^{-\frac{13}{4}}}$$

(ix) $\frac{-4}{x^{3/4}}$

$$y = -4x^{-3/4}$$

$$y' = (-4x^{-3/4})' = -4(x^{-3/4})'$$

$$y' = -\cancel{4}\left(\frac{-3}{\cancel{4}}x^{-3/4-1}\right) = 3x^{\frac{-7}{4}}$$

$$= \frac{3}{x^{7/4}}$$

(x) $\frac{3}{\sqrt[3]{x^2}}$

Let $y = \frac{3}{x^{2/3}} = 3x^{-2/3}$

$$y' = (3x^{-2/3})' = \cancel{3}\left(-\frac{2}{\cancel{3}}x^{-\frac{2}{3}-1}\right) = -2x^{-5/3}$$

$$y' = \frac{-2}{x^{5/3}} = \frac{-2}{\sqrt[3]{x^5}}$$

(i) Let $y = \frac{x^5}{a^2+b^2} + \frac{x^2}{a^2-b^2}$

$$y' = \left(\frac{x^5}{a^2+b^2} + \frac{x^2}{a^2-b^2}\right)'$$

$$y' = \left(\frac{x^5}{a^2+b^2}\right)' + \left(\frac{x^2}{a^2-b^2}\right)'$$

$$y' = \frac{1}{a^2+b^2}(x^5)' + \frac{1}{a^2-b^2}(x^2)'$$

$$y' = \frac{1}{a^2+b^2}(5x^4) + \frac{1}{a^2-b^2}(2x)$$

$$y' = \frac{5x^4}{a^2+b^2} + \frac{2x}{a^2-b^2}.$$

(ii) Let $y = 2x + \frac{1}{2}x^6$

$$y' = \left(2x + \frac{1}{2}x^6\right)'$$

$$= (2x)' + \left(\frac{1}{2}x^6\right)'$$

$$= 2(1) + \frac{1}{\cancel{2}}(\cancel{6}^{3}x^5)$$

$$= 2 + 3x^5$$

(iii) Let $y = \sqrt[3]{x^2} + \sqrt{x}$

$$y = (x^2)^{1/3} + x^{1/2}$$

$$y = x^{\frac{2}{3}} + x^{\frac{1}{2}}$$

$$y' = \left(x^{\frac{2}{3}} + x^{\frac{1}{2}}\right)'$$

$$y' = \left(x^{\frac{2}{3}}\right)' + \left(x^{\frac{1}{2}}\right)'$$

$$= \frac{2}{3}x^{\frac{2}{3}-1} + \frac{1}{2}x^{\frac{1}{2}-1}$$

$$= \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{2}{3x^{1/3}} + \frac{1}{2\sqrt{x}}$$

$$= \frac{2}{3\sqrt[3]{x}} + \frac{1}{2\sqrt{x}} \quad \text{Ans}$$

(iv) Let $y = \frac{1}{21}x^{21} + \frac{1}{22}x^{22}$

$$y' = \left(\frac{1}{21}x^{21} + \frac{1}{22}x^{22}\right)'$$

$$= \left(\frac{1}{21}x^{21}\right)' + \left(\frac{1}{22}x^{22}\right)'$$

$$= \frac{1}{21}(x^{21})' + \frac{1}{22}(x^{22})'$$

$$= \frac{1}{\cancel{21}}(\cancel{21}x^{20}) + \frac{1}{\cancel{22}}(\cancel{22}x^{21})$$

$$= x^{20} + x^{21}$$

(v) Let $y = -\frac{5}{4}x^{-\frac{4}{5}} + \frac{2}{3}x^{-\frac{3}{2}}$

$$y' = \left(-\frac{5}{4}x^{-4/5} + \frac{2}{3}x^{-3/2}\right)'$$

$$y' = \left(-\frac{5}{4}x^{-4/5}\right)' + \left(\frac{2}{3}x^{-3/2}\right)'$$

$$= -\frac{\cancel{5}}{\cancel{4}}\left(-\frac{\cancel{4}}{\cancel{5}}x^{-\frac{4}{5}-1}\right) + \frac{\cancel{2}}{\cancel{3}}\left(-\frac{\cancel{3}}{\cancel{2}}x^{-\frac{3}{2}-1}\right)$$

$$= x^{-\frac{9}{5}} - x^{-\frac{5}{2}}$$

Ans

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Suppose Math.
0332-6297570
Iftikhar Abbas.

③ (i) Let $y = 2ax^3 - \frac{x^2}{b} + 6$

$y' = \left(2ax^3 - \frac{x^2}{b} + 6\right)'$

$y' = (2ax^3)' - \left(\frac{x^2}{b}\right)' + (6)'$

$y' = 2a\,(x^3)' - \frac{1}{b}(x^2)' + 0$

$y' = 2a\,(3x^2) - \frac{1}{b}(2x)$

$y' = 6ax^2 - \frac{2x}{b}.$

(ii) Let $y = x^3 - \frac{3}{7}x^{7/3}$

$y' = \left(x^3 - \frac{3}{7}x^{7/3}\right)'$

$y' = (x^3)' - \left(\frac{3}{7}x^{\frac{7}{3}}\right)'$

$= 3x^2 - \frac{\cancel{3}}{\cancel{7}}\left(\frac{\cancel{7}}{\cancel{3}}x^{\frac{7}{3}-1}\right)$

$= 3x^2 - x^{\frac{4}{3}}$

(iii) Let $y = 5x^{\frac{3}{5}} - \frac{1}{7}x^{\frac{7}{3}}$

$y' = \left(5x^{\frac{3}{5}} - \frac{1}{7}x^{\frac{7}{3}}\right)'$

$y' = \left(5x^{3/5}\right)' - \left(\frac{1}{7}x^{7/3}\right)'$

$y' = \cancel{5}\left(\frac{3}{\cancel{5}}x^{\frac{3}{5}-1}\right) - \frac{1}{\cancel{7}}\left(\frac{\cancel{7}}{3}x^{\frac{7}{3}-1}\right)$

$y' = 3x^{\frac{-2}{5}} - x^{\frac{4}{3}}$

Ans.

(iv) Let $y = x^{10} - 10x^{15}$

$y' = \left(x^{10} - 10x^{15}\right)'$

$= (x^{10})' - (10x^{15})'$

$= 10x^9 - 10x^{14}$

Ans.

(v) Let $y = 3\left(\sqrt[3]{x^2}\right) - 4\left(\sqrt[4]{x}\right)$

$y = 3\,(x^2)^{\frac{1}{3}} - 4\,(x)^{\frac{1}{4}}$

$y = 3x^{\frac{2}{3}} - 4x^{\frac{1}{4}}$

$y' = \left(3x^{\frac{2}{3}} - 4x^{\frac{1}{4}}\right)'$

$y' = \left(3x^{\frac{2}{3}}\right)' - \left(4x^{\frac{1}{4}}\right)'$

$y' = \cancel{3}\left(\frac{2}{\cancel{3}}x^{\frac{2}{3}-1}\right) - \cancel{4}\left(\frac{1}{\cancel{4}}x^{\frac{1}{4}-1}\right)$

$y' = 2x^{-1/3} - x^{-3/4}$

Ans.

(4) (i) $P(x) = x^3 - 3x^2 + 2x + 1$

$$p'(x) = (x^3 - 3x^2 + 2x + 1)'$$
$$= (x^3)' - (3x^2)' + (2x)' + (1)'$$
$$= 3x^2 - 3(2x) + 2(1) + 0$$
$$= 3x^2 - 6x + 2$$

(ii) $P(x) = x^4 - 3x^2 + 2x - 3$

$$p'(x) = (x^4 - 3x^2 + 2x - 3)'$$
$$= (x^4)' - (3x^2)' + (2x)' - (3)'$$
$$= 4x^3 - 3(2x) + 2(1) + 0$$
$$= 4x^3 - 6x + 2$$

(iii) $P(x) = x^6 - x^4 + x^3 + x$

$$p'(x) = (x^6 - x^4 + x^3 + x)'$$
$$= (x^6)' - (x^4)' + (x^3)' + (x)'$$
$$= 6x^5 - 4x^3 + 3x^2 + 1$$

(iv) $P(x) = 9x^9 + 7x^7 + \frac{1}{5}x^5 - \frac{1}{4}x^4 + x + 1$

$$p'(x) = \left(9x^9 + 7x^7 + \frac{1}{5}x^5 - \frac{1}{4}x^4 + x + 1\right)'$$
$$= (9x^9)' + (7x^7)' + \left(\frac{1}{5}x^5\right)' - \left(\frac{1}{4}x^4\right)' + x' + 1'$$
$$= 9(9x^8) + 7(7x^6) + \frac{1}{\cancel{5}}(\cancel{5}x^4) - \frac{1}{\cancel{4}}(\cancel{4}x^3) + 1 + 0$$
$$= 81x^8 + 49x^6 + x^4 - x^3 + 1$$

Ans

(v) $P(x) = x^3 + x^2 + x + 1$

$$p'(x) = (x^3 + x^2 + x + 1)'$$
$$= (x^3)' + (x^2)' + x' + 1'$$
$$= 3x^2 + 2x + 1 + 0$$
$$= 3x^2 + 2x + 1$$

Ans

5. Find the derivative using product rule.

$(fg)' = fg' + gf'$

(i) $h(x) = (2x-5)\cdot(5x+7)$

$$h'(x) = [(2x-5)(5x+7)]'$$
$$= (2x-5)(5x+7)' + (5x+7)(2x-5)'$$
$$= (2x-5)(5+0) + (5x+7)(2-0)$$
$$= (2x-5)(5) + (5x+7)(2)$$
$$= 10x - 25 + 10x + 14$$
$$= 20x - 11$$

Ans.

(ii) $h(x) = x \cdot \sqrt{3x^2+4}$

$$h'(x) = \left[x \cdot (3x^2+4)^{1/2}\right]'$$
$$= x \cdot \left[(3x^2+4)^{1/2}\right]' + (3x^2+4)^{\frac{1}{2}} \cdot (x)'$$

$(f(x)^n)' = n f(x)^{n-1} \cdot f'(x)$

$$= x \cdot \frac{1}{2}(3x^2+4)^{\frac{1}{2}-1} \cdot (3x^2+4)' + (3x^2+4)^{1/2} \cdot 1$$
$$= \frac{x}{\cancel{2}}(3x^2+4)^{-\frac{1}{2}} \cdot (\overset{3}{\cancel{6}}x) + (3x^2+4)^{\frac{1}{2}}$$
$$= \frac{3x^2}{\sqrt{3x^2+4}} + \sqrt{3x^2+4}$$
$$= \frac{3x^2 + 3x^2+4}{\sqrt{3x^2+4}}$$
$$= \frac{6x^2+4}{\sqrt{3x^2+4}}$$
$$= \frac{2(3x^2+2)}{\sqrt{3x^2+4}}$$

Ans.

(iii) $h(x) = \sqrt[3]{x+1} \cdot \sqrt[5]{x^2+1}$

$$h'(x) = \left[(x+1)^{\frac{1}{3}} \cdot (x^2+1)^{\frac{1}{5}} \right]'$$

$$= (x+1)^{\frac{1}{3}} \cdot \left[(x^2+1)^{\frac{1}{5}}\right]' + (x^2+1)^{\frac{1}{5}} \left[(x+1)^{\frac{1}{3}}\right]'$$

$$= (x+1)^{\frac{1}{3}} \frac{1}{5}(x^2+1)^{\frac{1}{5}-1} (x^2+1)' + (x^2+1)^{\frac{1}{5}} \cdot \frac{1}{3}(x+1)^{\frac{1}{3}-1} (x+1)'$$

$$= (x+1)^{\frac{1}{3}} \cdot \frac{1}{5} (x^2+1)^{-\frac{4}{5}} (2x) + (x^2+1)^{\frac{1}{5}} \cdot \frac{1}{3} (x+1)^{-\frac{2}{3}} (1)$$

$$= (x+1)^{\frac{1}{3}} \cdot \frac{1}{5} \cdot \frac{2x}{(x^2+1)^{4/5}} + (x^2+1)^{\frac{1}{5}} \cdot \frac{1}{3} \cdot \frac{1}{(x+1)^{2/3}}$$

$$= \frac{2x (x+1)^{1/3}}{5 (x^2+1)^{4/5}} + \frac{(x^2+1)^{\frac{1}{5}}}{3 (x+1)^{2/3}}$$

$$= \frac{6x(x+1)^{1/3+2/3} + 5 (x^2+1)^{\frac{1}{5}+\frac{4}{5}}}{15 (x^2+1)^{4/5} (x+1)^{2/3}}$$

$$= \frac{6x (x+1) + 5 (x^2+1)}{15 (x^2+1)^{4/5} (x+1)^{2/3}} = \frac{6x^2 + 6x + 5x^2 + 5}{15 (x^2+1)^{4/5} (x+1)^{2/3}}$$

$$= \frac{11x^2 + 6x + 5}{15 (x^2+1)^{4/5} (x+1)^{2/3}}$$

Ans.

(iv) $h(x) = x^2 (\sqrt{x} + 1)$

$$h'(x) = \left[x^2 \cdot (\sqrt{x} + 1)\right]' = \left[x^2 \cdot (x^{\frac{1}{2}} + 1)\right]'$$

$$h'(x) = x^2 \cdot (x^{\frac{1}{2}} + 1)' + (x^{\frac{1}{2}} + 1) (x^2)'$$

$$= x^2 \left(\frac{1}{2} x^{\frac{1}{2}-1} + 0\right) + (x^{\frac{1}{2}} + 1)(2x)$$

$$= x^2 \left(\frac{1}{2} x^{-\frac{1}{2}}\right) + (x^{\frac{1}{2}} + 1)\, 2x$$

$$= \frac{1}{2} x^{2-\frac{1}{2}} + 2x^{\frac{1}{2}+1} + 2x$$

$a^{\frac{3}{2}} = a\, a^{\frac{1}{2}} = a\sqrt{a}$

$$= \frac{1}{2} x^{\frac{3}{2}} + 2x^{\frac{3}{2}} + 2x$$

$$= x^{\frac{2}{3}} \left(\frac{1}{2} + 2\right) + 2x$$

$$= \frac{5}{2} x^{\frac{2}{3}} + 2x = \frac{5}{2} x\sqrt{x} + 2x$$

$$= \frac{x}{2} \left(5\sqrt{x} + 4\right)$$

Ans.

(v) $h(x) = (x+1)^3 \cdot x^{-3/2}$

$$h'(x) = \left[\underline{(x+1)^3} \cdot \underline{x^{-3/2}} \right]'$$

$$= (x+1)^3 \cdot \left(x^{-3/2}\right)' + x^{-3/2} \cdot \left[(x+1)^3\right]'$$

$$= (x+1)^3 \cdot \left(-\frac{3}{2} x^{-\frac{3}{2}-1}\right) + x^{-\frac{3}{2}} \left(3(x+1)^2 (x+1)'\right)$$

$$= -\frac{3}{2}(x+1)^3 x^{-\frac{5}{2}} + 3x^{-\frac{3}{2}}(x+1)^2$$

$x^{5/2} = x \cdot x^{3/2}$

$$= \frac{-3(x+1)^3}{2x \cdot x^{3/2}} + \frac{3(x+1)^2}{x^{3/2}}$$

$$= \frac{-3(x+1)^3 + 3(x+1)^2(2x)}{2x^{5/2}}$$

$$= \frac{3(x+1)^2\left[-(x+1) + 2x\right]}{2x^{5/2}}$$

$$= \frac{3(x+1)^2(-x-1+2x)}{2x^{5/2}}$$

$$= \frac{3(x+1)^2(x-1)}{2x^{5/2}}$$

Ans.

6. Find the derivative using quotient rule.

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

(i) $h(x) = \dfrac{3x+4}{2x-3}$

$$h'(x) = \left(\frac{3x+4}{2x-3}\right)'$$

$$= \frac{(2x-3)\cdot(3x+4)' - (3x+4)(2x-3)'}{(2x-3)^2}$$

$$= \frac{(2x-3)(3) - (3x+4)(2)}{(2x-3)^2}$$

$$= \frac{\cancel{6x} - 9 - \cancel{6x} - 8}{(2x-3)^2} = \frac{-17}{(2x-3)^2}$$

(ii) $h(x) = (x^2-1)(x^2+1)^{-1}$

$$h'(x) = \left(\frac{x^2-1}{x^2+1}\right)'$$

$$h'(x) = \frac{(x^2+1)(x^2-1)' - (x^2-1)(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

(iii) $h(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$$h'(x) = \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)'$$

$$= \frac{(x^2+x+1)(x^2-x+1)' - (x^2-x+1)(x^2+x+1)'}{(x^2+x+1)^2}$$

$$= \frac{(x^2+x+1)(2x-1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{1}{(x^2+x+1)^2}\left[2x^3 - x^2 + 2x^2 - x + 2x - 1 - (2x^3 + x^2 - 2x^2 - x + 2x + 1)\right]$$

$$= \frac{1}{(x^2+x+1)^2}\left[\cancel{2x^3} - \cancel{x^2} + \cancel{2x^2} - \cancel{x} + \cancel{2x} - 1 - \cancel{2x^3} - \cancel{x^2} + 2x^2 + \cancel{x} - \cancel{2x} - 1\right]$$

$$= \frac{1}{(x^2+x+1)^2}\left(2x^2 - 2\right) = \frac{2(x^2-1)}{(x^2+x+1)^2}$$

Ans.

(iv) $h(x) = \dfrac{2x^4}{b^2 - x^2}$

$$h'(x) = \left(\frac{2x^4}{b^2 - x^2}\right)'$$

$$= \frac{(b^2-x^2)(2x^4)' - 2x^4(b^2-x^2)'}{(b^2-x^2)^2}$$

$$= \frac{(b^2-x^2)(8x^3) - 2x^4(-2x)}{(b^2-x^2)^2}$$

$$= \frac{8b^2x^3 - 8x^5 + 4x^5}{(b^2-x^2)^2}$$

$$= \frac{8b^2x^3 - 4x^5}{(b^2-x^2)^2} = \frac{4x^3(2b^2 - x^2)}{(b^2-x^2)^2}$$

Ans.

(v)

$$h(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$$

$$h'(x) = \left(\frac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}-1}\right)'$$

$$h'(x) = \frac{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)' - (x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}-1)'}{(x^{\frac{1}{2}}-1)^2}$$

$$h'(x) = \frac{1}{(\sqrt{x}-1)^2}\left[(x^{\frac{1}{2}}-1)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - (x^{\frac{1}{2}}+1)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)\right]$$

$$= \frac{1}{(\sqrt{x}-1)^2}\left[(\sqrt{x}-1)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x}+1)\left(\frac{1}{2\sqrt{x}}\right)\right]$$

$$= \frac{1}{2\sqrt{x}(\sqrt{x}-1)^2}\left[\cancel{\sqrt{x}}-1-\cancel{\sqrt{x}}-1\right] = \frac{-\cancel{2}}{\cancel{2}\sqrt{x}(\sqrt{x}-1)^2}$$

$$= \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$ Ans.