# Important Points for Exercise 3.1

$y = f(x)$

- $y$: dependent variable
- $x$: independent variable

$$y = 3x^3 + 7x^2 - 8$$

$x$ - independent variable
$y$ - dependent variable.

If $x$ is changed from $x$ to $x + \delta x$.

$\delta = \Delta$

Then average rate of $y$ w.r.t. $x$

$$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}.$$

If $x$ is changed from $a$ to $b$.

$$\frac{\delta y}{\delta x} = \frac{f(b) - f(a)}{b - a}$$

# Exercise 3.1

1. Find the average rate of change of the following functions when $x$ varies from $a$ to $b$.
   (i) $y = f(x) = x^2 + 4;$ $a = 2, b = 2.3$
   (ii) $y = f(x) = x^3 - 4;$ $a = 2, b = 2.3$
   (iii) $y = f(x) = x^3 - 8;$ $a = 3, b = 2.5$

(i) $y = f(x) = x^2 + 4$, $a = 2$, $b = 2.3$

$$\frac{\delta y}{\delta x} = \frac{f(b) - f(a)}{b - a}$$

$$\frac{\delta y}{\delta x} = \frac{f(2.3) - f(2)}{2.3 - 2} = \frac{\left[(2.3)^2 + 4\right] - \left[2^2 + 4\right]}{0.3}$$

$$\frac{\delta y}{\delta x} = \frac{5.29 + 4 - 4 - 4}{0.3} = \frac{1.29}{0.3} = 4.3.$$

(ii) $y = f(x) = x^3 - 4$, $a = 2$, $b = 2.3$

$$\frac{\delta y}{\delta x} = \frac{f(b) - f(a)}{b - a}$$

$$\frac{\delta y}{\delta x} = \frac{f(2.3) - f(2)}{2.3 - 2}$$

$$\frac{\delta y}{\delta x} = \frac{\left[(2.3)^3 - 4\right] - \left[2^3 - 4\right]}{0.3} = \frac{12.167 - 4 - 8 + 4}{0.3}$$

$$\frac{\delta y}{\delta x} = \frac{4.167}{0.3} = 13.89$$

(iii) $y = f(x) = x^3 - 8$, $a = 3$, $b = 2.5$

$$\frac{\delta y}{\delta x} = \frac{f(b) - f(a)}{b - a} = \frac{f(2.5) - f(3)}{2.5 - 3}$$

$$= \frac{\left[(2.5)^3 - 8\right] - \left[3^3 - 8\right]}{-0.5} = \frac{15.625 - 8 - 27 + 8}{-0.5}$$

$$= \frac{+11.375}{+0.5} = 22.75.$$

2. Find out the average rate of change when $x$ changes from $a$ to $b$.

(i) $A = \pi x^2$, where $x$ is the radius of the ~~sphere~~ circle; $a = 3, b = 3.1$

(ii) $V = \frac{4}{3}\pi x^3$, where $x$ is the radius of the ~~circle~~ sphere. $a = 2, b = 1.9$

(i) $A(x) = \pi x^2$ $\quad a = 3, \quad b = 3.1$

Average rate of change

$$\frac{\delta A}{\delta x} = \frac{A(b) - A(a)}{b - a}$$

$$\frac{\delta A}{\delta x} = \frac{\pi(3.1)^2 - \pi(3)^2}{3.1 - 3}$$

$$\frac{\delta A}{\delta x} = \frac{\pi[9.61 - 9]}{0.1} = \frac{\pi(0.61)_{\times 10}}{1 \times 10}$$

$$= 6.1\pi.$$

(ii) $V(x) = \frac{4}{3}\pi x^3$, $\quad a = 2, \quad b = 1.9$

$$\frac{\delta V}{\delta x} = \frac{V(b) - V(a)}{b - a}$$

$$= \frac{\frac{4}{3}\pi(1.9)^3 - \frac{4}{3}\pi(2)^3}{1.9 - 2}$$

$$= \frac{\frac{4}{3}\pi[6.859 - 8]}{-0.1}$$

$$= \frac{\frac{4}{3}\pi(-1.141) \times 10}{-01 \times 100}$$

$$= \frac{4}{3}\pi(11.41)$$

$$\approx 15.213\ \pi$$

3. The price p in rupees after "t" years is given by $p(t) = 3t^2 + t + 1$. Find the average rate of change of inflation from $t = 3$ to $t = 3.5$ years.

$$p(t) = 3t^2 + t + 1 \qquad a = 3, \quad b = 3.5$$

$$\frac{\delta p(t)}{\delta t} = \frac{P(b) - p(a)}{b - a}$$

$$= \frac{P(3.5) - P(3)}{3.5 - 3}$$

$$= \frac{\left[3(3.5)^2 + 3.5 + 1\right] - \left[3(3)^2 + 3 + 1\right]}{0.5}$$

$$= \frac{3(12.25) + 3.5 + \cancel{1} - 27 - 3 - \cancel{1}}{0.5}$$

$$= 14.5$$

Ans.

4. A ball is thrown vertically up, its height in metres after $t$ seconds is given by the formula $h(t) = -16t^2 + 80t$. Find the average velocity when $t$ changes from $a$ to $b$.
(a) $a = 2, b = 2.1$ (b) $a = 2, b = 2.01$

$$h(t) = -16t^2 + 80t$$

(a) $a = 2, \quad b = 2.1$

$$\text{Average velocity} = \frac{\delta h}{\delta t} = \frac{h(b) - h(a)}{b - a}$$

$$= \frac{[-16(2.1)^2 + 80(2.1)] - [-16(2)^2 + 80(2)]}{2.1 - 2}$$

$$= \frac{-16(4.41) + 168 + 64 - 160}{0.1}$$

$$= 14.4.$$

(b) $a = 2, \quad b = 2.01$

$$\text{Average velocity} = \frac{\delta h}{\delta t} = \frac{h(b) - h(a)}{b - a} = \frac{[-16(2.01)^2 + 80(2.01)] - [-16(2)^2 + 80(2)]}{2.01 - 2}$$

$$= \frac{-16(4.0401) + 160.8 + 64 - 160}{0.01}$$

$$= 15.84.$$