Important limits in Exercise 2.3

① $\lim_{x\to\infty} \frac{\text{number}}{x} = 0 \Rightarrow \frac{\text{number}}{\infty} = 0$ $\qquad \lim_{x\to a} f(x) = L$

② $\lim_{x\to 0} \frac{\text{number}}{x} = \infty$

③ $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{x} = e$ $\qquad e \approx 2.718281\ldots$

④ $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

⑤ $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$, where $a>0$, $a \neq 1$.

⑥ $\lim_{x\to 0} \frac{(1+x)^n - 1}{x} = n$, where $n \in \mathbb{Q}$.

⑦ $\lim_{x\to 0} \frac{\sin x}{x} = 1$

L-Hospital Rule.

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ is indeterminate form $\left(\frac{0}{0}, \frac{\infty}{\infty}, 0^{\infty}, \infty^{0}, 0^{0}, 1^{\infty}\right)$ then we use L-Hospital Rule, i.e.,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

① $(k)' = 0$, $\quad (x^n)' = nx^{n-1}$, $\quad (x)' = 1$

$$\left(f(x)^n\right)' = n\, f(x)^{n-1} \cdot f'(x).$$

$a>0, a\neq 1$ $\quad (a^x)' = a^x \ln a$, $\quad (e^x)' = 1.$

$(\sin ax)' = a \cos ax$

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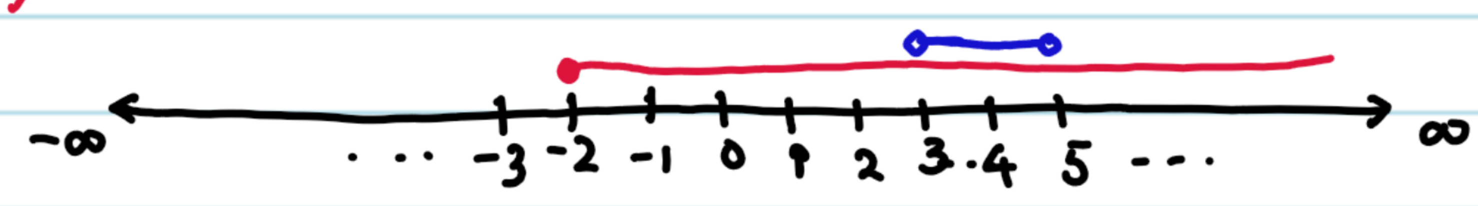
Exercise 2.3

Sindh Board.

Q # 01 Find the following:

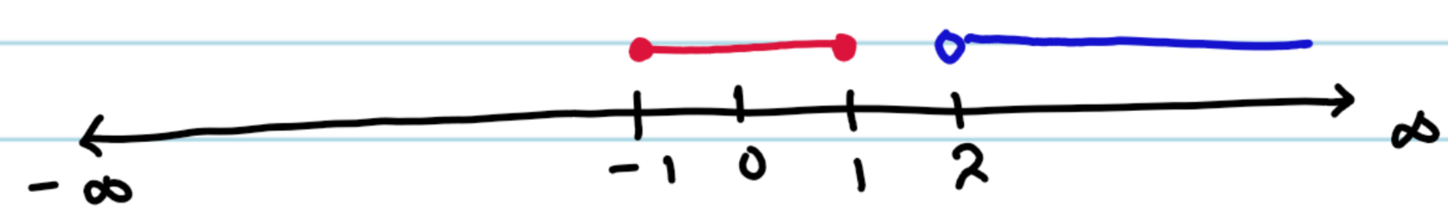
(i) $[2,\infty) \cup (3,5)$

$= [2,\infty)$



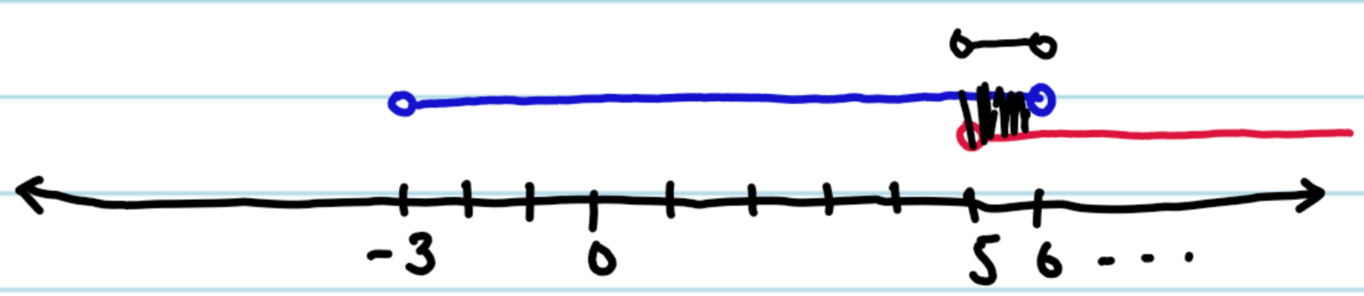
(ii) $[-1,1] - (2,\infty)$

$= [-1,1]$



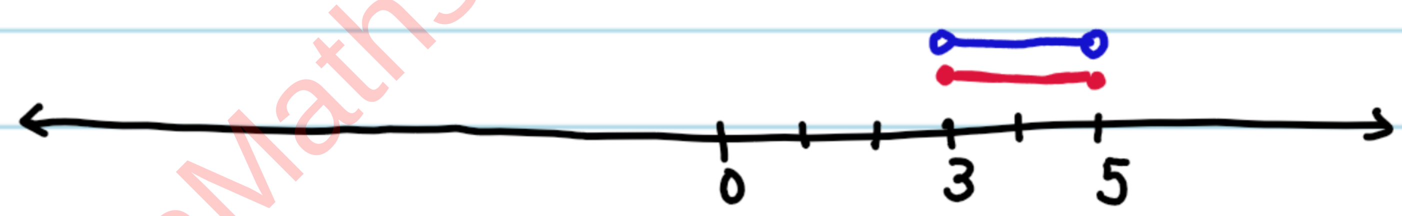
(iii) $(5,\infty) \cap (-3,6)$

$= (5,6)$



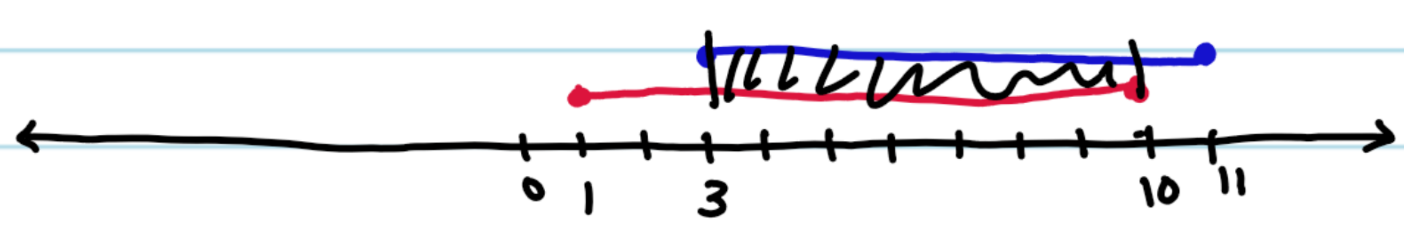
(iv) $[3,5] - (3,5)$

$= \{3, 5\}$



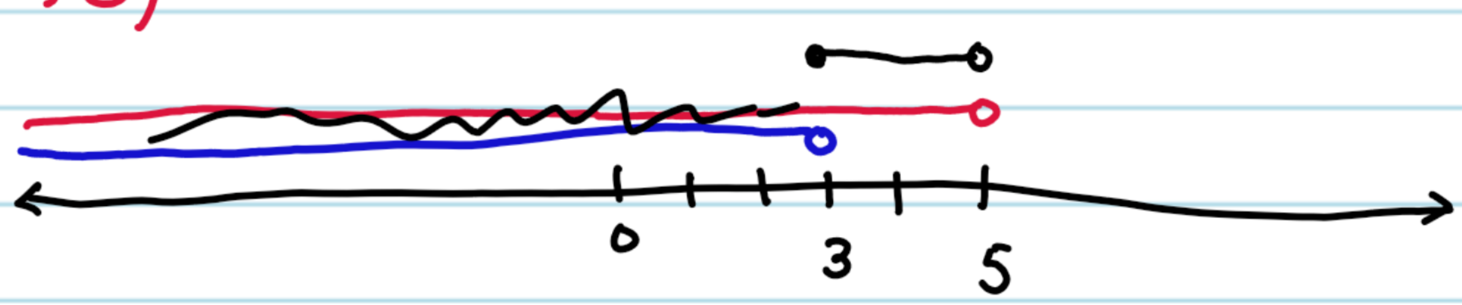
(v) $[1,10] \cap [3,11]$

$= [3, 10]$



(vi) $(-\infty,5) - (-\infty,3)$

$= [3, 5)$



Q # 02 Find the nth term and limit of the following sequences.

(i) $\frac{1}{2^0}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$

$$a_n = \frac{1}{2^{n-1}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2^{n-1}} = \frac{1}{2^{\infty-1}} = \frac{1}{2^\infty}$$

$$= \frac{1}{\infty} = 0$$

Ans.

(ii) $\frac{1\cdot 2}{3\cdot 4}, \frac{3\cdot 4}{5\cdot 6}, \frac{5\cdot 6}{7\cdot 8}, \ldots$

$1, 3, 5, \ldots$

$b_1 = 1$

$d = 2$

$b_n = b_1 + (n-1)d$

$= 1 + (n-1)2$

$= 1 + 2n - 2$

$= 2n - 1$

$$a_n = \frac{(2n-1)\cdot(2n)}{(2n+1)\cdot(2n+2)}$$

$$a_n = \frac{4n^2 - 2n}{4n^2 + 4n + 2n + 2}$$

$$a_n = \frac{4n^2 - 2n}{4n^2 + 6n + 2}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4n^2 - 2n}{4n^2 + 6n + 2}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{4n^2 - 2n}{n^2}\right)}{\left(\frac{4n^2 + 6n + 2}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{\left(4 - \frac{2}{n}\right)}{\left(4 + \frac{6}{n} + \frac{2}{n^2}\right)}$$

$$= \frac{4 - 0}{4 + 0 + 0} = \frac{4}{4} = 1$$

Ans.

Q # 03 Find the limit of the following sequences whose nth terms are

(i) $a_n = \dfrac{1+5n}{7n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1+5n}{7n}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{1+5n}{n}\right)}{\left(\frac{7n}{n}\right)} = \lim_{n\to\infty} \frac{\left(\frac{1}{n}+5\right)}{7}$$

$$= \frac{0+5}{7} = \frac{5}{7}$$

(ii) $a_n = \dfrac{(3n-1)(n^4-n)}{(n^2+5)(n^3-7)} = \dfrac{3n^5-3n^2-n^4+n}{n^5-7n^2+5n^3-35}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\left(\frac{3n^5-3n^2-n^4+n}{n^5}\right)}{\left(\frac{n^5-7n^2+5n^3-35}{n^5}\right)}$$

$$= \lim_{n\to\infty} \frac{\left(3-\frac{3}{n^3}-\frac{1}{n}+\frac{1}{n^4}\right)}{\left(1-\frac{7}{n^3}+\frac{5}{n^2}-\frac{35}{n^5}\right)} = \frac{3-0-0+0}{1-0+0-0}$$

$$= \frac{3}{1} = 3 \quad \text{Ans}$$

(iii) $a_n = \dfrac{(n+1)!}{n!-(n+1)!}$

$7! = 7\cdot 6!$

$(n+1)! = (n+1)\ n!$

$$a_n = \frac{(n+1)\,n!}{n!-(n+1)\,n!}$$

$$a_n = \frac{(n+1)\cancel{n!}}{\cancel{n!}[1-(n+1)]}$$

$$a_n = \frac{n+1}{\cancel{1}-n-\cancel{1}} = \frac{n+1}{n} = 1+\frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1+0 = 1 \quad \text{Ans}$$

Q # 04 Find the limit of the function

$$y = \frac{5x}{x+1} \quad \text{for} \quad x \to \infty.$$

$$\lim_{x \to \infty} y = \lim_{x \to \infty} \frac{5x}{x+1}$$

$$= \lim_{x \to \infty} \frac{\left(\frac{5\cancel{x}}{\cancel{x}}\right)}{\left(\frac{x+1}{x}\right)} = \lim_{x \to \infty} \frac{5}{\left(1 + \frac{1}{x}\right)}$$

$$= \frac{5}{1+0} = \frac{5}{1} = 5 \quad \text{Ans.}$$

Q # 05 Evaluate

(i)

polynomial.

$$\lim_{x \to 2} (x^5 + x^2 + x + 1)$$

$$= 2^5 + 2^2 + 2 + 1$$

$$= 32 + 4 + 2 + 1$$

$$= 39.$$

(ii)

$$\lim_{x \to 5} \left(\frac{1+x}{x^2} \right)$$

$$= \frac{1+5}{5^2}$$

$$= \frac{6}{25}.$$

(iii)

$$\lim_{x \to 1} [(2x^3 + 3x^2)(x+1)]$$

$$= (2(1)^3 + 3(1)^2)(1+1)$$

$$= (2+3)(2) = 10.$$

(iv)

$$\lim_{x \to 5} \{(x+1) - (x^2 + 2x + 3)\}$$

$$= (5+1) - (5^2 + 2(5) + 3)$$

$$= 6 - (25 + 10 + 3)$$

$$= 6 - 38$$

$$= -32.$$

(xi) $\lim_{x\to 0} \frac{17^x - 1}{x}$ $\qquad \lim_{x\to 0}\frac{a^x-1}{x} = \ln a$

$= \ln 17$

Ans.

(xii) $\lim_{h\to 0} \frac{(1+2h)^n - 1}{5h}$ $\qquad \lim_{x\to 0}\frac{(1+x)^n - 1}{x} = n.$

Let $x = 2h \Rightarrow h = \frac{x}{2}$

As $h\to 0$, $x \to 0$

$\lim_{x\to 0}\frac{(1+x)^n - 1}{5\frac{x}{2}} = \lim_{x\to 0}\frac{2[(1+x)^n - 1]}{5x}$

$= \frac{2}{5}\lim_{x\to 0}\frac{(1+x)^n - 1}{x} = \frac{2}{5}n$

Ans

(xiii) $\lim_{x\to 0}\frac{2^x - 4^x - 8^x - 1}{x+2}$ $\qquad \frac{2^0 - 4^0 - 8^0 - 1}{0+2} = \frac{1-1-1-1}{2}$

$= \frac{2^0 - 4^0 - 8^0 - 1}{0+2}$

$= \frac{\cancel{1} - \cancel{1} - 1 - 1}{2}$

$= \frac{-2}{2} = -1$

(xiv) $\lim_{x\to 0}\left(\frac{1+7x}{1-9x}\right)^{\frac{1}{x}}$

$(\ln f)' = \frac{1}{f}\cdot f'$

Let $y = \left(\frac{1+7x}{1-9x}\right)^{\frac{1}{x}}$

$\ln y = \ln\left(\frac{1+7x}{1-9x}\right)^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln\left(\frac{1+7x}{1-9x}\right) = \frac{1}{x}[\ln(1+7x) - \ln(1-9x)]$

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\left[\frac{\ln(1+7x)}{x} - \frac{\ln(1-9x)}{x}\right]$

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{\left(\frac{1}{1+7x}\cdot 7\right)}{1} - \lim_{x\to 0}\frac{\left(\frac{1}{1-9x}\cdot -9\right)}{1}$

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{7}{1+7x} - \lim_{x\to 0}\frac{-9}{1-9x}$

$\lim_{x\to 0}\ln y = \frac{7}{1+0} - \frac{-9}{1-0} = 7+9 = 16$

So $\lim_{x\to 0} y = e^{16}$

$\lim_{x\to 0}\left(\frac{1+7x}{1-9x}\right)^{\frac{1}{x}} = e^{16}$

Ans.

$(a^x)' = a^x \ln a$

(xv) $\lim_{x\to 0} \frac{a^x - b^x}{x}$ $\left(\frac{a^0 - b^0}{0} = \frac{0}{0}\right)$

Using L-Hospital Rule

$= \lim_{x\to 0} \frac{a^x \ln a - b^x \ln b}{1}$

$= \lim_{x\to 0} a^x \ln a - b^x \ln b$

$= a^0 \ln a - b^0 \ln b = \ln a - \ln b = \ln\left(\frac{a}{b}\right).$

$(e^{kx})' = ke^{kx}$

(xvi) $\lim_{x\to 0} \frac{e^{-2x} - e^{-11x}}{x}$ $\left(\frac{e^0 - e^0}{0} = \frac{0}{0}\right)$

Using L-Hospital Rule

$= \lim_{x\to 0} \frac{-2e^{-2x} - (-11e^{-11}x)}{1}$

$= \lim_{x\to 0} \left(-2e^{-2x} + 11e^{-11}x\right) = -2e^0 + 11e^0$

$= -2 + 11 = 9$ Ans

(xvii) $\lim_{x\to 0} \frac{3e^{-5x} - 5e^{-2x} + 2}{x}$ $\left(\frac{0}{0}\right)$

Use L-Hospital Rule

$= \lim_{x\to 0} \frac{3(-5e^{-5x}) - 5(-2e^{-2x}) + 0}{1}$

$= \lim_{x\to 0} -15e^{-5x} + 10e^{-2x}$

$= -15e^0 + 10e^0$

$= -15 + 10 = -5.$

(xviii) $\lim_{x\to 0} (1+3\tan x)^{\cot x}$

$(\ln f)' = \frac{1}{f} \cdot f'$

Let $y = (1+3\tan x)^{\cot x}$

$\ln y = \ln(1+3\tan x)^{\cot x}$

$\ln y = \cot x \ln(1+3\tan x)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln(1+3\tan x)}{\tan x}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\left(\frac{1}{1+3\tan x} \cdot 3\cancel{\sec^2 x}\right)}{\cancel{\sec^2 x}}$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{3}{1+3\tan x} = \frac{3}{1+0} = 3$

$\lim_{x\to 0} y = e^3$

So

$\lim_{x\to 0} (1+3\tan x)^{\cot x} = e^3$

Ans

Q # 07

(i) $\lim_{x\to 0} \dfrac{a\sin ax}{x}$

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

(x – radian)

$= a \lim_{x\to 0} \dfrac{\sin ax}{ax} \cdot a$

$= a^2 \left(\lim_{x\to 0} \dfrac{\sin ax}{ax} \right)$

$= a^2 (1) = a^2$ Ans.

(ii) $\lim_{x\to 0} \dfrac{\sin \sqrt{a}\, x}{\frac{x}{\sqrt{a}}}$

$= \lim_{x\to 0} \sqrt{a}\, \dfrac{\sin \sqrt{a}\, x}{\sqrt{a}\, x} \cdot \sqrt{a}$ $= \sqrt{a}\sqrt{a} \left(\lim_{x\to 0} \dfrac{\sin \sqrt{a}\, x}{\sqrt{a}\, x} \right)$

$= a\ (1)$

$= a$ Ans.

(iii) $\lim_{x\to 0} (3\cos x + 2\tan x)^3$

$= (3\cos 0 + 2\tan 0)^3$

$= (3 + 0)^3$

$= 3^3 = 27$ Ans

(iv) $\lim_{x\to 0} \dfrac{3\sin x - x^3}{2x}$

$= \lim_{x\to 0} \dfrac{3\sin x}{2x} - \lim_{x\to 0} \dfrac{x^3}{2x}$

$= \dfrac{3}{2} \left(\lim_{x\to 0} \dfrac{\sin x}{x} \right) - \lim_{x\to 0} \dfrac{x^2}{2}$

$= \dfrac{3}{2}(1) - \dfrac{0^2}{2}$

$= \dfrac{3}{2}$ Ans

OR

$= \lim_{x\to 0} \dfrac{3\cos x - 3x^2}{2}$

$= \dfrac{3\cos 0 - 0}{2}$

$= \dfrac{3}{2}$ Ans

OR

(v) $\lim_{x\to 0} \dfrac{\sin px}{\sin qx}$

$$= \lim_{x\to 0} \frac{\frac{\sin px}{px}\cdot p\cancel{x}}{\frac{\sin qx}{qx}\cdot q\cancel{x}}$$

$$= \frac{p}{q}\cdot \frac{\left(\lim_{x\to 0}\frac{\sin px}{px}\right)}{\left(\lim_{x\to 0}\frac{\sin qx}{qx}\right)} = \frac{p}{q}\frac{(1)}{(1)} = \frac{p}{q}.$$

$$= \lim_{x\to 0} \frac{p\cos px}{q\cos qx} = \frac{p\cos 0}{q\cos 0}$$

$$= \frac{p}{q}.$$

(vi) $\lim_{x\to 0} \dfrac{(2\pi - x)\sec(\pi - x)}{\pi/2}$

$$= \frac{(2\pi - 0)\sec(\pi - 0)}{\frac{\pi}{2}} = \frac{2\times 2\pi(-1)}{\pi}$$

$$= -\frac{4\cancel{\pi}}{\cancel{\pi}} = -4.$$

(vii) $\lim_{x\to 0} \dfrac{\sin x^\circ}{x}$

$1^\circ = \frac{\pi}{180}$ rad

$x^\circ = \frac{\pi}{180}x$ rad

$$= \lim_{x\to 0} \frac{\sin\left(\frac{\pi}{180}x\right)}{\frac{\pi}{180}x}\cdot\frac{\pi}{180}$$

$$= \frac{\pi}{180}\left(\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{180}x\right)}{\frac{\pi}{180}x}\right)$$

$$= \frac{\pi}{180}(1) = \frac{\pi}{180}.$$

$(\cos nx)'$

$= -n\sin nx$

(viii) $\lim_{x\to 0} \dfrac{1-\cos nx}{1-\cos mx}$ $\left(\frac{1-1}{1-1} = \frac{0}{0}\right)$

$$= \lim_{x\to 0} \frac{0 + n\sin nx}{0 + m\sin mx} = \lim_{x\to 0} \frac{n\sin nx}{m\sin mx} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{n\,n\cos nx}{m\,m\cos mx} = \frac{n^2\cos 0}{m^2\cos 0} = \frac{n^2}{m^2}$$ Ans

(ix) $\lim_{x \to 0} \frac{\sin 3x \sin 5x}{7x^2}$

$$= \lim_{x \to 0} \frac{\sin 3x}{7x} \cdot \lim_{x \to 0} \frac{\sin 5x}{x}$$

$$= \frac{1}{7}\left(\lim_{x \to 0} \frac{\sin 3x}{3x}\right)3 \cdot \left(\lim_{x \to 0} \frac{\sin 5x}{5x}\right) \cdot 5$$

$$= \frac{1}{7}(1)\,3 \cdot (1) \cdot 5$$

$$= \frac{15}{7}$$ Ans.

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Q # 08 (i) $\lim_{x\to 0} \dfrac{\sin^2\left(\frac{x}{2}\right)}{4x^2}$

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

$(x - \text{radian})$

$= \frac{1}{4} \lim_{x\to 0} \dfrac{\left(\sin \frac{x}{2}\right)^2}{\frac{x^2}{4} \times 4}$

$= \frac{1}{16} \lim_{x\to 0} \dfrac{\left(\sin \frac{x}{2}\right)^2}{\left(\frac{x}{2}\right)^2} = \frac{1}{16} \lim_{x\to 0} \left(\dfrac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2$

$= \frac{1}{16} \left[\lim_{x\to 0} \dfrac{\sin x/2}{x/2}\right]^2 = \frac{1}{16} (1)^2 = \frac{1}{16}.$

(ii) $\lim_{x\to\infty} x \sin \frac{1}{x}$

$= \lim_{x\to\infty} \dfrac{\left(\sin \frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$

Let $\frac{1}{x} = \theta$ ✓

As $x \to \infty$, $\theta \to 0$

$= \lim_{\theta\to 0} \dfrac{\sin\theta}{\theta}$

$= 1.$

(iii) $\lim_{x\to\infty} \left[\sqrt{x^2+x+1} - x\right]$ $(\infty - \infty)$

$= \lim_{x\to\infty} \sqrt{x^2+x+1} - x \times \dfrac{\sqrt{x^2+x+1} + x}{\sqrt{x^2+x+1} + x} = \lim_{x\to\infty} \dfrac{\left(\sqrt{x^2+x+1}\right)^{\cancel{2}} - x^2}{\sqrt{x^2+x+1} + x}$

$= \lim_{x\to\infty} \dfrac{\cancel{x^2} + x + 1 - \cancel{x^2}}{\sqrt{x^2+x+1} + x} = \lim_{x\to\infty} \dfrac{x+1}{\sqrt{x^2+x+1} + x}$

$= \lim_{x\to\infty} \dfrac{x\left(1+\frac{1}{x}\right)}{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + x} = \lim_{x\to\infty} \dfrac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\left[\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + 1\right]}$

$= \dfrac{1+0}{\sqrt{1+0+1} + 1} = \dfrac{1}{1+1} = \dfrac{1}{2}$ Ans.

(iv) $\lim_{x\to 0} \dfrac{1-\cos^3 x}{\sin^2 x}$ $\left(\dfrac{1-\cos^3 0}{\sin^2 0} = \dfrac{0}{0}\right)$

$(\cos^3 x)'$

$= 3\cos^2 x \cdot (-\sin x)$

$(\sin^2 x)'$

$= 2\sin x \cdot \cos x$

Use L-Hospital Rule

$= \lim_{x\to 0} \dfrac{0 - 3\cos^2 x(-\sin x)}{2\sin x \cos x}$

$= \lim_{x\to 0} \dfrac{3 \cancel{\sin x} \cos^{\cancel{2}} x}{2 \cancel{\sin x} \cancel{\cos x}} = \lim_{x\to 0} \frac{3}{2} \cos x = \frac{3}{2} \cos 0$

$= \frac{3}{2}$ Ans.

(v) $\lim_{x\to 0} \dfrac{a^x + a^{-x} - 2}{x^2}$ $\left(\dfrac{a^0 + a^0 - 2}{0} = \dfrac{0}{0}\right)$

Use L-Hospital Rule

$= \lim_{x\to 0} \dfrac{a^x \ln a + (-a^{-x}\ln a) - 0}{2x} = \lim_{x\to 0} \dfrac{a^x \ln a - a^{-x}\ln a}{2x}$ $\left(\dfrac{0}{0}\right)$

Use L-Hospital Rule

$= \lim_{x\to 0} \dfrac{a^x \ln a . \ln a - (-a^{-x}\ln a)\ln a}{2} = \lim_{x\to 0} \dfrac{a^x(\ln a)^2 + a^{-x}(\ln a)^2}{2}$

$= \dfrac{a^0(\ln 2)^2 + a^0(\ln 2)^2}{2} = \dfrac{\cancel{2}(\ln 2)^2}{\cancel{2}} = (\ln 2)^2$ Ans.

(vi) $\lim_{x\to 0} \dfrac{6^x - 3^x - 2^x + 1}{x^2}$

Use L-Hospital Rule $= \lim_{x\to 0} \dfrac{6^x \ln 6 - 3^x \ln 3 - 2^x \ln 2}{2x}$ $\left(\dfrac{\ln 6 - \ln 3 - \ln 2}{0}\right.$

Use L-Hospital Rule $= \lim_{x\to 0} \dfrac{6^x \ln 6 . \ln 6 - 3^x \ln 3 . \ln 3 - 2^x \ln 2 . \ln 2}{2}$ $= \dfrac{\ln 6 - (\ln 3 + \ln 2)}{0}$

$= \dfrac{6^0(\ln 6)^2 - 3^0(\ln 3)^2 - 2^0(\ln 2)^2}{2} = \dfrac{(\ln 6)^2 - (\ln 3)^2 - (\ln 2)^2}{2}$ $\left.= \dfrac{\ln 6 - \ln 6}{0} = \dfrac{0}{0}\right)$

(vii) $\lim_{x\to 3} \dfrac{\frac{x}{x+2} - \frac{3}{5}}{x-3}$ $\left(\dfrac{\frac{3}{5} - \frac{3}{5}}{3-3} = \dfrac{0}{0}\right)$

$= \lim_{x\to 3} \dfrac{\left(\frac{5x - 3x - 6}{5(x+2)}\right)}{x-3} = \lim_{x\to 3} \dfrac{2x - 6}{5(x+2)(x-3)}$

$= \lim_{x\to 3} \dfrac{2\cancel{(x-3)}}{5(x+2)\cancel{(x-3)}} = \dfrac{2}{5(3+2)} = \dfrac{2}{5(5)}$

$= \dfrac{2}{25}$ Ans.

(viii) $\lim_{y\to 4} \dfrac{y^{\frac{5}{2}} - 16y^{\frac{1}{2}}}{y-4}$ $\left(\dfrac{0}{0}\right)$

Use L-Hospital Rule

$= \lim_{y\to 4} \dfrac{\frac{5}{2}y^{\frac{5}{2}-1} - 16\left(\frac{1}{2}y^{\frac{1}{2}-1}\right)}{1-0} = \lim_{y\to 4}\left(\frac{5}{2}y^{\frac{3}{2}} - 8y^{-\frac{1}{2}}\right)$

$= \dfrac{5}{2}(4)^{\frac{3}{2}} - 8(4)^{-\frac{1}{2}} = \dfrac{5}{2}(2^2)^{\frac{3}{\cancel{2}}} - 8(2^2)^{-\frac{1}{\cancel{2}}} = \dfrac{5}{\cancel{2}}(\cancel{8})^{4} - \dfrac{\cancel{8}^{4}}{\cancel{2}}$

$= 20 - 4 = 16$ Ans.

(ix) $\lim_{x\to 1} \dfrac{\frac{1}{\sqrt{x}}-1}{1-x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 1} \dfrac{x^{-1/2}-1}{1-x}$

Use L-Hospital Rule

$= \lim_{x\to 1} \dfrac{-\frac{1}{2}x^{-\frac{1}{2}-1}-0}{0-1} = \lim_{x\to 1} \frac{1}{2}x^{-3/2} = \frac{1}{2}(1)^{-3/2} = \frac{1}{2}$ Ans

(x) $\lim_{x\to \pi} \dfrac{\sqrt{5+\cos x}-2}{\pi-x}$ $\left(\dfrac{\sqrt{5-1}-2}{\pi-\pi} = \dfrac{0}{0}\right)$

$= \lim_{x\to\pi} \dfrac{\frac{1}{2}(5+\cos x)^{\frac{1}{2}-1}(0-\sin x)-0}{0-1}$

$= \lim_{x\to\pi} -\frac{1}{2}(5+\cos x)^{-\frac{1}{2}}(-\sin x) = -\frac{1}{2}(5-1)^{-\frac{1}{2}}(-\sin\pi)$

$= -\frac{1}{2}(2^2)^{-1/2}(0) = 0$

(xi) $\lim_{x\to 1} x^{\frac{1}{x-1}}$

Let $y = x^{\frac{1}{x-1}}$

$\ln y = \ln x^{\frac{1}{x-1}} = \frac{1}{x-1}\ln x$

$\lim_{x\to 1}\ln y = \lim_{x\to 1}\dfrac{\ln x}{x-1}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 1}\ln y = \lim_{x\to 1}\dfrac{1/x}{1-0}$

$= \lim_{x\to 1}\frac{1}{x}$

$\lim_{x\to 1}\ln y = \frac{1}{1} = 1$

$\lim_{x\to 1} y = e^1$

So $\lim_{x\to 1} x^{\frac{1}{x-1}} = e$

(xii) $\lim_{x\to 1}\dfrac{x^2-\sqrt{x}}{\sqrt{x}-1}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 1}\dfrac{2x-\frac{1}{2}x^{\frac{1}{2}-1}}{\frac{1}{2}x^{\frac{1}{2}-1}-0} = \lim_{x\to 1}\dfrac{2x-\frac{1}{2}x^{-\frac{1}{2}}}{2\,x^{-\frac{1}{2}}}$

$= \dfrac{2(1)-\frac{1}{2}(1)^{-1/2}}{2(1)^{-1/2}} = \dfrac{2-\frac{1}{2}}{2} = \dfrac{\left(\frac{4-1}{2}\right)}{2}$

$= \dfrac{3}{4}$ Ans.

(xiii) $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sqrt{2} - \cos x - \sin x}{(4x-\pi)^2} \qquad \left(\dfrac{0}{0}\right)$

$$= \lim_{x \to \frac{\pi}{4}} \frac{0 + \sin x - \cos x}{2(4x-\pi)4} = \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{8(4x-\pi)} \qquad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x + \sin x}{8(4)} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{32} = \frac{1}{\underset{16}{\cancel{32}}}\left(\frac{\cancel{2}}{\sqrt{2}}\right)$$

$$= \frac{1}{16\sqrt{2}} \quad \text{Ans.}$$

(xiv) $\lim\limits_{x \to e} \dfrac{\ln x - 1}{x - e} \qquad \left(\dfrac{0}{0}\right)$

$$= \lim_{x \to e} \frac{1/x - 0}{1 - 0}$$

$$= \lim_{x \to e} \frac{1}{x} = \frac{1}{e} \quad \text{Ans.}$$

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