Course Outline

Limit and continuity;

→ Introduction to functions
→ Introduction to limits
→ Techniques of finding limits
→ Indeterminate form of limits
→ Continuous and discontinuous functions and their applications

Differential Calculus;

→ Concept and idea of differentiation
→ Geometrical and physical meaning of derivatives
→ Rules of differentiation
→ Techniques of differentiation
→ Rates of change, Tangents and Normal lines
→ Chain Rule → linear approximation
→ Implicit differentiation → Application of differentiation
→ Extreme value functions
→ Mean value theorems.
→ Maxima and minima of functions for single-variable
→ Concavity

Integral Calculus;

→ Concept and idea of integration
→ Indefinite integrals
→ Techniques of integration
→ Riemann sums and Definite integrals
→ Application of definite integrals
→ improper integrals
→ Applications of integration

Set of natural numbers $= N = \{1, 2, 3, 4, \ldots\}$

whole " $= W = \{0, 1, 2, 3, \ldots\}$

Integers " $= Z = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

Even " $= E = \{0, \pm 2, \pm 4, \ldots\}$

Even Numbers: The integers of the form $2n$ are called even numbers where $n \in Z$.

ODD Numbers: The integers of the form $2n+1$, where $n \in Z$.

$\Gamma = \{\pm 1, \pm 3, \pm 5, \ldots\}$

Positive Number: A number n is said to be +ve number if $n > 0$.

Negative Number: A number n is if $n < 0$.

Note: 0 is neither -ve nor +ve.

Prime Number: A number $p > 1$ is said to be prime if 1 and p are only its divisor.

$P = \{2, 3, 5, 7, 11, 13, \ldots\}$

Note: 2 is the only even prime number all other prime number are odd.

Non-negative integers $= \{0, 1, 2, 3, \ldots\}$

non-positive $= \{0, -1, -2, \ldots\}$

Rational number:

A number of the form p/q wher $p, q \in Z$ and $q \neq 0$

e.g $1 = \frac{1}{1}$, $2 = \frac{2}{1}$, $3/2$.

$0 = \frac{0}{1}$ $\therefore$ 0 is rational

$\frac{1}{0}$ not rational

Irrational number:

The ~~union~~ of numbers which not be written in the form of p/q wher $p, q \in Z$ and $q \neq 0$

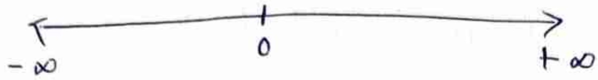
e.g: $\sqrt{2}$, $\sqrt{3}$, $\ldots$, π

Real Number:

The union of rational and irrational numbers is called set of real numbers.

$$R = Q \cup Q'$$
$$= \{0, \pm 1, \pm\sqrt{2}, \ldots\}$$

Note: Real line denote the set of real numbers.



Absolute value of a real number:

Let $x \in R$.

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Theorem:

Let $x, y \in R$, then

i. $|x| = 0$ iff $x = 0$

ii. $|-x| = |x|$ for all $x \in R$.

iii. $|xy| = |x| \cdot |y|$ $\forall\, x, y \in R$.

iv. if $a \geq 0$ then $|x| \leq a$ iff $-a \leq x \leq a$

v. $|x+y| \leq |x| + |y|$ $\forall\, x, y \in R$.

Intervals:

Open Interval: Let $a, b \in R$ and $a < b$. The set

$$(a, b) \text{ or }]a, b[= \{x \in R : a < x < b\}$$

is called open interval determined by a and b.

Closed Interval:

$$[a, b] = \{x \in R : a \leq x \leq b\}$$

Half Open / Half Closed:

$$]a, b] = \{x \in R : a < x \leq b\}$$
$$[a, b[= \{x \in R : a \leq x < b\}$$

Prove that $||a|-|b|| \le |a-b|$ for every $a,b \in R$.

Consider $|a| = |a-b+b|$

$\le |a-b| + |b|$

$|a|-|b| \le |a-b| \longrightarrow (1)$

now

$|b| = |b-a+a|$

$\le |b-a| + |a|$

$-|a|+|b| \le |-(a-b)|$

$-|a|+|b| \le |a-b|$

Multiplying both sides by $(-)$

$|a|-|b| \ge -|a-b| \longrightarrow (2)$

By combining (1) and (2)

$-|a-b| \le |a|-|b| \le |a-b| \quad \forall a,b \in R.$ $\quad$ $|x| \le a$ iff. $(\because -a \le x \le a)$

Q Express $3 < x < 7$ in modulus notation.

Subtracting 5

$3-5 < x-5 < 7-5$

$-2 < x-5 < 2$

$|x-5| < 2.$

Q If $\delta > 0$ and $a \in R$. Show that $a-\delta < x < a+\delta$.

iff $|x-a| < \delta$

Consider $a-\delta < x < a+\delta$

Subtracting a.

$a-\delta-a < x-a < a+\delta-a$

$-\delta < x-a < \delta$

Conversly

$$|x-a| < \delta$$

$$-\delta < x-a < \delta$$

Adding a.

$$a-\delta < x-a+a < \delta+a$$

$$a-\delta < x < \delta+a$$