Working Rules for Solution of inequalities.

Step I
convert the inequality into an eq. that is called associated eq.

Step II
Solve the associated eq. these solution is called boundary numbers of inequality.

Step III
Locate boundary numbers on the real line and real line divided into distinct region.

Step IV
Now check these regions by using orbitrary pt. (test pts) from the region.
The region whose test pts Satisfy the inequality are in the solution set.

Step V
Union of all those regions which belong to solution set makes the solution set of inequality.

3) Solve the inequalities. (Solutions should be in form of intervals)

Q $|2x+5| > |2-5x| \longrightarrow ①$

the associated eq.

$|2x+5| = |2-5x|$

$\pm(2x+5) = (2-5x)$

$2x+5 = 2-5x$; $-(2x+5) = 2-5x$

$2x+5x = 2-5$; $-2x-5 = 2-5x$

$7x = -3$; $-2x+5x = 2+5$

$\boxed{x = -3/7}$; $3x = 7$

$= -0.4$ $\boxed{x = 7/3}$

$-\infty$ $-3/7$ $7/3$ $+\infty$

Here possible intervals

$(-\infty, -3/7)$ $(-3/7, 7/3)$ $(7/3, +\infty)$

Check $(-\infty, -\frac{3}{7})$

Put $x = -1$ in eq ①

$|2(-1)+5| > |2-5(-1)|$

$|-2+5| > |2+5|$

$3 > 7$ false

Check $(-3/7, 7/3)$

Put $x = 0$ in eq ①

$|2(0)+5| > |2-5(0)|$

$|5| > |2|$ true.

$5 > 2$

Check $(7/3, +\infty)$

Put $x = 3$ in eq ①

$|2(3)+5| > |2-5(3)|$

$|11| > |-13|$ false.

$11 > 13$.

$S.S = (-3/7, 7/3)$

$\left|\frac{x+8}{12}\right| < \frac{x-1}{10}$

the associated eq is.

$\left|\frac{x+8}{12}\right| = \frac{x-1}{10}$

$\pm\left(\frac{x+8}{12}\right) = \frac{x-1}{10}$

$\frac{x+8}{12} = \frac{x-1}{10}$

$10(x+8) = 12(x-1)$

$10x+80 = 12x-12$

$80+12 = 12x-10x$

$92 = 2x$

$x = 46$

Now

$-\left(\frac{x+8}{12}\right) = \frac{x-1}{10}$

$-10(x+8) = 12(x-1)$

$-10x-80 = 12x-12$

$-10x-12x = -12+80$

$-22x = 68$

$x = -\frac{68}{22}$

$x = -\frac{34}{11}$ $= -3.\ldots$

$(-\infty, -\frac{34}{11})$ $-34/11$ $+46$ $+\infty$

$(-\infty, -\frac{34}{11})$ $(-\frac{34}{11}, 46)$ $(46, +\infty)$

$S.S = (46, +\infty)$

$|x| + |x-1| > 1$

the associated eq, is.

$|x| + |x-1| = 1$

$\pm(x) \pm (x-1) = 1$

Case I $\quad x + x - 1 = 1$

$2x = 2 \Rightarrow x = 1$

Case II

$-x - (x-1) = 1$

$-2x + 1 = 1 \Rightarrow x = 0$

Case III

$+x - (x-1) = 1$

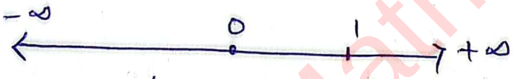
$x - x + 1 = 1$

$1 = 1$

Case IV

$-x + x - 1 = 1$

$-1 = 1$ not possible



Here possible intervals are

$(-\infty, 0)\ (0,1)\ (1, \infty)$

Check. $(-\infty, 0)$

put $x = -1$ in eq ①

$|-1| + |-1-1| > 1$

$1 + 2 > 1$

$3 > 1$ true

Check $(0,1)$

put $x = 0.5$ in eq ①

$|0.5| + |0.5 - 1| > 1$

$0.5 + 0.5 > 1$

$1 > 1$ false.

Check $(1, \infty)$

Put $x = 2$.

$|2| + |2-1| > 1$

$2 + 1 > 1 \Rightarrow 3 > 1$ true.

S.S $= (-\infty, 0) \cup (1, \infty)$.

Completeness property of R.

Upper Bound:

Let S be a non-empty subset of real numbers.
An element $M \in R$ is called an upper bound of S if
$x \leq M$ for all $x \in S$.

Lower Bound:

An element $m \in R$ is called lower bound of S if
$m \leq x$ for all $x \in S$.

Note:

→ If S is bounded above, then an upper bound M of S is called least upper bound. or <u>Supremum</u>. if it is less than any other upper bound of S. $M = \sup S$

→ If S is bounded below, then a lower bound m of S is called greatest lower bound. or <u>infimum</u>.

$$m = \inf S.$$

Example:

$S = \{1, 2, 3, \dots, 20\}$

$M \geq 20$ is an upper bound of S.

$m \leq 1$ is a lower bound of S.

$S = \{1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}\}$

upper bound is 1

Lower bound is 0.

Bounded set:

Let $S: N \to R$ be defined by $S_n = (-1)^n + (1)^n$ then S is bounded, since $\text{Rng } S = \{0, 2\}$ is bounded.

Unbounded set:

Let $f: R \to R$ be defined by $f(x) = x^2$

f is not bounded.

$\text{Rng } f$ has no upper bound so its unbounded set.

Binary Relation:

Let $A = \{2, 4, 6\}$ $B = \{1, 3\}$

Then Cartesian product $A \times B$ of A and B is

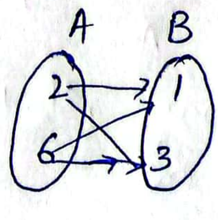
$A \times B = \{(2,1), (2,3), (4,1), (4,3), (6,1), (6,3)\}$

Then any subset of $A \times B$ is called B.R of $A \times B$.

e.g.

$R_1 = \{(2,1)(2,3), (6,1)(6,3)\}$

This is B.R of $A \times B$.



Function :

A function is defined as a relation between a set of inputs having one output each. (each input is related to exactly one output).

Every function has a domain and codomain or range.

A function is denoted by $f(x)$. where x is input.

General representation.

$$y = f(x).$$

Domain ;

The set of elements of A That occurs in the first position of member of f.

Range :

The set of element of B.

Into function: Let $f: A \rightarrow B$ be a function such that $Rng\ f = B$. then f is called onto or surjective function.

If $Rng\ f \neq B$. then f is called into function.

one-one function :(injective)

when there is mapping for a range for each domain between two sets

Bijective Function :

A function $f: A \rightarrow B$ which is both one-to-one and onto is called bijective function, or one-to-one correspondence.

Let A and B be any two non-empty sets. f is a binary relation from set A to B then f is called function from A to B if

i- $Dom f = A$

ii- In binary relation f every element of set A is attached only one

Identify Domain and Range of given functions.

Function	Domain	Range
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$

$y = x^2$ gives a real y-value for any real number x, so the domain is $(-\infty, \infty)$. The range is $[0, \infty)$ because square of any real number is non-negative.

⇒ $y = 1/x$ gives a real y-value for any real number x except $x = 0$. We cannot divide any number by zero. The range of $y = 1/x$, the set of all non-zero real number.

⇒ The formula $y = \sqrt{1-x^2}$ gives a real y-value for every x in the closed interval from -1 to 1. Outside this domain, $1-x^2$ is negative and its square root is not a real number. The values of $1-x^2$ vary from 0 to 1 on the given domain.
So the range of $\sqrt{1-x^2}$ is $[0, 1]$.

Definition:

Let $f: A \to B$ and $g: B \to C$ be two given functions. The composite function $g \circ f$ (or the composition $g \circ f$) is defined by the rule

$$(g \circ f)x = g(f(x))$$

The domain of $g \circ f$ is the set of all x in the domain of f for which $f(x)$ is in the domain of g.

Example:

Let f, g be functions on R to R. defined by

$f(x) = 2x$, $g(x) = 3x^2 - 1$, then

$$(g \circ f)x = g(f(x))$$
$$= 3(2x)^2 - 1 = 3(4x^2) - 1 = 12x^2 - 1$$

$$(f \circ g)x = f(g(x))$$
$$= f(3x^2 - 1) = 2(3x^2 - 1) = 6x^2 - 2$$

thus $f \circ g \neq g \circ f$

Further note that

$$(fg)(x) = f(x)\, g(x)$$
$$= 2x(3x^2 - 1) = 6x^3 - 2x$$

$$(gf)(x) = g(x) f(x)$$
$$= (3x^2 - 1)(2x) = 6x^3 - 2x$$

Thus $fg = gf$.

Monotone Sequences and functions:

A sequence $\{S_n\}$ of real number is said to be

i) nondecreasing if $S_n \leq S_{n+1}$

ii) non increasing if $S_n \geq S_{n+1}$

iii) increasing if $S_n < S_{n+1}$

iv) decreasing if $S_n > S_{n+1}$

for all $n \in N$.

These sequences are called monotone (or monotic).

A function $f : R \to R$ is said to be

nondecreasing if $x_1 < x_2$ implies $f(x_1) \leq f(x_2)$

non increasing if $x_1 > x_2$ implies $f(x_1) \geq f(x_2)$

for all $x_1, x_2 \in \text{Dom} f$.

increasing if $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

decreasing if $x_1 > x_2$ implies $f(x_1) > f(x_2)$.

for all $x_1, x_2 \in \text{Dom} f$.

Example:

The function $f : [0, \infty[\to R$ given by $f(x) = x^2$ is increasin

on $[0, \infty[$.

while $f :]-\infty, 0] \to R$ defined by $f(x) = x^2$

is decreasing on $]-\infty, 0]$.