27 October 2021 Wednesday

Scaler Quantity:

The quantity that have or posses only magnitude. like mass, work, force. etc.

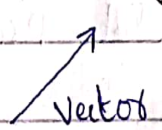
Vector Quantity:

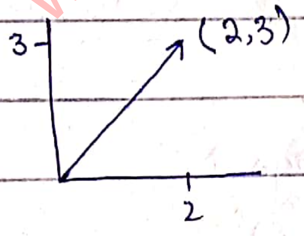
The Quantities that Posses both magnitude & direction are Called Vector Quantity.

like displacement, acceleration, velocity Weight etc.

Mathematical interprotation or representation of Vector Quantity:

$\vec{V}$, $\underline{V}$, $\mathbf{V}$





Parallel Vector:

Two Vectors are called parallel if and only if they have both positive Scaler Multiply.

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Scaler tive number (Multiply)

$\vec{u}, \vec{v}$

$\vec{\mu} = 3\vec{v}$

Same Direction: $\rightrightarrows$

Two vectors are parallel in the same direction make 0° angle.

Opposite Direction: $\rightleftarrows$

Two vectors are parallel in the opposite direction make 180° angle.

perpendicular : 90°

Two vectors are parallel in the perpendicular make 90° angle.

Orthognal :

If the dot product of two vectors are equal to zero then it is called orthognal.

$\vec{u} \cdot \vec{v} = 0$ orthognal

$\vec{u} = 2\hat{i} + 5\hat{j}$, $\vec{v} = 5\hat{i} - 2\hat{j}$

$\vec{u} \cdot \vec{v} = (2\hat{i} + 5\hat{j})(5\hat{i} - 2\hat{j})$ $\quad i \cdot i = 1$

$\vec{u} \cdot \vec{v} = (2 \cdot 5) - (5 \cdot 2)$ $\quad j \cdot j = 1$

$\vec{u} \cdot \vec{v} = 10 - 10$

$\vec{u} \cdot \vec{v} = 0$ Ans

These two vectors are orthognal.

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Positive Vector

if $\vec{u} = 2\hat{i} + 3\hat{j}$, $\vec{v} = 4\hat{i} + 6\hat{j}$

$$\vec{v} = 4\hat{i} + 6\hat{j}$$

$$\vec{v} = 2(2\hat{i} + 3\hat{j})$$

$$\vec{v} = 2(\vec{u})$$

These two vectors are parallel.

Negative Vector:

Suppose: $\vec{u} = 2\hat{i} + 3\hat{j}$, $\vec{v} = -4\hat{i} + 6\hat{j}$

$$\vec{v} = -4\hat{i} + 6\hat{j}$$

$$\vec{v} = -2(2\hat{i} + 3\hat{j})$$

$$\vec{v} = -2(\vec{u})$$

Negative vector.

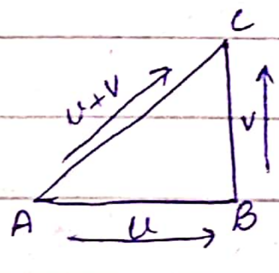
Addition of Vector:

1) Triangular
2) Parallelogram

i) Triangular:

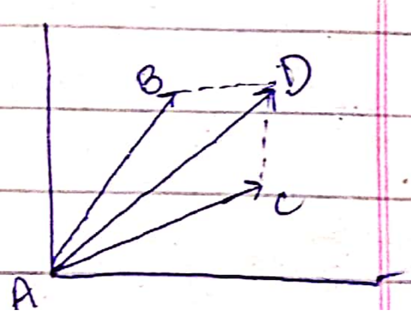
$\vec{AB} + \vec{BC} = \vec{AC}$

$u + v = \vec{AC}$



ii) Parallelogram:

$\vec{AB} + \vec{AC} = \vec{AD}$



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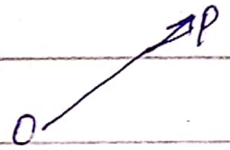
Subtraction of Vector:

Position Vector:

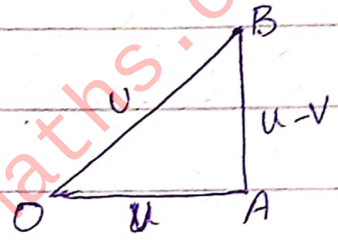
The Vector whose initial point is origon (0) and terminal point is P is called Position Vector.

$\vec{OP}$



Starting point O

Terminal Point P



$\vec{OB} = \vec{OA} + \vec{AB}$

$\vec{OB} - \vec{OA} = \vec{AB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB} = u - v$

Origon is always zero.

Unit Vector:

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{\text{Vector}}{\text{Magnitude}}$$

$$\vec{u} = (2,3) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Null Vector:

The Vector whose magnitude is zero are called null vectors.

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$\vec{v} = 0 \Rightarrow |\vec{v}| = 0$

$|\vec{v}| = \sqrt{x^2 + y^2}$ where $x = 0$, $y = 0$

If Power of Vector is 1 (Linear),
Power of Vector is 2 (Quadratic)
—— Power of n is (Polynomial).

Example : $\hat{u} = ?$

$\vec{u} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Solution,

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|}$$

$$|\vec{u}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\vec{u}| = \sqrt{(3)^2 + (4)^2 + (5)^2}$$

$$|\vec{u}| = \sqrt{9 + 16 + 25}$$

$$|\vec{u}| = \sqrt{50} \Rightarrow |\vec{u}| = 5\sqrt{2}$$

$$\hat{u} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

$$\hat{u} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} + \frac{\cancel{5}}{\cancel{5}\sqrt{2}}\hat{k}$$

$$\hat{u} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{2 \times 2}{5\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

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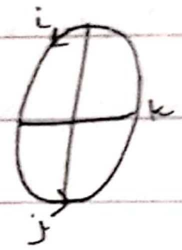
$$\hat{u} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{\cancel{\sqrt{2}}\,\sqrt{2}\cdot 2}{5\cancel{\sqrt{2}}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\hat{u} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{2\sqrt{2}}{5}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

Answer



$i \cdot i = \cos\theta$

$i \cdot i = \cos 0$

$i \cdot i = 1$

$i \times i = \sin\theta$

$i \times i = \sin 0$

$i \times i = 1$

General form:

$i = (1, 0, 0)$

$j = (0, 1, 0)$

$k = (0, 0, 1)$

$i \cdot i = (1,0,0)\cdot(1,0,0)$

$i \cdot i = 1 + 0 + 0 = \boxed{1}$

$i \cdot j = (1,0,0)\cdot(0,1,0)$

$i \cdot j = 0 + 0 + 0 = \boxed{0}$

$$i \times j = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$A_{ij} = (-1)^{i+j} M_{ij}$

$(-1)^{1+2}$

$-1\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix}$

$-$

$= i(0-0) - j(0-0) + k(1-0)$

$\boxed{i \times j = k}$

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28 October 2021 Thursday

Linear Combination Of Vectors

if $V_1, V_2, V_3 \ldots V_n \in \mathbb{R}^n$ and $c_1, c_2, c_3, \ldots c_n \in \mathbb{R}$ the linear combination of vector is:

$$\vec{w} = c_1\vec{V_1} + c_2\vec{V_2} + c_3\vec{V_3} + \ldots + c_n\vec{V_n}$$

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Example:

$V_1 = (1,2)$, $V_2 = (2,1)$ are the linear combination of $w = (2,3)$ then find the constant number:

Solution:

where $n = 2$

$$\vec{w} = c_1\vec{V_1} + c_2\vec{V_2}$$

$$(2,3) = c_1(1,2) + c_2(2,1)$$

$$(2,3) = (c_1, 2c_1) + (2c_2, c_2)$$

$$(2,3) = c_1 + 2c_2, 2c_1 + c_2$$

Now Comparing:

$$c_1 + 2c_2 = 2 \quad \text{—— (i)}$$

$$2c_1 + c_2 = 3 \quad \text{—— (ii)}$$

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We solve these equation by matrix.

By Matrix:

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

A X B

$$AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

We make this matrix Augmented.

$$A = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 1 & 3 \end{array}\right]$$

Now apply row operation:

$R_2 - 2R_1$

$$A = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2-2 & 1-4 & 3-4 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -3 & -1 \end{array}\right]$$

Now again make equation.

$-3c_2 = -1$, $c_1 + 2c_2 = 2$ — (i)

$c_2 = \cancel{+}\,1/\cancel{+}3$

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$\boxed{C_2 = 1/3}$ Answer.

Put the Value of C_2 in (i)

$C_1 + 2C_2 = 2$

$C_1 + 2(1/3) = 2$

$C_1 + 2/3 = 2$

$C_1 = 2 - \frac{2}{3}$

$C_1 = \frac{6-2}{3}$

$\boxed{C_1 = 4/3}$ Answer.

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VERIFICATION:

$C_1 + 2C_2 = 2$

$\frac{4}{3} + 2(1/3) = 2$

$\frac{4}{3} + \frac{2}{3} = 2$

$\frac{4+2}{3} = 2$

$\frac{6}{3} = 2$

$2 = 2$

L.H.S = R.H.S

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By adding | Subtraction two equation

$2 \times i - (ii)$

$C_1 + 2C_2 = 2$ — (i

$2C_1 + C_2 = 3$ — (ii

$\cancel{2C_1} + 4C_2 = 4$

$\underline{-\cancel{2C_1} \pm C_2 = -3}$

$3C_2 = +1$

$\boxed{C_2 = 1/3}$ Answer

Put the Value of C_2 in (i

$C_1 + 2 \cdot 1/3 = 2$

$C_1 + 2/3 = 2$

$C_1 = 2 - \frac{2}{3}$

$C_1 = \frac{6-2}{3}$

$\boxed{C_1 = 4/3}$ Answer.

Example:

$V_1 = (2, 3, \lambda)$, $V_2 = (1, 2, 4)$, $V_3 = (2, 1, 3)$

$C_i = (2, 1, 3)$, $i = (1, 2, 3)$, $w = (2, 4, 6)$

Find the Value of λ.

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(Vector always write into Column form)

Solution: For Non-trivial Solution

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ \lambda & 4 & 3 \end{bmatrix}$$

To Find λ Value Take determinant:

$$A = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ \lambda & 4 & 3 \end{vmatrix}$$

$= 2(6-4) - 1(9-\lambda) + 2(12-2\lambda) = 0$

$\Rightarrow 12 - 8 - 9 + \lambda + 24 - 4\lambda = 0$

$\Rightarrow 4 - 9 + 24 - 3\lambda = 0$

$\Rightarrow 19 - 3\lambda = 0 \Rightarrow 19 = 3\lambda$

$$\boxed{\lambda = 19/3}$$

Length and Dot Matrix:

Length is always equal to the Square of $\vec{u} \cdot \vec{u}$.

$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}}$

Unit Vector:

Unit Vector is the Vector whose length is always one.. (Unity).

* Non-trivial Solution $= 0$, trivial $\neq 0$

Singular $= 0$ Non Singular $\neq 0$

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29 October 2021 Friday

Cosine formula:

$\vec{a}\cdot\vec{b} = ||a||\cdot||b|| \cos\theta$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{||a||\cdot||b||}$ (For $\cos\theta$)

$\cancel{\cos^{-1}}\, \cancel{\cos}\theta = \cos^{-1} \dfrac{\vec{a}\cdot\vec{b}}{||a||\,||b||}$

$\theta = \cos^{-1} \dfrac{\vec{a}\cdot\vec{b}}{||a||\cdot||b||}$ (For angle)

Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|b|}$

Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|a|}$

INEQUALITIES:

Schwarz inequality (Cauchy Schwarz Bunikowsky inequality:

$||a\,b|| \leq ||a||\cdot||b||$

Triangular inequality:

$||a+b|| \leq ||a|| + ||b||$

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Find A^{-1} different method ?

$$AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

Adj A = ?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\text{Coo-factor of } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{ij} = (-1)^{i+j} M_{ij}$$

$$= (-1)^{1+1} \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} \Rightarrow 45-48 \Rightarrow -3$$

$$= (-1)^{1+2} \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} \Rightarrow -(36-42) \Rightarrow 6$$

$$= (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} \Rightarrow -(18-24) \Rightarrow 6$$

$$= (-1)^{1+3} \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} \Rightarrow 32-35 \Rightarrow -3$$

$$= (-1)^{2+2} \begin{vmatrix} 1 & 7 \\ 3 & 9 \end{vmatrix} \Rightarrow 9-21 \Rightarrow -12$$

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$$=(-1)^{2+3}\begin{vmatrix}1 & 2\\ 7 & 8\end{vmatrix} \Rightarrow -(8-14) \Rightarrow 6$$

$$=(-1)^{3+1}\begin{vmatrix}2 & 3\\ 5 & 6\end{vmatrix} \Rightarrow 12-15 \Rightarrow -3$$

$$=(-1)^{3+2}\begin{vmatrix}1 & 3\\ 4 & 6\end{vmatrix} \Rightarrow -(6-12) \Rightarrow 6$$

$$=(-1)^{3+3}\begin{vmatrix}1 & 2\\ 4 & 5\end{vmatrix} \Rightarrow 5-8 \Rightarrow -3$$

$$A^{*} = \begin{bmatrix}-3 & 6 & -3\\ 6 & -12 & 6\\ -3 & 6 & -3\end{bmatrix}^{t}$$

$$A^{-1} = \begin{bmatrix}-3 & 6 & -3\\ 6 & -12 & 6\\ -3 & 6 & -3\end{bmatrix}$$

Now we find determinant:

$$|A| = \begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{vmatrix}$$

$$= 1(-3)-2(-6)+3(-3)$$

$$= -3+12-9$$

$$= 0$$

Solution does not exist.

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ii) Row matrix

iii) Column matrix

Scaler Matrix:

A Square matrix having same elements in its principal diagonal except 1 is called a Scalar matrix.

E.g: $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

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3rd November 2021 Wednesday

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ $\cos\theta = ?$, Proj of

$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ $\vec{a}$ on $\vec{b}$, $\vec{b}$ on $\vec{a}$

Solution: Also find Schwarz & triangular inequalities :

Solution:

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{||a||\,||b||}$$

$$\vec{a}\cdot\vec{b} = (2\hat{i} + 3\hat{j} - \hat{k})(3\hat{i} - \hat{j} + 2\hat{k})$$

$$= 6 - 3 - 2$$

$$= 1$$

$$||a|| =$$

$$= \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$||a|| = \sqrt{14}$$

$$||b|| = \sqrt{(3)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{9 + 1 + 4}$$

$$||b|| = \sqrt{14}$$

$$\cos\theta = \frac{1}{\sqrt{14}\cdot\sqrt{14}}$$

$$\cos\theta = \frac{1}{14}$$

$$\cancel{\cos}\ \cancel{\cos}^{-1}\theta = \cos^{-1}\frac{1}{14}$$

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$$\theta = \cos^{-1}\frac{1}{14} \Rightarrow \boxed{\theta = 85.90} \text{ Ans.}$$

$$\text{Projection } \vec{b} \text{ on } \vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}$$

$$= \frac{1}{\sqrt{14}} \quad \text{Ans}$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}$$

$$= \frac{1}{\sqrt{14}} \quad \text{Ans}$$

Inequalities:

i) $|\vec{a}\cdot\vec{b}| \leq \|a\|\cdot\|b\|$ (Schwarz Ineq)

$$|1| \leq (\sqrt{14})(\sqrt{14})$$

$$1 \leq 14$$

This inequality is satisfied.

ii) $\|a+b\| \leq \|a\| + \|b\|$

$$\sqrt{5^2+2^2+1^2} \leq \sqrt{14} + \sqrt{14}$$

$$\sqrt{25+4+1} \leq 2\sqrt{14}$$

$$\sqrt{30} \leq 2(3.74)$$

$$5.47 \leq 7.48$$

This inequality is satisfied

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MATRICES:

A rectangular matrix of array of number or rows & column enclosed by a set of brackets is called Matrix.

Raw Matrix:

A Matrix having single row is called row matrix.

e.g: $A = \begin{bmatrix} 1 & 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 6 \end{bmatrix}$

Column Matrix:

A matrix having single column is called column matrix.

e.g: $A = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Diagonal Matrix:

$$\begin{bmatrix} \overset{a_{11}}{1} & 0 \\ 0 & \underset{a_{22}}{2} \end{bmatrix} \qquad \begin{matrix} i = j \\ \longrightarrow i = j \end{matrix}$$

A square matrix having each of its elements equal to zero except at least one element in its diagonal is called diagonal Matrix.

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e.g.:

$A = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 9 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Square Matrix:

A matrix in which no of rows are equal to no of column is called Square Matrix.

e.g: $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 7 \\ 4 & 5 & 1 \\ 7 & 4 & 6 \end{bmatrix}$

Rectangular Matrix:

The matrix in which no of rows are not equal to no of column is called Rectangular Matrix.

e.g: $A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & 7 & 6 \end{bmatrix}$

Identity matrix or Unit Matrix:

A Scaler matrix having 1 as its elements in the diagonal is called an identity or Unit Matrix.

e.g: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Null Matrix:

A matrix in which all element are equal to zero is Called null matrix or zero matrix.

e.g: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Equal Matrix:

Two matrix are said to be equal if they are of Same order with the same Correspondence elements.

e.g: $A = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

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Upper Triangular Matrix:

if all elements below the main diagonal of a Square matrix are zero then it is Called upper Triangular Matrix.

e.g: $A = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$

Lower Triangular Matrix:

if all elements above the main diagonal of a Square Matrix or zero then it is Called

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lower triangular Matrix.

e.g: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 8 & 0 \\ 3 & 5 & 7 \end{bmatrix}$

Triangular Matrix:

A Matrix which is either upper triangular or lower triangular is Called Triangular Matrix.

Symmetric Matrix:

Let A be a Square matrix if $A^t = A$ then A is Called Symmetric matrix.

Skew-Symmetric Matrix:

Let A be a Square Matrix if $A^t = -A$ then A is Called Skew-Symmetric Matrix or Anti Symmetric Matrix.

Harmitian Matrix:

Let A be Square Matrix if $(\bar{A})^t = A$ then A is Called harmitian Matrix.

Skew-Harmetian Matrix:

Let A be Square Matrix if $(\bar{A})^t = -A$ then A is Called Skew harmitian Matrix or Anti harmitian Matrix.

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Leading Entry (L.E):

The first non zero entry in any non zero row of an matrix is Called leading entry.

Echelon form:

First non zero element of each row should be 1.

All element under this 1 should be zero.

e.g:

Reduce Echelon form:

First two Conditions are Same of echelon form.

All elements above leading entry 1 should be zero.

e.g. etc.

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Prove $(A+A)^t$ is Symmitric Matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^t \Rightarrow A^t = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} a_{11}+a_{11} & a_{12}+a_{21} & a_{13}+a_{31} \\ a_{21}+a_{12} & a_{22}+a_{22} & a_{23}+a_{32} \\ a_{31}+a_{13} & a_{32}+a_{23} & a_{33}+a_{33} \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} a_{11}+a_{11} & a_{12}+a_{21} & a_{13}+a_{31} \\ a_{12}+a_{21} & a_{22}+a_{22} & a_{23}+a_{32} \\ a_{13}+a_{31} & a_{23}+a_{32} & a_{33}+a_{33} \end{bmatrix}^t$$

$$(A + A^t)^t = \begin{bmatrix} a_{11}+a_{11} & a_{12}+a_{21} & a_{13}+a_{31} \\ a_{21}+a_{12} & a_{22}+a_{22} & a_{23}+a_{32} \\ a_{13}+a_{31} & a_{32}+a_{23} & a_{33}+a_{33} \end{bmatrix}$$

Hence $(A+A^t)^t = A + A^t$ is Symmetric.

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Prove that: if a and b are symmetric then prove that AB is symmetric:

$$(AB)^t = AB$$

Prove: A is symmetric and B is symmetric, Mean to say:

$$A^t = A \quad , \quad B^t = B$$

$$(AB)^t = B^t A^t$$

$$(AB)^t = BA \qquad BA = AB$$

$$(AB)^t = AB$$

Hence prove that AB is symmetric.

Conjugate:

Conjugate Apply in Case of Complex number where i, j, k exist.

Sign change where imaginary number exist.

e.g: $A = \begin{bmatrix} 3+i & 3i \\ 2-i & -i \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 3-i & -3i \\ 2+i & i \end{bmatrix}$

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Solving System Of Linear Equation

Inverse Method:

$$x + 2y + 3z = 6$$
$$2x - 3y + 2z = 14$$
$$3x + y - z = -2$$

Solution:

$$AX = B$$
$$X = A^{-1}B$$
$$A^{-1} = \frac{Adj A}{|A|}$$

Adj A = (Matrix of Cofactor)^t

Now we get first determinant:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{vmatrix}$$

$$|A| = 1(3-2) - 2(-2-6) + 3(2-(-9))$$
$$|A| = 1 - 2(-8) + 3(11)$$

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$|A| = 1 + 16 + 33$

$|A| = 50 \neq 0$ Solution exist

Now we find Adj A:

Adj A = (Cofactor of Matrix)^t

$$\text{Adj}A = \begin{bmatrix} \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ -3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} \end{bmatrix}^t$$

$$= \begin{bmatrix} (3-2) & -(-2-6) & (2-(-9)) \\ -(-2-3) & (-1-9) & -(1-6) \\ (4-(-9)) & -(2-6) & (-3-4) \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 8 & 11 \\ 5 & -10 & 5 \\ 13 & 4 & -7 \end{bmatrix}^t$$

$$\text{Adj}A = \begin{bmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} \Rightarrow \frac{1}{50}\begin{bmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{bmatrix}$$

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$|A| = 1 + 16 + 33$

$|A| = 50 \neq 0$ Solution exist.

Now we find Adj A :

Adj A = (Cofactor of Matrix)^t

$$\text{Adj}A = \begin{bmatrix} \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ -3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} \end{bmatrix}^t$$

$$= \begin{bmatrix} (3-2) & -(-2-6) & (2-(-9)) \\ -(-2-3) & (-1-9) & -(1-6) \\ (4-(-9)) & -(2-6) & (-3-4) \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 8 & 11 \\ 5 & -10 & 5 \\ 13 & 4 & -7 \end{bmatrix}^t$$

$$\text{Adj}A = \begin{bmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} \Rightarrow \frac{1}{50}\begin{bmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{bmatrix}$$

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$$X = A^{-1}B$$

$$= \frac{1}{50}\begin{bmatrix} 1 & 5 & 13 \\ 8 & -10 & 4 \\ 11 & 5 & -7 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 1\times6 + 5\times14 + 13\times(-2) \\ 8\times6 + (-10\times14) + (4\times(-2)) \\ 11\times6 + 5\times14 + (-7\times-2) \end{bmatrix}$$

$$X = \frac{1}{50}\begin{bmatrix} 50 \\ -100 \\ 150 \end{bmatrix}$$

$$X = \begin{bmatrix} 50/50 \\ -100/50 \\ 150/50 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$\boxed{x = 1}$ Answer

$\boxed{y = -2}$ Answer

$\boxed{z = 3}$ Answer.

Verification:

To Varification put x, y, z's Value in any equation:

$$x + 2y + 3z = 6$$

$$1 + 2(-2) + 3(3) = 6$$

$$1 - 4 + 9 = 6$$

$\boxed{6 = 6}$ Hence prove:

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CRAMER RULE:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} 6 & 2 & 3 \\ 14 & -3 & 2 \\ -2 & 1 & -1 \end{vmatrix}}{|A|}$$

$$= \frac{6(3-2) - 2(-14-(-4)) + 3(14-6)}{50}$$

$$x = \frac{6+20+24}{50} \Rightarrow \frac{50}{50}$$

$$\boxed{x = 1}\ ; \text{ Answer}$$

$$y = \frac{\begin{vmatrix} 1 & 6 & 3 \\ 2 & 14 & 2 \\ 3 & -2 & -1 \end{vmatrix}}{50}$$

$$= \frac{1(-14+4) - 6(-2-6) + 3(-4-42)}{50}$$

$$= \frac{(-10) - 6(-8) + 3(-46)}{50}$$

$$= \frac{-10+48-138}{50}$$

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$$y = \frac{-100}{50}$$

$$\boxed{y = -2}$$ Answer.

$$z = \frac{\begin{vmatrix} 1 & 2 & 6 \\ 2 & -3 & 14 \\ 3 & 1 & -2 \end{vmatrix}}{50}$$

$$z = \frac{1(6-14) - 2(-4-42) + 6(2+9)}{50}$$

$$z = \frac{-8 - 2(-46) + 6(11)}{50}$$

$$z = \frac{-8 + 92 + 66}{50}$$

$$z = \frac{150}{50}$$

$$\boxed{z = 3}$$ Answer.

Method of Elimination/Backword Subtitution/ Echolon form:

First we do we make augomented Matrix:

$$A_b = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 14 \\ 3 & 1 & -1 & -2 \end{array}\right]$$ Augmented

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Apply this Augmented Matrix Row operation.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2-2 & -3-(4) & 2-6 & 14-12 \\ 3-3 & 1-6 & -1-9 & -2-18 \end{array}\right] \quad \begin{array}{l} R_2-2R_1 \to R_2 \\ R_3-3R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 12 \\ 0 & -5 & -10 & -20 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0/-7 & -7/-7 & -4/-7 & 12/-7 \\ 0 & -5 & -10 & -20 \end{array}\right] \quad R_2/-7 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 4/7 & -12/7 \\ 0 & -5 & -10 & -20 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 4/7 & -2/7 \\ 0 & -5+5 & -10+20/7 & -20-\frac{10}{7} \end{array}\right] \quad R_3+5R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 4/7 & -2/7 \\ 0 & 0 & -50/7 & -150/7 \end{array}\right]$$

Upper triangular Echolon form

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Now we make equation:

$$\frac{-50}{7}z = \frac{-150}{7}$$

$$z = \frac{-150}{\cancel{7}} \times \frac{\cancel{7}}{-50}$$

$$z = \frac{150}{50}$$

$$\boxed{z = 3}$$ **Answer**

$$y + \frac{4}{7}z = \frac{-2}{7}$$

$$y + \frac{4}{7}(3) = \frac{-2}{7}$$

$$y + \frac{12}{7} = \frac{-2}{7}$$

$$y = \frac{-2}{7} - \frac{12}{7}$$

$$y = \frac{-14}{7}$$ $$\boxed{y = -2}$$ **Answer**

$$x + 2y + 3z = 6$$

$$x + 2(-2) + 3(3) = 6 \Rightarrow x - 4 + 9 = 6$$

$$x = 6 + 4 - 9$$

$$x = 10 - 9$$

$$\boxed{x = 1}$$ **Answer**

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Reduce Echolon form / Forward Subtitution / J Method:

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 4/7 & -2/7 \\ 0 & 0 & -50/7 & -150/7 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2-2 & 3-\frac{8}{7} & 6+\frac{4}{7} \\ 0 & 1 & 4/7 & -2/7 \\ 0 & 0 & -50/7 & -150/7 \end{array}\right] \quad R_1 - 2R_2 \rightarrow R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 13/7 & 46/7 \\ 0 & 1 & 4/7 & -2/7 \\ 0 & 0 & -50/7 & -150/7 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 13/7 & 46/7 \\ 0 & 1 & 4/7 & -2/7 \\ 0 & 0 & \frac{-50}{7} \times \frac{7}{-50} & \frac{-150}{7} \times \frac{7}{-50} \end{array}\right] \quad R_3 \Big/ \frac{-50}{7} \rightarrow R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 13/7 & 46/7 \\ 0 & 1 & 4/7 & -2/7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 13/7 & 46/7 \\ 0 & 1 & \frac{4}{7}-\frac{4}{7} & -\frac{2}{7}-\frac{12}{7} \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_2 - \frac{4}{7}R_3 \rightarrow R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 13/7 & 46/7 \\ 0 & 1 & 0 & -14/7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

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$$= \left[\begin{array}{ccc|c} 1 & 0 & 13/7 & 46/7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & \frac{13}{7}-\frac{13}{7} & \frac{46}{7}-\frac{39}{7} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 - \frac{13}{7}R_3 \to R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7/7 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\boxed{x = 1}$ Answer

$\boxed{y = -2}$ Answer

$\boxed{z = 3}$ Answer.

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5th November 2021 Friday

LU Decomposition

Decompose upper and lower triangular.

$$AX = B$$

$$A = LU$$

$$LUX = B$$

$$UX = Y$$

$$LY = B$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad X \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = LU$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 2 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

$\boxed{u_{11} = 1}$, $\boxed{u_{12} = 2}$ $\boxed{u_{13} = 3}$

$l_{21}u_{11} = 2$

$l_{21}(1) = 2$ $\quad \boxed{l_{21} = 2}$

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$l_{21} u_{12} + u_{22} = -3$

$2(2) + u_{22} = -3$

$4 + u_{22} = -3$

$u_{22} = -3 - 4$

$\boxed{u_{22} = -7}$

$l_{21} u_{13} + u_{23} = 2$

$(2)(3) + u_{23} = 2$

$6 + u_{23} = 2$

$u_{23} = 2 - 6$

$\boxed{u_{23} = -4}$

$l_{31} u_{11} = 3$

$l_{31}(1) = 3$

$\boxed{l_{31} = 3}$

$l_{31} u_{12} + l_{32} u_{22} = 1$

$3(2) + l_{32}(-7) = 1$

$6 - 7 l_{32} = 1$

$-7 l_{32} = 1 - 6$

$-7 l_{32} = -5$

$l_{32} = \cancel{-}5 / \cancel{-}7$

$\boxed{l_{32} = 5/7}$

$l_{31} u_{13} + l_{32} u_{23} + u_{33} = -1$

$3(3) + l_{32}(-4)(5/7) + u_{33} = -1$

$9 - \frac{20}{7} + u_{33} = -1$

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$$\frac{63-20}{7} + U_{33} = -1$$

$$\frac{43}{7} + U_{33} = -1$$

$$U_{33} = \frac{-1}{1} - \frac{43}{7}$$

$$U_{33} = \frac{-7-43}{7}$$

$$\boxed{U_{33} = -50/7}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/7 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -4 \\ 0 & 0 & -50/7 \end{bmatrix}$$

$$LY = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/7 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ 2y_1 + y_2 \\ 3y_1 + \frac{5}{7}y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

Forward Subtitution / Lower Triangular

$$\boxed{y_1 = 6}$$

$$2y_1 + y_2 = 14$$

$$2(6) + y_2 = 14$$

$$12 + y_2 = 14$$

$$y_2 = 14 - 12$$

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$$\boxed{y_2 = 2}$$

$$3y_1 + \frac{5}{7}y_2 + y_3 = -2$$

$$3(6) + \frac{5}{7}(2) + y_3 = -2$$

$$18 + \frac{10}{7} + y_3 = -2$$

$$\frac{126+10}{7} + y_3 = -2$$

$$\frac{136}{7} + y_3 = -2$$

$$y_3 = \frac{-2}{1} - \frac{136}{7}$$

$$y_3 = \frac{-14-136}{7}$$

$$\boxed{y_3 = \frac{-150}{7}}$$

$$Ux = y$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -7 & -4 \\ 0 & 0 & -50/7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ \frac{-150}{7} \end{bmatrix}$$

Backword Subtitution / upper Triangular :

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$x_1 + 2x_2 + 3x_3 = 6$ —— (i)

$-7x_2 - 4x_3 = 2$ —— (ii)

$\frac{-50}{7}x_3 = \frac{-150}{7}$ —— (iii)

$x_3 = \frac{\cancel{+}150}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{+}50}$

$\boxed{x_3 = 3}$ Answer

put the Value of x_3 in (ii)

$-7x_2 - 4(3) = 2$

$-7x_2 - 12 = 2$

$-7x_2 = 2 + 12$

$-7x_2 = 14$

$+x_2 = \frac{14}{-7}$

$\boxed{x_2 = -2}$ Answer.

Put the Value of x_2 and x_3 in (i)

$x_1 + 2(-2) + 3(3) = 6$

$x_1 - 4 + 9 = 6$

$x_1 = 6 + 4 - 9$

$\boxed{x_1 = 1}$ Answer.

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10th Nov 2021 WEDNESDAY

TRANSPOSE & PERMUTATION

TRANPOSE PROPERTIES:

- $(A+B)^t = A^t + B^t$
- $(AB)^t = B^t A^t \neq A^t B^t$
- $(kA)^t = k(A)^t$
- $(A-B)^t = A^t - B^t$
- $(A^{-1})^t = (A^t)^{-1}$
- $(A^t)^t = A$

PERMUTATION:

- Identity matrix / Unit matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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$$P_{123} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- If we interchange the rows of identity matrix $\longrightarrow$ permutation Matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

$$A = LU$$

- Interchange Rows because first entry is zero, we make first entry non zero

$$PA = LU$$

$$P_{12} A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

$$P_{12} A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

- Basically permutation are used for Rows interchange

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LU-Decomposition By Permutation :

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 7 \\ 2 & 7 & 9 \end{bmatrix}$$

- Pivot element $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 7 \\ 2 & 7 & 9 \end{bmatrix}$
- Non zero element

$$PA = LU$$

$$P_{12}A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 7 \\ 2 & 7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 1 \\ 2 & 7 & 9 \end{bmatrix}$$

$\boxed{l_{21} = 0}$ **Ans.**

- Rows Operation for upper diagonal

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 1 \\ 2-2 & 7-4 & 9-14 \end{bmatrix} \quad R_3 - 2R_1 \to R_3$$

$$= \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 1 \\ 0 & 3 & -5 \end{bmatrix} \quad \boxed{l_{31} = 2} \text{ Ans.}$$

$$= \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 1 \\ 0 & 3-3 & -5-3 \end{bmatrix} \quad R_3 - 3R_2 \to R_3$$

$$= \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 1 \\ 0 & 0 & -8 \end{bmatrix} \quad \boxed{l_{32} = 3} \text{ Ans.}$$

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- We take l_{31} and l_{32} Values of previous Matrix after Apply rows operation:

$$U = \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

RANK :

Number of non-zero in the echolon form is called Rank.

$$2x_1 + y + 3z = x + \lambda x$$
$$3x + y + 2z = \lambda y$$
$$3x + 4y + 3z = 2z + \lambda z$$

$$2x_1 - x - \lambda x + y + 3z = 0$$
$$3x + y - \lambda y + 2z = 0$$
$$3x + 4y + 3z - 2z - \lambda z = 0$$

$$x(1-\lambda) + y + 3z = 0$$
$$3x + y(1-\lambda) + 2z = 0$$
$$3x + 4y + z(1-\lambda) = 0$$

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11 Nov 2021 Thursday

Solution Of Homogeneous Equation

$AX = B$ & $B \neq 0$

Then equation will be Homogeneous equation.

If B matrix of solution not be equal to zero then equation will be Non homogeneous.

Trivial :

If Rank(A) = No of unknown Variable the solution will be trivial.

$$X = 0 \ ; Y = 0 \ ; z = 0$$

Infinite :

If Rank (A) < No of unknown Variable then solution will be infinite.

Rank(A) = m , n = no of unknown Variable

$m < n$ $\quad |A| = 0$

$(n-m)$ Variable will be independent.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 3 \end{bmatrix}$$

Unknown Variable = 3 , (x, y, z)

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When we ~~do~~ make this matrix in echelon form or upper triangular through row operation.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank (A) = 2

Unknown Variable = m = 3

Rank (A) = n = 2

$n < m$ $m - n$

$2 < 3$ $3 - 2 = 1$ (Independent)

Solution is infinite.

if we have three equation:

$x + y + z = 0$ — (i)

$y + 5z = 0$ — (ii)

$y = -5z$ — (iii)

Then we will take one independent variable

$z = t$ $y = -5t$

Put $z = t$ in (iii) and (i)

$x + (-5t) + t = 0$

$x - 5t + t = 0$

$x - 4t = 0$ $\boxed{x = 4t}$

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EXAMPLE:

$x_1 + x_2 + x_3 = 0$

$x_1 - 2x_2 - x_3 = 0$

$2x_1 - 2x_2 - x_3 = 0$

$AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$n = 3$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & -4 & -3 \end{bmatrix} \quad R_2 - R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -2 \\ 0 & -4 & -3 \end{bmatrix} \quad R_3 - 2R_1 \rightarrow R_3$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2/3 \\ 0 & -4 & -3 \end{bmatrix} \quad R_2/_{-3} \rightarrow R_2$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2/3 \\ 0 & 0 & -1/3 \end{bmatrix} \quad R_3 + 4R_2 \rightarrow R_3$$

Rank $= m = 3$

Solution will be trivial.

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EXAMPLE 2: 12 Nov 2021 FRIDAY

$2x + y + u = 0$

$6x + 3y + 4z + 2u = 0$

$4x + 2y + z + 3u = 0$

System of Homogenous equation:

$$AX = B$$

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 2 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & -1 \\ 4 & 2 & 1 & 3 \end{bmatrix} \quad R_2 - 3R_1 \rightarrow R_2$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 - 2R_1 \rightarrow R_3$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix} \quad R_3 - 1/4 R_2 \Rightarrow R_3$$

- $m = 3$, $n = 4$

  $m < n$

  $3 < 4$

- $(n-m)$ Variable independent

  $4 - 3 = 1$

  $u = t$ ——— (i)

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$$4z - u = 0$$
$$4z - t = 0$$
$$4z = t$$
$$z = t/4 \quad \text{———} (2)$$

$$2x + y + 0z + u = 0 \quad \text{———} (i)$$
$$0x + 0y + 4z - u = 0 \quad \text{———} (ii)$$
$$5/4\, u = 0$$
$$\boxed{u = 0} \text{ Ans} \quad \boxed{z = 0} \text{ Ans.}$$

- put the value u and z in i)

$$2x + y = 0$$
$$y = -2x$$
if $x = t$
$$y = -2t$$

- when $t = 1$

$$\boxed{x = 1} \text{ Ans}$$
$$\boxed{y = -2} \text{ Ans}$$

- put these value in (1)

$$2x + y + u = 0$$
$$\cancel{2}(1) - \cancel{2} + 0 = 0$$
$$0 = 0 \quad \textbf{Satisfied.}$$

- when $t = -1$

$$\boxed{x = -1} \text{ Ans}$$
$$\boxed{y = 2} \text{ Ans.}$$

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$2x + y + u = 0$

$2(-1) + 2 + 0 = 0$

$-\cancel{2} + \cancel{2} = 0$

$0 = 0$ **Satisfied**

- Each and every Value satisfied this equation.
- Equation are homogenous.

RANK OF MATRIX AND ROW REDUCE FORM

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 5 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 2 \\ 4 & 2 & 1 \end{bmatrix} \quad R_2 - R_1 \to R_2$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & -5 \end{bmatrix} \quad R_3 - 2R_1 \to R_3$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & -5 \end{bmatrix} \quad R_2/2 \to R_2$$

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$$= \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad R_3 + 5R_2 \to R_3$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 - 3R_2 \to R_1$$

$\boxed{\text{RANK} = 2}$

- This Matrix is called echolon form and Reduced echolon form.

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Assignment No1:

If $\vec{u} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{v} = 3\hat{i} + 5\hat{j} + 3\hat{k}$

Find the angle b/w $\vec{u}$ and $\vec{v}$ and also find the inequalities.

Solution:

$\vec{u} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{V} = 3\hat{i} + 5\hat{j} + 3\hat{k}$

For angle:

$$\theta = \cos^{-1} \frac{\vec{u}\cdot\vec{v}}{||u||\,||v||}$$

$$\vec{u}\cdot\vec{v} = (2\hat{i} + 3\hat{j} - \hat{k})(3\hat{i} + 5\hat{j} + 3\hat{k})$$

$$= 6 + 15 - 3$$

$$\vec{u}\cdot\vec{v} = 18$$

$$\vec{u} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$|\vec{u}| = \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$|\vec{u}| = \sqrt{14}$$

$|\vec{v}| = ?$

$$\vec{v} = 3\hat{i} + 5\hat{j} + 3\hat{k}$$

$$|\vec{v}| = \sqrt{(3)^2 + (5)^2 + (3)^2}$$

$$|\vec{v}| = \sqrt{9 + 25 + 9}$$

$$|\vec{v}| = \sqrt{43}$$

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$$\theta = \cos^{-1} \frac{18}{\sqrt{14}\cdot\sqrt{43}}$$

$$\theta = \cos^{-1} \frac{18}{(3.74)(6.56)}$$

$$\theta = \cos^{-1} \frac{18}{24.535}$$

$$\theta = \cos^{-1} 0.734$$

$$\theta = 42.7^\circ$$

INEQUALITES:

Schwarz inequality (Cauchy Schwarz) Bunikawsky inequalities:

$$\|\underset{u}{a}\cdot\underset{v}{b}\| \leq \|\underset{u}{a}\|\cdot\|\underset{v}{b}\|$$

$$\|\underset{u}{a}\cdot\underset{v}{b}\| = ?$$

$$\underset{u}{a}\cdot\underset{v}{b} = 18$$

$$\|\underset{u}{a}\cdot\underset{v}{b}\| = \sqrt{18^2}$$

$$\|\underset{u}{a}\cdot\underset{v}{b}\| = \sqrt{324} \Rightarrow 18.$$

$$\|u\| = \sqrt{14}$$

$$\|v\| = \sqrt{43}$$

Putting Values,

$$18 \leq \sqrt{14}\cdot\sqrt{43}$$

$$18 \leq 24.535$$

Proved.

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Triangular Inequalities:

$|a+b| \leq |a| + |b|$

$|u+v| \leq |u| + |v|$

$u+v = (2\hat{i} + 3\hat{j} - \hat{k}) + (3\hat{i} + 5\hat{j} + 3\hat{k})$

$u+v = 5\hat{i} + 8\hat{j} + 2\hat{k}$

$|u+v| = \sqrt{5^2 + 8^2 + 2^2}$

$= \sqrt{25+64+4}$

$= \sqrt{93}$

$|u+v| = 9.64$

$|u+v| \leq |u| + |v|$

$9.64 \leq 3.74 + 6.56$

$9.64 \leq 10.3$

Hence proved.

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Question No 2:

if $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$

then Show that $A - A^t$ is skew symetric.

Solution:

$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$

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$$A^t = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A - A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-3 & 0-(-1) \\ 3-2 & 2-2 & -1-3 \\ -1-0 & 3-(-1) & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$$(A - A^t)^t = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$-(A - A^t)^t = (-1)\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$$A - A^t = -(A - A^t)^t$$

Hence Prove that $A - A^t$ is Skew

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symmetric

Question No 4:

$$(1-m)x + 2y + 3z = 0$$
$$3x + (1-m)y - z = 0$$
$$5x + 2y + (1-m)z = 0$$

Find the value of 'm' when Homogenous system of Equation have infinite solution?

Solution:

$$A = \begin{bmatrix} 1-m & 2 & 3 \\ 3 & 1-m & -1 \\ 5 & 2 & 1-m \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1-m & 2 & 3 \\ 3 & 1-m & -1 \\ 5 & 2 & 1-m \end{vmatrix}$$

$$\cancel{= 1-m\,((1-m)(1-m)) - 2\,(3-3m}$$

$$= 1-m \begin{vmatrix} 1-m & -1 \\ 2 & 1-m \end{vmatrix} - 2 \begin{vmatrix} 3 & -1 \\ 5 & 1-m \end{vmatrix} + 3 \begin{vmatrix} 3 & 1-m \\ 5 & 2 \end{vmatrix}$$

$$= 1-m\,(1-2m+m^2-(-2)) - 2\,(3-3m-(-5)) + 3\,(6-(5-5m))$$

$$= 1-m\,(1-2m+m^2+2) - 2\,(3-3m+5)$$

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$+3(6-5+5m)$

$= 1-m(m^2-2m+3)-2(8-3m)+3(5m+1)$

$= -m^3+3m^2-5m+3-16+6m+15m+3$

$= -m^3+3m^2+16m-10$

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17 Nov 2021 Wednesday

PROPERTIES OF DETERMINANTS :-

CONSISTANCY OF MATRIX

① "If Rank of A Matrix is equal to Rank of Augomented then matrix is Called Consistance & Solution will be exist."

$$\text{Rank}(A) = \text{Rank}(A|b)$$

i) UNIQUE Matrix:

"Rank (A) = Rank(Ab) = Unknown Variable then solution will be Consistant and Unique." = 0

ii) Infinite Matrix:

"Rank (A) = Rank (Ab) < no of unknown Variable then solution will be constant and infinite."

② "if Rank(A) ≠ Rank (Ab)

Matrix will be INCONSISTANT and Solution does not exist"

Augomented MATRIX

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$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 22$$

Solution:

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 13 \\ 22 \end{bmatrix}$$

$$A_b = \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 22 \end{array}\right] \quad R_1/2 \to R_1$$

$$A_b = \left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 22 \end{array}\right]$$

$$A_b = \left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 1+\frac{9}{2} & -3-21/2 & 13-15/2 \\ 0 & 19+3 & -47-7 & 22-5 \end{array}\right] \quad \begin{array}{l} R_2 - 3R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \end{array}$$

$$A_b = \left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 22 & -54 & 17 \end{array}\right]$$

$$A_b = \left[\begin{array}{ccc|c} 1 & -3/2 & 7/2 & 5/2 \\ 0 & 1 & -27/11 & 1 \\ 0 & 22 & -54 & 17 \end{array}\right] \quad 2/11\, R_2 \to R_2$$

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$$A_b = \left[\begin{array}{ccc|c} 0 & -3 & 7 & 5 \\ 0 & 1 & -27/11 & 1 \\ 0 & 0 & 0 & -5 \end{array}\right] \begin{array}{l} 2R_1 \to R_1 \\ R_3 - 22R_2 \to R_3 \\ \end{array}$$

Rank(A) = 2

Rank(A_b) = 3

Rank(A) $\neq$ Rank(A_b)

Then solution does not exist.

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EXERCISES 1.1

1) $x + 2y = 8$
$3x - 4y = 4$

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}, B \begin{bmatrix} 8 \\ 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}.$$

Now we make augment matrix:

$$A_b = \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 3 & -4 & 4 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 3-3 & -4-6 & 4-24 \end{array}\right] R_2 - 3R_1 \rightarrow R_2$$

$$= \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & -10 & -20 \end{array}\right]$$

$$-10y = -20$$

$$y = \frac{\cancel{-20}\,2}{\cancel{-10}} \qquad \boxed{y = 2}$$

$$x + 2y = 8$$
$$x + 2(2) = 8$$
$$x + 4 = 8$$
$$x = 8 - 4 \Rightarrow \boxed{x = 4}$$

Verification Put the value of x and y in (i)

$$4 + 2(2) = 8$$
$$4 + 4 = 8 \Rightarrow 8 = 8 \text{ Ans}$$

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2) $2x-3y+4z=-12$

$x-2y+z=-5$

$3x+y+2z=1$

Solution:

$$A=\begin{bmatrix}2 & -3 & 4\\ 1 & -2 & 1\\ 3 & 1 & 2\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\ B=\begin{bmatrix}-12\\ -5\\ 1\end{bmatrix}$$

First we make augmented matrix and apply row operation:

$$A_b=\left[\begin{array}{ccc|c}2 & -3 & 4 & -12\\ 1 & -2 & 1 & -5\\ 3 & 1 & 2 & 1\end{array}\right]$$

$$A_b=\left[\begin{array}{ccc|c}2 & -3 & 4 & -12\\ 1 & -2 & 1 & -5\\ 3-3 & 1+6 & 2-3 & 1+15\end{array}\right] \quad R_3-3R_2\rightarrow R_3$$

$$A_b=\left[\begin{array}{ccc|c}2 & -3 & 4 & -12\\ 1 & -2 & 1 & -5\\ 0 & 7 & -1 & 16\end{array}\right]$$

$$A_b=\left[\begin{array}{ccc|c}2/2 & -3/2 & 4/2 & -12/2\\ 1 & -2 & 1 & -5\\ 0 & 7 & -1 & 16\end{array}\right] \quad R_1/2\rightarrow R_1$$

$$A_b=\left[\begin{array}{ccc|c}1 & -3/2 & 2 & -6\\ 1 & -2 & 1 & -5\\ 0 & 7 & -1 & 16\end{array}\right]$$

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$-2 + \frac{3}{2}$

$\frac{-4+3}{2} \Rightarrow \frac{-1}{2}$

$$Ab = \left[\begin{array}{ccc|c} 1 & -3/2 & 2 & -6 \\ 1-1 & -2+3/2 & 1-2 & -5+6 \\ 0 & 7 & -1 & 16 \end{array}\right] \quad R_2 - R_1 \to R_2$$

$$Ab = \left[\begin{array}{ccc|c} 1 & -3/2 & 2 & -6 \\ 0 & -1/2 & -1 & 1 \\ 0 & 7 & -1 & 16 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 2 & -6 \\ 0 & 1 & 2 & -2 \\ 0 & 7 & -1 & 16 \end{array}\right] \quad -2 \times R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 2 & -6 \\ 0 & 1 & 2 & -2 \\ 0 & 7-7 & -1-14 & 16+14 \end{array}\right] \quad R_3 - 7R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 2 & -6 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & -15 & 30 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & -12 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & -15 & 30 \end{array}\right] \quad R_1 \times 2 \to R_1$$

$-15z = 30$

$\boxed{z = -2}$

$y + 2z = -2$

$y + 2(-2) = -2$

$y - 4 = -2$

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$y = -2 + 4$

$\boxed{y = 2}$

$2x - 3y + 4z = -12$

$2x - 3(2) + 4(-2) = -12$

$2x - 6 - 8 = -12$

$2x = -12 + 8 + 6$

$2x = 2$

$\boxed{x = 1}$

Verification:

Put the Value of x, y, z.

$2(1) - 3(2) + 4(-2) = -12$

$2 - 6 - 8 = -12$

$-6 - 6 = -12$

$-12 = -12$

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6) $x + y - 2z = 5$

$2x + 3y + 4z = 2$

Solution:

$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

First we make augmented Matrix:

$A_b = \left[\begin{array}{ccc|c} 1 & 1 & -2 & 5 \\ 2 & 3 & 4 & 2 \end{array}\right]$

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For what value of λ and μ the system:

$x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$

have (i) No Solution

ii) Unique Solution

iii) infinite solution

Solution:

$$A_b = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$$A_b = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1-1 & 2-1 & 3-1 & 10-6 \\ 1-1 & 2-1 & \lambda-1 & \mu-6 \end{array}\right] \begin{array}{l} \\ R_2-R_1 \to R_2 \\ R_3-R_1 \to R_3 \end{array}$$

$$A_b = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0-0 & 1-1 & \lambda-1-2 & \mu-6-4 \end{array}\right] \quad R_3-R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

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i) When $\lambda = 3$ & $\mu \neq 10$ then no solution.

ii) When $\lambda \neq 3$ & $\mu \neq 10$ then solution is Consistant.

Rank(A) = RANK(Ab) = Unknown Variable

No Unknown Variable = x, y, z = 3

Rank(A) = 3 , Rank(Ab) = 3

iii) When $\lambda = 3$ & $\mu = 10$ then solution is infinite solution

Rank(A) = Rank(Ab) < no of unknown Variable

Rank(A) = 2

Rank(Ab) = 2

Unknown Variable = 3

$2 = 2 < 3$

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UNIQUE IN Homogenous System:

$x = 0 , y = 0 , z = 0$

UNIQUE IN NON-HOMOGENOUS System:

Unknown Variable x, y, z have Unique Value mean $x = 1, y = 2$ $z = 3$. Not a infinit Value.

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$x+2y+z=3$
$2x+3y+2z=5$
$3x-5y+5z=2$
$3x+9y+z=4$

Solution:

$$=\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 3 & -5 & 5 & 2 \\ 3 & 9 & 1 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2-2 & 3-4 & 2-2 & 5-6 \\ 3-3 & -5-6 & 5-3 & 2-9 \\ 3-3 & 9-6 & 1-3 & 4-9 \end{array}\right] \begin{array}{l} R_2 \to R_2-2R_1 \\ R_3 \to R_3-3R_1 \\ R_4 \to R_4-3R_1 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & -11 & 2 & -7 \\ 0 & 3 & -2 & -5 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & -11 & 2 & -7 \\ 0 & 3 & -2 & -5 \end{array}\right] \quad R_2 \to -R_2$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & -11+11 & 2+0 & -7+11 \\ 0 & 3 & -2 & -5 \end{array}\right] \quad R_3 \to R_3 + 11R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 3 & -2 & -5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 3-3 & -2-0 & -5-3 \end{array}\right] \quad R_4 \to R_4 - 3R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -2 & -8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -2+2 & -8+4 \end{array}\right] \quad R_4 \to R_4 + R_3$$

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19-

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

$\text{RANK}(A) = 3$

$\text{RANK}(A_b) = 4$

$\text{RANK}(A) \neq \text{RANK}(A_b)$ Ans.

Solution is Inconsistant, does not exist.

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:-

$x + y + z = 6$

$2x + y + 3z = 13$

$5x + 2y + z = 12$

Solution:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & 3 & 13 \\ 5 & 2 & 1 & 12 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2-2 & 1-2 & 3-2 & 13-12 \\ 5-5 & 2-5 & 1-5 & 12-30 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & -4 & -18 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -3+3 & -4-3 & -18-3 \end{array}\right] \quad R_3 \to R_3 - 3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -7 & -21 \end{array}\right]$$

RANK(A) = 3

RANK(A_b) = 3

RANK(A) = RANK(A_b) Ans = Unknown Variable

Solution is Consistant / Unique Solution.

$$-7z = -21$$

$$z = \frac{-21}{-7}$$

$$\boxed{z = 3} \text{ Ans}$$

$$-y + z = 1$$

$$-y + 3 = 1$$

$$-y = 1 - 3 \Rightarrow -y = -2 \Rightarrow \boxed{y = 2} \text{ Ans.}$$

$$x + y + z = 6$$

$$x + 2 + 3 = 6 \Rightarrow x + 5 = 6$$

$$x = 6 - 5 \Rightarrow \boxed{x = 1} \text{ Ans.}$$

UNIQUE SOLUTION: In Unique solution we have x, y, z Variables of Unique / one Value.

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$x+y+z=6$
$2x+y+3z=13$
$5x+2y-z=12$

Solution:

$$A=\begin{bmatrix}1&1&1\\2&1&3\\5&2&-1\end{bmatrix} \quad B=\begin{bmatrix}6\\13\\12\end{bmatrix}$$

$$A_b=\left[\begin{array}{ccc|c}1&1&1&6\\2&1&3&13\\5&2&-1&12\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&1&1&6\\0&-1&1&1\\0&-3&-4&-18\end{array}\right]$$

(Pehly sy ho gia hy yeh)

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Properties Of Determinants

① For a Square Matrix A.

$|A| = |A^t|$

$A = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$

$|A| = 6 - 24 \Rightarrow |A| = -18$

$A^t = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}^t \Rightarrow \begin{bmatrix} 2 & 6 \\ 4 & 3 \end{bmatrix}$

$|A^t| = 6 - 24$

$|A^t| = -18$

$|A| = |A^t|$ Hence proved.

② If in a Square matrix A, two rows or two Column are interchange the determinants of resulting matrix $-|A|$.

$A = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$ $|A| = -18$

$A = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}$ $R_1 \leftrightarrow R_2$ interchange

$|A| = 24 - 6$

$|A| = 18$ $|A| = -|A|$

$-|A| = -18$ Hence proved.

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$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 5 & 2 \\ 2 & 3 & -4 \end{bmatrix} = 2(-26) - 3(-8) + 1(-7)$$

$$= -52 + 24 - 7 \Rightarrow -35$$

$C_1 \leftrightarrow C_2$

$$\begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 2 \\ 3 & 2 & -4 \end{bmatrix} \quad = 3(-8) - 2(-26) + 1(7)$$

$$= -24 + 52 + 7$$

$$|A| = 35 \Rightarrow -|A| = -35$$

$|A| = -|A|$: **Hence proved.**

3) If a matrix A has two identical rows or columns then $|A| = 0$.

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 3 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

$|A| = 0$

- Two rows are identical

4) If all the enteries of row or columns are zero then $|A| = 0$

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix} \Rightarrow |A| = 0$$

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5) If the enteries of rows or columns in a square matrix A are multiplied by a constant number $k \in R$ then the determinants of resultant matrix is $k|A|$.

$$\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 5 \\ 9 & 4 & 2 \end{vmatrix} \Rightarrow 3 \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{vmatrix}$$

$$3(6-20)-1(12-45)+2(24-27) = 3[1(6-20)-1(4-15)+2(8-9)]$$

$$3(-14)-1(-33)+2(-3) = 3[(-14)-(-11)+2(-1)]$$

$$-42+33+(-6) = 3[-14+11-2]$$

$$-15 = 3(-5)$$

$$-15 = -15$$

$$\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3/4 & -5 \\ 9 & 4 & 2 \end{vmatrix} = 3 \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3/4 & -5 \\ 3 & 4 & 2 \end{vmatrix}$$

$$3\left(\frac{3}{2}+20\right)-1(12+45)+2\left(24-\frac{27}{4}\right) = 3\left(1\left(\frac{3}{2}+20\right) -1(4+15)+2(8-9/4)\right)$$

$$3\left(\frac{43}{2}\right)-(57)+2\left(\frac{96-27}{4}\right) = 3\left(\frac{43}{2}-19+2\left(\frac{23}{4}\right)\right)$$

$$\frac{129}{2}-57+\frac{69}{2} = 3\left(\frac{43}{2}-19+\frac{23}{2}\right)$$

$$\frac{129-114+69}{2} = 3\left(\frac{43-38+23}{2}\right)$$

$$\frac{64}{2} \Rightarrow 42 = 3(14) \Rightarrow 42$$

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6) If each row or Column of a Square matrix Consist of two terms then the determinants Can be written as the Sum of two determinants.

$$\begin{vmatrix} \alpha_{11}+a_{11} & a_{11} \\ \alpha_{12}+a_{12} & b_{12} \end{vmatrix} = \begin{vmatrix} \alpha_{11} & b_{11} \\ \alpha_{12} & b_{12} \end{vmatrix} + \begin{vmatrix} a_{11} & b_{11} \\ a_{12} & b_{11} \end{vmatrix}$$

$$\begin{vmatrix} 5 & 1 & 2 \\ 6 & 5 & 1 \\ 3 & 3 & -9 \end{vmatrix}$$

$$\begin{vmatrix} 2+3 & 1 & 2 \\ 3+3 & 5 & 1 \\ 2+1 & 3 & -9 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 5 & 1 \\ 2 & 3 & -9 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 2 \\ 3 & 5 & 1 \\ 1 & 3 & -9 \end{vmatrix}$$

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

without expansion $|A|=0$

$C_1 = C_1 + C_2 + C_3$

$$\begin{vmatrix} \cancel{a}-\cancel{b}+\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a} & b-c & c-a \\ \cancel{b}-\cancel{c}+\cancel{c}-\cancel{a}+\cancel{a}-\cancel{b} & c-a & a-b \\ \cancel{c}-\cancel{a}+\cancel{a}-\cancel{b}+\cancel{b}-\cancel{c} & a-b & b-c \end{vmatrix}$$

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$$\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$$

All enteries of 1st Column are zero then determinant of $|A| = 0$.

7) If to each entry of a row or Column of a Square matrix A is added a non-zero multiple of the Corresponding entry of other row or Columns then the determinants of resulting matrix is $|A|$.

$$\begin{vmatrix} a_{11} & a_{12} + k a_{11} \\ a_{21} & a_{22} + k a_{21} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

EXAMPLE:

$$\begin{vmatrix} 2 & 3 & 1 \\ 3 & 5 & 2 \\ 1 & 7 & 1 \end{vmatrix}$$

$= 2(5-14) - 3(3-2) + 1(21-5)$

$= 2(-9) - 3(1) + 1(16)$

$= -18 - 3 + 16$

$= -5$

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$$\begin{vmatrix} 2 & 3 & 5 \\ 3 & 5 & 8 \\ 1 & 7 & 3 \end{vmatrix}$$ Multiply by 2 of C_1 and add in C_3 $(2 \times C_1 + C_3)$

$= 2(15-56)+3(9-8)+5(21-5)$

$= 2(-41)+3(1)+5(16)$

$= -82+3+80$

$= -5$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$= 1(45-48)-2(36-42)+3(32-35)$

$= -3-2(-6)+3(-3)$

$= -3+12-9 \Rightarrow 12-12 = 0$

$$= \begin{vmatrix} 1 & 2 & 4 \\ 4 & 5 & 10 \\ 7 & 8 & 16 \end{vmatrix}$$ Multiply by 1 of C_1 and add in C_3 $(1 \times C_1 + C_3)$

$= 1(80-80)-2(64-70)+4(32-35)$

$= 0-2(-6)+4(-3)$

$= 12-12$

$= 0$

$$\begin{vmatrix} 4 & 3 \\ 5 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 3+8 \\ 5 & 2+10 \end{vmatrix}$$

$8-15 = 48-55$

$-7 = -7$

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8) If a matrix is in triangular form then the value of its determinants is the product of the entries on its main diagonal.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 4 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{vmatrix}$$

$|A| = 1(6-0) - 4(0) + 5(0)$

$= 6 - 0 + 0$

$= 6$

Main Diagonal:

$\Rightarrow 1 \times 2 \times 3 = 6$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 5 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

$1 \times 5 \times 1 = 5$

$= 1(5-0) - 0(3-0) + 0(9-10)$

$= 5$

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Without Expansion Verify that:

1) $$\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$$

L.H.S :

$$= \begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \alpha+\beta+\gamma & \beta+\gamma & 1 \\ \alpha+\beta+\gamma & \gamma+\alpha & 1 \\ \alpha+\beta+\gamma & \alpha+\beta & 1 \end{vmatrix} \quad \text{Add } C_2 \text{ in } C_1$$

$$= \alpha+\beta+\gamma \begin{vmatrix} 1 & \beta+\gamma & 1 \\ 1 & \gamma+\alpha & 1 \\ 1 & \alpha+\beta & 1 \end{vmatrix} \quad \text{Take Common } (\alpha+\beta+\gamma) \text{ from } C_1$$

$$= (\alpha+\beta+\gamma)(0)$$

$= 0$ Hence proved

2) $$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

$$= \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

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Adding C_2, C_3 in C_1

$$= \begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$$

$$= \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$$

$= 0$ (Because C_1 is zero)

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AFTER WINTER VOCATION

Eigen Values And Eigen Vectors:

Let A be a Square matrix of order $n \times n$, then a numbers (real or Complex) λ is Said to eigen Values of matrix A if there exist a Column matrix X of order $n \times 1$ Such that

$$AX = \lambda X$$

$$AX - \lambda X = 0$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 5$$

$$\underbrace{X}_{\text{Eigen Vector}}(\underbrace{A}_{\text{Square Matrix}} - \overbrace{\lambda}^{\text{Eigen Values}} \underbrace{I}_{\text{Unit Matrix}}) = 0$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For Eigen Values:

$|A - \lambda I| = 0 \Rightarrow$ Characteristic Equation

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Find eigen Values And Eigen Vector of matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

Solution:

General Equation:

$$(A - \lambda I)X = 0 \quad \text{---(1)}$$

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Characteristic Equation:

$|A - \lambda I| = 0$ — (2

$$\left| \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{vmatrix} = 0.$$

$$(1-\lambda)(4-\lambda) - 10 = 0$$

$$4 - \lambda - 4\lambda + \lambda^2 - 10 = 0$$

$$\lambda^2 - 5\lambda - 6 = 0 \quad \Rightarrow \lambda^2 - 6\lambda + \lambda - 6 = 0$$

$$\lambda(\lambda - 6) + 1(\lambda - 6) = 0$$

$$(\lambda - 6)(\lambda + 1) = 0$$

$$\lambda - 6 = 0$$

$$\underbrace{\boxed{\lambda = 6} \qquad \boxed{\lambda = -1}}_{\text{Eigen Values}}$$

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ be the eigen Vector of $\lambda = 6$.

$$\begin{vmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{vmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

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Put the Value of λ

$$\begin{bmatrix} 1-6 & -2 \\ -5 & 4-6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-5x - 2y = 0$

$-5x = 2y$

$\frac{x}{2} = \frac{y}{-5} = k$ (we say $k=1$)

$\frac{x}{2} = \frac{y}{-5} = 1$

$\frac{x}{2} = 1$ & $\frac{y}{-5} = 1$

$\boxed{x = 2}$ $\boxed{y = -5}$ for $\lambda = 6$

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ be the eigen Vector

for $\lambda = -1$

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$$\begin{bmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-(-1) & -2 \\ -5 & 4-(-1) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2/2 & -2/2 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 \to \frac{R_1}{2}$$

$$\begin{bmatrix} 1 & -1 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_2 \to R_2 + 5R_1$$

$x - y = 0$

$x = y = k$ (we say $k = 1$)

$$\underbrace{\boxed{x = 1} \ \& \ \boxed{y = 1}}_{\lambda = -1}$$

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Find eigen Values and eigen Vectors?

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Solution:

$(A - \lambda I)X = 0$ — (i) General Equation

$|A - \lambda I| = 0$ — (ii) Characteristic Equation

Put the Values of A and Identity matrix:

$$\left| \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda)-2\right] - 2\left[(2-\lambda)-1\right] + 1\left[2-(3-\lambda)\right] = 0$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda)-2\right] - 2\left[2-\lambda-1\right] + 1\left[2-3+\lambda\right] = 0$$

$$(2-\lambda)(\lambda^2 - 5\lambda + 6 - 2) - 2(1-\lambda) + (\lambda - 1) = 0$$

$$2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda - 2 + 2\lambda + \lambda - 1 = 0$$

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$\lambda - 1 = 0$

$\boxed{\lambda = 1}$

$-\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$

$$1\lambda^3 - \begin{bmatrix}\text{Sum of} \\ \text{Diagonal} \\ \text{enteries}\end{bmatrix}\lambda^2 + \begin{bmatrix}\text{Sum of} \\ \text{Diagonal} \\ \text{Minors}\end{bmatrix}\lambda - |A| = 0$$

$\lambda = 1$

$(-1)^3 + 7(1)^2 - 11(1) + 5 = 0$

$-1 + 7 - 11 + 5 = 0$

$12 - 12 = 0$

$0 = 0$ [redacted]

$(\lambda - 1)(-\lambda^2 + 6\lambda - 5) = 0$

$(\lambda - 1) = 0 \qquad -\lambda^2 + 6\lambda - 5 = 0$

$\lambda = 1 \qquad -(\lambda^2 - 6\lambda + 5) = 0$

1	-1	7	-11	5
	$\downarrow$	-1	6	-5
	-1	6	-5	$0 \rightarrow$ Rem

$-1\lambda^2 + 6\lambda - 5$

$\boxed{\lambda = 1}$

$\lambda^2 - 6\lambda + 5 = 0$

$\lambda^2 - 5\lambda - \lambda + 5 = 0$

$\lambda(\lambda - 5) - 1(\lambda - 5) = 0$

$(\lambda - 1)(\lambda - 5) = 0$

& $\lambda - 1 = 0 \qquad \lambda - 5 = 0$

$\boxed{\lambda = 1} \qquad \boxed{\lambda = 5}$

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Eigen Values:

$\lambda = 1 \quad , \lambda = 1 \quad ; \lambda = 5$

When eigen Values are repeated then use only one Values for find out eigen Vector.

In 3x3 matrix Equation becomes always Cubic.

MAP Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ For $\lambda = 5$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By using Two equation

$-3x + 2y + z = 0$ —— (i)

$x - 2y + z = 0$ —— (ii)

$x + 2y - 3z = 0$ —— (iii)

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Now Applying CRAMERS Rules :

$$\frac{x}{\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -3 & 2 \\ 1 & -2 \end{vmatrix}} = k$$

$$\frac{x}{4} = \frac{y}{-4} = \frac{z}{4} = k$$ We say $k=1$

$$\frac{x}{4} = k$$

$k=1$

$\boxed{x=4}$ Answer

$$\frac{y}{-4} = k$$

$\boxed{y=-4}$ Answer

$$\frac{z}{4} = k$$

$\boxed{z=4}$ Answer

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for $\lambda = 1$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

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$\lambda = 1$

$$\begin{bmatrix} 2-1 & 2 & 1 \\ 1 & 3-1 & 1 \\ 1 & 2 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

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$x + 2y + z = 0$

Let suppose $y = k_1$ & $z = k_2$

$x + 2k_1 + k_2 = 0$

$x = -2k_1 - k_2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2k_1 - k_2 \\ k_1 \\ k_2 \end{bmatrix}$$

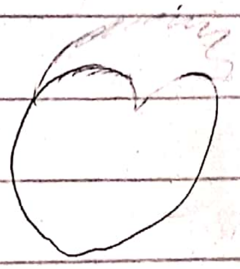
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} k_1 + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} k^2$$

Let suppose $k_1 = 1$

$k_2 = 1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} s$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$



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$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Solution:

$$(A - \lambda I)X = 0$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \begin{bmatrix} \text{sum of diognal} \\ \text{elements} \end{bmatrix} \lambda^2 + \begin{bmatrix} \text{sum of} \\ \text{diognal} \\ \text{minors} \end{bmatrix} \lambda - |A| = 0$$

Sum of diognal elements:

$$2 + 3 + 2 = 7$$

Sum of diognal minors:

$$\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$$

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$$\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 6 - 2 = 4$$

$$4 + 3 + 4 = \underline{11}$$

$|A|$:

$$2(4) - 2(1) + 1(-1) = \phi$$

$$8 - 2 + 1 = \phi$$

$$8 - 3 = \phi$$

$$= 5$$

$$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

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Find The Eigen Value And Eigen Vector?

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Solution:

Characteristic Equation:

$$|A - \lambda I| = 0$$

General Equation:

$$(A - \lambda I)X = 0$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = 0$$

$$((-\lambda)(-\lambda)) - (-1) = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\sqrt{\lambda^2} = \sqrt{-1}$$

$$\boxed{\lambda = \pm i}$$

$$\boxed{\lambda = i} \qquad \boxed{\lambda = -i}$$

Eigen Value

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Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ For $\lambda = +i$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2$

$$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -i \times i & 1 \times i \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad i \times R_1 \to R_1$$

$$\begin{bmatrix} -i^2 & i \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & i \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 + R_2 \to R_2$$

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By Using Cramer's Rule:

$x + iy = 0$

$x = -iy$

$\frac{x}{-i} = \frac{y}{1} = k$ We say $k = 1$

$\frac{x}{-i} = k$ $\quad \frac{y}{1} = 1$

$\boxed{x = -i}$ $\quad \boxed{y = 1}$

Eigen Vector

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Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ for $\lambda = -i$

$(A - \lambda I)X = O$

$$\begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -(-i) & 1 \\ -1 & -(-i) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -i^2 & -i \\ -1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad -i \times R_1 \to R_1$$

$$\begin{bmatrix} -(\sqrt{-1})^2 & -i \\ -1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -i \\ -1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 + R_2 \to R_2$$

By using CRAMER'S RULE:

$x - iy = 0$

$x = iy$

$\frac{x}{i} = \frac{y}{1} = k$ We say $k = 1$

$\frac{x}{i} = k$

$$\underbrace{\boxed{x = i} \qquad \boxed{y = 1}}_{\text{Eigen vector}}$$

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FIND EIGEN VALUE ?

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix}$$

Solution:

$(A - \lambda I)X = 0$ (GENREL EQUATION)

$|A - \lambda I| = 0$ (CHARACTERISTIC EQUATION)

$$\left| \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 4-\lambda & 5 \\ 0 & 4 & 3-\lambda \end{vmatrix} = 0$$

$(2-\lambda)[(4-\lambda)(3-\lambda)-20] - 0 + 0 = 0$

$(2-\lambda)[12-4\lambda-3\lambda+\lambda^2-20] = 0$

$(2-\lambda)(\lambda^2-7\lambda-8) = 0$

$2\lambda^2-14\lambda-16-\lambda^3+7\lambda^2+8\lambda = 0$

$-\lambda^3+9\lambda^2-6\lambda-16 = 0$

When $\lambda = 2$

$-(2)^3+9(2)^2-6(2)-16 = 0$

$-8+9(4)-12-16 = 0$

$-20+36-16 = 0$

$36-36 = 0$

$0 = 0$

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$(\lambda-2)$ (Quadratic Eq)

Synthetic Division:

$$\begin{array}{c|cccc} & -1 & 9 & -6 & -16 \\ 2 & \downarrow & -2 & 14 & 16 \\ \hline & -1 & 7 & 8 & \boxed{0} \end{array}$$

$$-\lambda^2+7\lambda+8=0$$

$$-(\lambda^2-7\lambda-8)=0$$

$$(\lambda-2)(\lambda^2-7\lambda-8)=0$$

$$\lambda-2=0 \qquad \lambda^2-7\lambda-8=0$$

$\boxed{\lambda=2}$ Eigen Value

$$\lambda^2-7\lambda-8=0$$

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \text{(QUADRATIC FORMULA)}$$

$$\lambda=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(-8)}}{2(1)}$$

$$=\frac{7\pm\sqrt{49+32}}{2}$$

$$=\frac{7\pm\sqrt{81}}{2} \Rightarrow \frac{7\pm9}{2}$$

$$\lambda=\frac{7+9}{2} \qquad \lambda=\frac{7-9}{2}$$

$$\lambda=\frac{16}{2} \qquad \lambda=\frac{-2}{2}$$

$\boxed{\lambda=8}$ Eigen Value $\boxed{\lambda=-1}$

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Diagonalize The Matrix

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$$

$(A-\lambda I)x = 0$ General Equation

$|A-\lambda I| = 0$ Characteristic Equation

$$\left| \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{vmatrix} = 0$$

$$[(4-\lambda)(3-\lambda)] - 6 = 0$$

$$12 - 4\lambda - 3\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\lambda(\lambda-1) - 6(\lambda-1) = 0$$

$$(\lambda-1)(\lambda-6) = 0$$

$\lambda - 1 = 0$

$\boxed{\lambda = 1}$ Eigen Value

$\lambda - 6 = 0$

$\boxed{\lambda = 6}$ Eigen Value

- if eigen values are repeated then we can't diagonalize the matrix

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Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ for $\lambda = 6$

$$\begin{bmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4-6 & 2 \\ 3 & 3-6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_2 \to 2R_2 + 3R_2$$

$-2x + 2y = 0$

$+2(-x + y) = 0$

$-x + y = 0$

$-x = -y \qquad \cancel{y = k}$

$\cancel{x = k}$

$x = y = k \qquad x = k \qquad$ we say $k = 1$

$\boxed{x = 1}$, $\boxed{y = 1}$ Eigen Vector

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} k$$

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ for $\lambda = 1$

$$\begin{bmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 4-1 & 2 \\ 3 & 3-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x + 2y = 0$$

$$3x = -2y$$

$$\frac{x}{-2} = \frac{y}{3} = k$$

$$x = -2k$$

$$y = 3k$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = k\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$ Eigen Vector

$$P = \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} \text{Adj } P$$

$$|P| = \begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} \Rightarrow -5$$

$$\text{Adj } P = \begin{bmatrix} 1 & -1 \\ -3 & -2 \end{bmatrix}$$

$$D = P^{-1}AP$$

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$$= \frac{1}{-5}\begin{bmatrix} 1 & -1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} 1 & -1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & 6 \end{bmatrix}$$

$$= \frac{-1}{5}\begin{bmatrix} -5 & 0 \\ 0 & -30 \end{bmatrix}$$

$$= \begin{bmatrix} -5/-5 & 0/-5 \\ 0/-5 & -30/-5 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} \text{Adj } P$$

$$|P| = \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix} \Rightarrow 5$$

$$\text{Adj } P = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$P^{-1} = \frac{1}{5}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

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(3x3)

$D = P^{-1}AP$

$$= \frac{1}{5}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ -3 & 6 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 0 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$$

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Application Of Differential Equation

$$\frac{du}{dt} = \lambda u$$

then Solution will be

$u = ce^{\lambda t}$ at $t = 0$

$u(0) = c$

$$\frac{du}{dt} = \lambda u$$

$$\frac{du}{u} = \lambda dt$$

$$\int \frac{du}{u} = \int \lambda dt$$

$\ln u = \lambda t + \ln c$

$\ln u - \ln c = \lambda t$

$\ln \frac{u}{c} = \lambda t$

$\cancel{e^{\ln \frac{u}{c}}} = e^{\lambda t}$

$\frac{u}{c} = e^{\lambda t}$

$u = ce^{\lambda t}$

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$$u(t) = Ce^{\lambda t} \qquad t = 0$$

$$u(0) = Ce^{\lambda(0)} \qquad u(0) = 5$$

$$u(0) = Ce^{0}$$

$$u(0) = C$$

$$5 = C$$

Matrix

$$\frac{du}{dt} = Au$$

Any function

Eigen Value

$$u(t) = C_i V_i e^{\lambda_i t} \qquad i = 1, 2, 3, \ldots n$$

Eigen Vector

For 2x2 $\quad u(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t}$

For 3x3 $\quad u(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t} + C_3 V_3 e^{\lambda_3 t}$

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Question 1

Find $\frac{du}{dt} = Au$ when $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

& $u(0) = (4, 2)$ Boundary Condition

Solution:

$$u(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t}$$

$$|A - \lambda I| = 0$$

Scanned with CamScanner

$u(t) = ce^{\lambda t}$ $\qquad t = 0$

$u(0) = ce^{\lambda(0)}$ $\qquad u(0) = 5$

$u(0) = ce^{0}$

$u(0) = c$

$5 = c$

$\dfrac{du}{dt} = A\,u$

Matrix

Any function

$u(t) = c_i V_i e^{\lambda_i t}$ $\qquad i = 1, 2, 3, \ldots n$

Eigen Value

Eigen Vector

For 2x2 $\quad u(t) = c_1 V_1 e^{\lambda_1 t} + c_2 V_2 e^{\lambda_2 t}$

For 3x3 $\quad u(t) = c_1 V_1 e^{\lambda_1 t} + c_2 V_2 e^{\lambda_2 t} + c_3 V_3 e^{\lambda_3 t}$

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Question 1

Find $\dfrac{du}{dt} = AU$ when $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

& $\underbrace{u(0) = (4, 2)}$ Boundary Condition

Solution:

$u(t) = c_1 V_1 e^{\lambda_1 t} + c_2 V_2 e^{\lambda_2 t}$

$|A - \lambda I| = 0$

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$$\left| \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\cancel{\lambda^2 = 1}$$

$$(\lambda - 1)(\lambda + 1) = 0$$

$$\lambda = -1 \qquad \lambda = 1$$

$$\lambda_1 = 1 \quad , \lambda_2 = -1$$

for $\lambda_1 = 1$ $\quad V_1 = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} -\lambda & 0 \\ 0 & -\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x + y = 0$$

$$-x = -y = k$$

$$x = y = k$$

$$x = 1 \quad , y = 1$$

$$\vec{V_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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for $\lambda_2 = -1$ $\vec{V_2} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} -\lambda & \phi \\ \phi & -\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x + y = 0$$

$$x = -y = k$$

$\boxed{x = 1}$ $\boxed{y = -1}$

$$u(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t}$$

$$u(t) = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{1t} + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

$$u(t) = \begin{bmatrix} C_1 e^{t} + C_2 e^{-t} \\ C_1 e^{t} - C_2 e^{-t} \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$= C_1 e^0 + C_2 e^0 = 4 \quad \text{--- (1)}$$

$$C_1 - C_2 = 2 \quad \text{--- (2)}$$

$$C_1 + \cancel{C_2} = 4$$

$$C_1 - \cancel{C_2} = 2$$

$$2C_1 = 6$$

$$C_1 = 6/2$$

$$\boxed{C_1 = 3}$$

$$C_1 + C_2 = 4$$

$$3 + C_2 = 4$$

$$C_2 = 4 - 3$$

$$\boxed{C_2 = 1}$$

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FIND $\frac{du}{dt} = Au$ when $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$u(0) = (6, 4)$

Solution:

$|A - \lambda I| = 0$ Characteristic

$$\left| \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\sqrt{\lambda^2} = \sqrt{-1}$$

$$\lambda = \pm i \qquad \boxed{\lambda_1 = i, \ \lambda_2 = -i} \text{ Eigen Value}$$

for $\lambda_1 = i$ $\quad v_1 = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i^2 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad R_1 \times i \to R_1$$

$$\begin{bmatrix} -1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad R_1 + R_2 \to R_2$$

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$-x + iy = 0$

$-x = -iy$

$x = iy = k$

$\frac{x}{i} = \frac{y}{1} = k$

$x = i \quad y = 1$

$$\boxed{V_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}}$$ Eigen Vector

For $\lambda_2 = -i \quad V_2 = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -(-i) & -1 \\ 1 & -(-i) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i^2 & -i \\ 1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 \times i \Rightarrow R_1$$

$$\begin{bmatrix} -1 & -i \\ 1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_1 + R_2 \rightarrow R_2$$

$-x - iy = 0$

$-x = iy = k$

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$$\frac{+x}{i} = \frac{y}{-1} = k$$

$+x = i(1)$ $\quad y = 1$

$x = +i$, $\quad y = -1$

$$\boxed{V_2 = \begin{bmatrix} +i \\ -1 \end{bmatrix}}$$ Eigen Vector.

* $u(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t}$

$$u(t) = C_1 \begin{bmatrix} i \\ 1 \end{bmatrix} e^{i(0)} + C_2 \begin{bmatrix} +i \\ -1 \end{bmatrix} e^{-i(0)}$$ at $t=0$

$$= C_1 \begin{bmatrix} i \\ 1 \end{bmatrix} e^{0} + C_2 \begin{bmatrix} +i \\ -1 \end{bmatrix} e^{0}$$

$$= C_1 \begin{bmatrix} i \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} +i \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} C_1 i + C_2 i \\ C_1 - C_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$ ~~where i=1~~

$C_1 i + C_2 i = 6$ —— (1)

$C_1 - C_2 = 4$ —(2) (Multiply by i)

$C_1 i - C_2 i = 4i$ —— (3)

Add eq (i) and (iii)

$$C_1 i + \cancel{C_2 i} = 6$$
$$C_1 i - \cancel{C_2 i} = 4i$$
$$2C_1 i = 10$$

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$$\frac{+x}{i} = \frac{y}{-1} = k$$

$+x = i(1)$ $y = 1$

$x = +i$, $y = -1$

$$\boxed{V_2 = \begin{bmatrix} +i \\ -1 \end{bmatrix}}$$ Eigen Vector.

* $u(t) = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t}$

$$u(t) = C_1 \begin{bmatrix} i \\ 1 \end{bmatrix} e^{i(0)} + C_2 \begin{bmatrix} +i \\ -1 \end{bmatrix} e^{-i(0)}$$ at $t=0$

$$= C_1 \begin{bmatrix} i \\ 1 \end{bmatrix} e^{0} + C_2 \begin{bmatrix} +i \\ -1 \end{bmatrix} e^{0}$$

$$= C_1 \begin{bmatrix} i \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} +i \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} C_1 i + C_2 i \\ C_1 - C_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$ ~~where i=1~~

$C_1 i + C_2 i = 6$ —— (1)

$C_1 - C_2 = 4$ —— (2) (Multiply by i)

$C_1 i - C_2 i = 4i$ —— (3)

Add eq (i) and (iii)

$$C_1 i + \cancel{C_2 i} = 6$$
$$C_1 i - \cancel{C_2 i} = 4i$$
$$\overline{2C_1 \cancel{i} = 10\cancel{i}}$$

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$$2C_1 = 10$$

$$C_1 = 10/2$$

$$\boxed{C_1 = 5}$$

Put the Value of C_1 in (ii)

$$C_1 \mp C_2 = 4$$

$$5 \mp C_2 = 4$$

$$\boxed{C_2 = +1}$$

$$u(t) = C_1 V_1 e^{\lambda t} + C_2 V_2 e^{\lambda t}$$

$$u(t) = 5\begin{bmatrix} i \\ 1 \end{bmatrix} e^{+it} + 1\begin{bmatrix} +i \\ -1 \end{bmatrix} e^{-it}$$

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Diagonalize the Matrix:

$$A = \begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix}$$

Solution:

$$D = P^{-1}AP$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 6 \\ 0 & -1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(-1-\lambda) - 0 = 0$$

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$$-2-2\lambda+\lambda+\lambda^2=0$$
$$\lambda^2-\lambda-2=0$$
$$\lambda^2+\lambda-2\lambda-2=0$$
$$\lambda(\lambda+1)-2(\lambda+1)=0$$
$$(\lambda+1)(\lambda-2)=0$$
$$\boxed{\lambda=-1} \qquad \boxed{\lambda=2}$$

Eigen Value

For $\lambda=-1$ $\quad X=\begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} 2-\lambda & 6 \\ 0 & -1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=0$$

$$\begin{bmatrix} 2-(-1) & 6 \\ 0 & -1-(-1) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+1 & 6 \\ 0 & -1+1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x+6y=0$$
$$3(x+2y)=0$$
$$x+2y=0$$
$$x=-2y=k$$
$$\frac{x}{-2}=\frac{y}{1}=k$$

$$x=-2 \qquad y=1$$

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$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} k}$$ Eigen Vector

For $\lambda = 2$

$$\begin{bmatrix} 2-2 & 6 \\ 0 & -1-2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_2 \to 2R_1$$

$$\begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad R_2 \to R_1 + R_2$$

$$0x + 6y = 0$$

$$0x = -6y$$

$$\frac{x}{-6} = \frac{y}{0} = k \quad \text{we say } k = 1$$

$$x = -6 \quad , \quad y = 0$$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \end{bmatrix} k}$$ Eigen Vector.

$$P = \begin{bmatrix} -2 & -6 \\ 1 & 0 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} \text{Adj } P$$

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$$|P| = \begin{vmatrix} -2 & -6 \\ 1 & 0 \end{vmatrix} \Rightarrow 6$$

$$\text{Adj } P = \begin{bmatrix} 2 & 6 \\ -1 & -2 \end{bmatrix}$$

$$P^{-1} = \frac{1}{6}\begin{bmatrix} 0 & 6 \\ -1 & -2 \end{bmatrix}$$

DIAGONAL FORMULA:

$$D = P^{-1}AP$$

$$D = \frac{1}{6}\begin{bmatrix} 0 & 6 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & -6 \\ 1 & 0 \end{bmatrix}$$

$$D = \frac{1}{6}\begin{bmatrix} 0 & 6 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ -1 & 0 \end{bmatrix}$$

$$D = \frac{1}{6}\begin{bmatrix} -6 & 0 \\ 0 & 12 \end{bmatrix}$$

$$\boxed{D = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}}$$ ANSWER

DIAGONALIZE THE MATRIX 3×3

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Solution:

$$D = P^{-1}AP$$

Eigen Value:

$|A - \lambda I| =$

$\left|\begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & - \end{bmatrix}\right.$

$\left|\begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 0 & 0 \end{bmatrix}\right.$

$\left|\begin{bmatrix} 1-\lambda & 3 \\ 3 & 1- \\ 0 & 0 \end{bmatrix}\right.$

$(1-\lambda)\left[(1-\lambda)\right.$

$(1-\lambda)\left[(-2-\right.$

$(1-\lambda)(-2+$

$-2+\lambda+\lambda^2+$

$-\lambda^3+12\lambda+$

when $\lambda =$

$-(-2)^3+12($

$-(-8)-2$

$8-$

0

Eigen Value:

$$|A-\lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-\lambda & 3 & 0 \\ 3 & 1-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{bmatrix} \right| = 0$$

$$(1-\lambda)\left[(1-\lambda)(-2-\lambda)-0\right] - 3\left(3(-2-\lambda)-0\right) + 0$$

$$(1-\lambda)\left[(-2-\lambda+2\lambda+\lambda^2)\right] - 3(-6-3\lambda) = 0$$

$$(1-\lambda)(-2+\lambda+\lambda^2) + 18 + 9\lambda = 0$$

$$-2+\lambda+\cancel{\lambda^2}+2\lambda-\cancel{\lambda^2}-\lambda^3+18+9\lambda = 0$$

$$-\lambda^3 + 12\lambda + 16 = 0$$

When $\lambda = -2$

$$-(-2)^3 + 12(-2) + 16 = 0$$

$$-(-8) - 24 + 16 = 0$$

$$8 - 8 = 0$$

$$0 = 0$$

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Synthetic Division:

-2	-1	0	12	16
	$\downarrow$	2	-4	-16
	-1	2	8	0

$$-\lambda^2+2\lambda+8$$

$$(\lambda+2)(-\lambda^2+2\lambda+8)=0$$

$$\lambda+2=0$$

$$\lambda=-2$$

$$-\lambda^2+2\lambda+8=0$$

$$-\lambda^2-2\lambda+4\lambda+8=0$$

$$-\lambda(\lambda+2)+4(\lambda+2)=0$$

$$(-\lambda+4)(\lambda+2)=0$$

$$\lambda+2=0 \qquad -\lambda+4=0$$

$$\lambda=-2 \qquad \lambda=4$$

$$\boxed{\lambda=-2,\ \lambda=-2,\ \lambda=4}$$

Eigen Values:

Two eigen values are same then we can't diagonalize this matrix. $(\lambda=-2,\ \lambda=-2)$

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Solution:

Eigen Values:

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(4-\lambda)(6-\lambda) - 0\right] - 2(0) + 3(0) = 0$$

$$(1-\lambda)\left[24 - 4\lambda - 6\lambda + \lambda^2\right] - 0 + 0 = 0$$

$$(1-\lambda)(\lambda^2 - 10\lambda + 24) = 0$$

$$\lambda^2 - 10\lambda + 24 - \lambda^3 + 10\lambda^2 - 24\lambda = 0$$

$$-\lambda^3 + 11\lambda^2 - 34\lambda + 24 = 0$$

when $\lambda = 1$:

$$-(1)^3 + 11(1)^2 - 34(1) + 24 = 0$$

$$-1 + 11 - 34 + 24 = 0$$

$$10 - 34 + 24 = 0$$

$$34 - 34 = 0$$

$$0 = 0$$

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$(\lambda-1)$ (Quadratic Equation)

Synthetic Division:

$$\begin{array}{c|cccc} 1 & -1 & 11 & -34 & 24 \\ & \downarrow & -1 & 10 & -24 \\ \hline & -1 & 10 & -24 & \multicolumn{1}{|c}{0} \end{array}$$

$$-\lambda^2+10\lambda-24=0$$

$$-(\lambda^2-10\lambda+24)=0$$

$$\lambda^2-10\lambda+24=0$$

$$(\lambda-1)(\lambda^2-10\lambda+24)=0$$

$$\lambda-1=0$$

$$\lambda=1$$

$$\lambda^2-10\lambda+24=0$$

$$\lambda^2-4\lambda-6\lambda+24=0$$

$$\lambda(\lambda-4)-6(\lambda-4)=0$$

$$(\lambda-4)(\lambda-6)=0$$

$\lambda-4=0$ $\quad\quad$ $\lambda-6=0$

$\lambda=4$ $\quad\quad$ $\lambda=6$

$$\boxed{\lambda=1,\ \lambda=4,\ \lambda=6}$$

Eigen Vector:

Let $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for $\lambda=1$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1-1 & 2 & 3 \\ 0 & 4-1 & 5 \\ 0 & 0 & 6-1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x + 2y + 3z = 0$

$0x + 3y + 5z = 0$

$0x + 0y + 5z = 0$

let $z = k$

$3y + 5k = 0$

$y = -5$

$$\frac{x}{\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 0 & 3 \\ 0 & 5 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}}$$

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0} = k$$

$\frac{x}{1} = k$

$x = 1k$

$y = 0k$

$z = 0k$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} k}$$

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Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\lambda = 4$

$$\begin{bmatrix} 1-4 & 2 & 3 \\ 0 & 4-4 & 5 \\ 0 & 0 & 6-4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x + 2y + 3z = 0$

$0x + 0y + 5z = 0$

$0x + 0y + 2z = 0$

$$\frac{x}{\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -3 & 3 \\ 0 & 5 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -3 & 2 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x}{10} = \frac{y}{-15} = \frac{z}{0} = k$$

$x = 10$, $y = -15k$, $z = 0k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -15 \\ 0 \end{bmatrix} k$$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\lambda = 6$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x + 2y + 3z = 0,$

$0x - 2y + 5z = 0$

$0x + 0y + 0z = 0$

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Positive Definite:

A Square matrix A is positive definite if A is Symmetric matrix any One of the following is true.

i) All eigen Value are Positive. (>0)

ii) All its Pivots are positive (without changing row. (in echolon form)

iii) All upper left determinant of 1, 2, — n of an nxn matrix A are +tive.

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Test whatever A^TA is positive definite if $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$

Solution:

$A^T A = ?$

- First we find A^T

$$A^t = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

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$$A^TA = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$B = A^TA = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$$

- This matrix is Symmetric and Square.

Eigen Value :

$|B - \lambda I| = 0$

$$\left| \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 6-\lambda & 5 \\ 5 & 6-\lambda \end{vmatrix} = 0$$

$(6-\lambda)^2 - (5)^2 = 0$

$(6-\lambda-5)(6-\lambda+5) = 0$

$(1-\lambda)(11-\lambda) = 0$

$1-\lambda = 0 \qquad 11-\lambda = 0$

$\lambda = 1 > 0 \qquad \lambda = 11 > 0$

- Both eigen Values are greater than ~~posi~~ 0 mean positive eigen Value.

Make Echolen form:

$$B = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 5/6 \\ 5 & 6 \end{bmatrix} \quad R_1 \Rightarrow R_1/6$$

$$\begin{bmatrix} 1 & 5/6 \\ 0 & 11/6 \end{bmatrix} \quad R_2 \rightarrow R_2 - 5R_1$$

- Pivots element are positive 1 & $11/6$

All upper left determinant Checking:

$$\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$$

$|6| = 6 > 0$ First order det positive

$\begin{vmatrix} 6 & 5 \\ 5 & 6 \end{vmatrix}$ Second order determinant Positive.

$36 - 25 = 11 > 0$

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WEDNESDAY 25 JANUARY 2022

SINGULAR VALUE DECOMPOSITION

- We have one matrix and this matrix is always Triangular Matrix. Order of this matrix $A_{m\times n}$.
- $A_{m\times n} \leftarrow A = U \Sigma V^t$ $\quad U_{m\times m}$ $\quad V^t_{n\times n}$

  U = Orthogonal matrix Rotation

  Σ = Diagonal matrix Stretching

  V^t = Orthonormal matrix Rotation

- $U_i = \frac{AV_i}{\delta_i}$ $\quad \delta_i = \sqrt{\lambda_i}$

  A → Given Matrix

  V_i → First column of Orthonormal

  δ_i → Greater value Sigma

  $A_{2\times 3}$ $B_{3\times 2}$

  $AB_{2\times 2}$

- U_r $m\times m$ $\quad \Sigma_{m\times n}$ $\quad V^t_{n\times n}$

$A_{m\times n} = U_{m\times \cancel{m}} \cdot \Sigma_{\cancel{m}\times n} \cdot V^t_{n\times n}$

$A_{m\times n} = U \cdot \Sigma_{m\times \cancel{n}} \cdot V^t_{\cancel{n}\times n}$

$A_{m\times n} = U \Sigma V^t_{m\times n}$

$\Sigma \rightarrow$ Stretching

U and $V^t \rightarrow$ Rotation

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Σ

Taking Transpose (U and V^t ...

- if $A = U \Sigma V^t$ $\quad (AB)^t = B^t A^t$

$A^t = V \Sigma^t U^t$

$A^t A = V \Sigma^t \underbrace{U^t \cdot U}_{\text{IDENTITY}} \Sigma V^t$

$\because \Sigma^t \cdot \Sigma = \Sigma^2$

$A^t A = V \Sigma^2 V^t$

$V = \boxed{A^tA \text{ ky eigen Vector}}$

A^tA ky eigen Vector isy yeh matrix V hoga.

$\delta i \quad \delta_i = \sqrt{\lambda_i}$

$\delta_1 > \delta_2 > \delta_3 \qquad \lambda_1 = 1, \lambda_2 = 2$

$\lambda_1^2 = \Sigma^2 = \Sigma_i$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$$

$$A^tA = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$$

3 eigen Value and 3 Sigma Value.

- First A^tA
- Then eigen value and Then find eigen Vector $\rightarrow V$

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- $V_1 = [1, 2, 4] = \sqrt{1^2+2^2+4^2} \Rightarrow \sqrt{21}$

$V \leftarrow$ • $V_1 = \frac{1}{\sqrt{21}}[1, 2, 4]$ ORTHONORMAL

We Take Transpose V.

- $AA^T = U\Sigma \underbrace{V^t \cdot V}_{\text{IDENTITY}} \Sigma^t U^t$
- $AA^t = U\Sigma^2 U^t$
- AA^t find (Squar Matrix)
- eigen Value
- eigen Vector (U). and this Vector make orthonormal.

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21 Jan 2022 THURSDAY

Q: $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

Solution:

$$A^t = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}^t$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^t A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$A^t A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Eigen Value:

$|A^t A - \lambda I| = 0$ Characteristic Equation:

$$\left| \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} = 0$$

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$$\lambda^3 - \lambda^2(3) + \lambda(1+0+1) - (1+(-1)) = 0$$
$$\lambda^3 - 3\lambda^2 + 2\lambda + 0 = 0$$
$$\lambda^3 - 3\lambda^2 + 2\lambda = 0$$
$$\lambda(\lambda^2 - 3\lambda + 2) = 0$$
$$\boxed{\lambda_3 = 0}$$
$$\lambda^2 - 3\lambda + 2 = 0$$
$$\lambda^2 - \lambda - 2\lambda + 2 = 0$$
$$\lambda(\lambda-1) - 2(\lambda-1) = 0$$
$$(\lambda-1)(\lambda-2) = 0$$
$$\lambda - 2 = 0 \quad , \quad \lambda - 1 = 0$$
$$\boxed{\lambda_1 = 2} \quad , \quad \boxed{\lambda_2 = 1}$$

Taking Greater Value of λ is δ_1 :

$$\boxed{\delta_1 = \sqrt{2}}$$
$$\boxed{\delta_2 = \sqrt{1} \Rightarrow 1}$$
$$\boxed{\delta_3 = \sqrt{0} \Rightarrow 0}$$

Sigma. $\Sigma = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (2×3)

Eigon Vector:

$$(A - \lambda I)X = 0$$
$$\begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix}$$

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$$V = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{pmatrix}$$

$$\begin{bmatrix} 1-2 & 0 & 1 \\ 0 & 1-2 & 0 \\ 1 & 0 & 1-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$y = 0$

$-x + z = 0$

$x = 1,\ y = 1$

$\sqrt{1^2 + 0^2 + 1^2}$

$$\vec{V_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad V = \begin{pmatrix} \lambda=2 & \lambda=1 & \lambda=0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}^t$$

$$V_1 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \qquad (\vec{V})^t = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

When $\lambda = 1$

$$\begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 1-1 & 0 & 1 \\ 0 & 1-1 & 0 \\ 1 & 0 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x = 0$

$z = 0$

let Suppose $y = k$

We Say $k = 1$

$y = 1$

$$\vec{V_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \sqrt{1+0+0} \Rightarrow \sqrt{1} = 1$$

$$\vec{V_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

When $\lambda = 0$

$$\begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-0 & 0 & 1 \\ 0 & 1-0 & 0 \\ 1 & 0 & 1-0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$y = 0$

$x = -1$

$z = 1$

$$\vec{V_3} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{V_3} = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$(\vec{V})^t = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}^t$$

$$\vec{V}^t = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

Now we find U:

$$U_i = \frac{Av_i}{s_i} \qquad i = 1, 2, 3$$

$$U_1 = \frac{Av_1}{s_1}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

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$\frac{2}{\sqrt{2}}$ $\quad \frac{\sqrt{2}\cdot\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$ $\quad \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$ 1

$$U_1 = \frac{\begin{bmatrix} 1/\sqrt{2} + 0 + 1/\sqrt{2} \\ 0 + 0 \end{bmatrix}}{\sqrt{2}}$$

$$U_1 = \frac{\begin{bmatrix} 1/\sqrt{2} + \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}}{\sqrt{2}}$$

$$U_1 = \frac{\begin{bmatrix} 2/\sqrt{2} \\ 0 \end{bmatrix}}{\sqrt{2}}$$

$$U_1 = \frac{\begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}}{\sqrt{2}}$$

$$\boxed{U_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$U_2 = \frac{A V_2}{\delta_2}$$

$$U_2 = \frac{\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{1}$$

$$U_2 = \frac{\begin{bmatrix} 0 + 0 + 0 \\ 0 - 1 \quad 0 \end{bmatrix}}{1}$$

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$$U_2 = \frac{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}{1}$$

$$U_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\boxed{U = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1+0+0+\cancel{\sqrt{2}}(-1) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Big[\quad \Big]$$

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$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}_{3\times 2}$$

Solution

$A_{3\times 2}$, $\Sigma_{3\times 2}$, $V^t_{2\times 2}$, $U_{3\times 3}$

$$A^T = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

Eigen Value:

$$|A^t A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) = 0$$

$2-\lambda = 0$ $\qquad$ $3-\lambda = 0$

$\boxed{\lambda_2 = 2}$ $\qquad$ $\boxed{\lambda_1 = 3}$

$\sigma_1 = \sqrt{3}$, $\sigma_2 = \sqrt{2}$

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Sigma Matrix:

$$\boxed{\Sigma = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix}}$$ Answer

Eigen Vector:

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ for $\lambda = 3$

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-3 & 0 \\ 0 & 3-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x = 0$

$y = k$ $\quad k = 1$

$y = 1$

$$\boxed{\vec{V_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$V_1 = \sqrt{0^2 + 1^2}$

$= \sqrt{1} \Rightarrow 1$

$$\boxed{V_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

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Eigen Vector:

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ for $\lambda = 2$

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-2 & 0 \\ 0 & 3-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x = k \qquad x = 1$

$y = 0$

$$\boxed{V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$V_2 = \sqrt{1^2 + 0^2}$

$V_2 \quad \sqrt{1} \Rightarrow 1$

$$\boxed{\vec{V_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\boxed{\vec{V}^t = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$ Answer

Now we find U:

$$u_i = \frac{Av_i}{\delta_i}$$

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$$U_1 = \frac{AV_1}{\delta_1}$$

$$U_1 = \frac{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\sqrt{3}}$$

$$U_1 = \frac{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\sqrt{3}}$$

$$\boxed{U_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}}$$

$$U_2 = \frac{AV_2}{\delta_2}$$

$$U_2 = \frac{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\sqrt{2}}$$

$$U_2 = \frac{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}{\sqrt{2}} \Rightarrow \boxed{U_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}}$$

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To Find 3RD Eigen Vector And Vector U:

$$AA^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Eigen Value:

$$|AA^T - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(1-\lambda)(2-\lambda)-1\right] - 1(2-\lambda-0)$$

$$(2-\lambda)\left[2-\lambda-2\lambda+\lambda^2-1\right] - 2+\lambda$$

$$(2-\lambda)(\lambda^2-3\lambda+1) - 2+\lambda$$

$$2\lambda^2 - 6\lambda + \cancel{2} - \lambda^3 + 3\lambda^2 - \cancel{\lambda} - \cancel{2} + \cancel{\lambda}$$

$$-\lambda^3 + 5\lambda^2 - 6\lambda = 0$$

$$-(\lambda^3 - 5\lambda^2 + 6\lambda) = 0$$

$$\lambda^3 - 5\lambda^2 + 6\lambda = 0$$

$$\lambda(\lambda^2 - 5\lambda + 6) = 0$$

$\boxed{\lambda = 0}$ $\qquad \lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$$\lambda(\lambda-2) - 3(\lambda-2) = 0$$

$$(\lambda-2)(\lambda-3) = 0$$

$$\boxed{\lambda = 2}, \quad \boxed{\lambda = 3}$$

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Eigen Vector:

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for $\lambda = 0$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x}{\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{x}{1} = \frac{-y}{2} = \frac{z}{1}$$

$x = 1, \quad y = -2, \quad z = 1$

$$\boxed{X = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}$$ Answer

$= \sqrt{(1)^2 + (-2)^2 + (1)^2}$

$= \sqrt{1 + 4 + 1}$

$\sqrt{6}$

$$= \frac{1}{\sqrt{3}} / \frac{1}{\sqrt{2}} + 0 - \frac{1}{\sqrt{3}} / \frac{1}{\sqrt{2}}$$

$= 0$

$$\boxed{u_2 = \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}}$$ Answer

$$\boxed{U = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}}$$

Final U

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2 Feb 2022 Wednesday

Condition Of Matrix & Norms Of Matrix :

i) ill Pose Condition

ii) Well pose Condition $AX = C \rightarrow$ Solution Set

Small change in A → Small Change in C this is called well pose Condition.

~~Small~~ $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $x_1 = 2, x_2 = 3$

$\begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$x_1 = 2.3, \; x_2 = 3.2$

Small change → Well pose Condition

Small $\Delta A \rightarrow$ Large ΔA = ill pose Condition.

Norms of Matrix :

Sum of rows is called Norm of Matrix.

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Maximum

$\|A\|_\infty$ = Maximum rows of Sum

$$\begin{bmatrix} 2 & 2 \\ -4 & -3 \end{bmatrix}$$

$\|A\|_1 = |2| + |2| = 4$

$\|A\| = |-4| + |-3| = 7$

- Sign are not consider because absolute value is always +ive.

FORMULA FOR CONDITION NUMBER:

$\text{Cond}[A] = \|A\|_\infty \times \|A^{-1}\|_\infty$

- If $\text{Cond}[A] \approx 1$ then matrix will be well pose condition.
- If $\text{Cond}[A] > 1$ then matrix will be ill pose condition.

$$\begin{array}{ccc} A & X & = \quad C \\ \begin{bmatrix} 400 & -201 \\ -800 & 401 \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} & = \begin{bmatrix} 200 \\ -200 \end{bmatrix} \end{array}$$

Matrix will be ill or well condition:

Solution:

$400x_1 - 201x_2 = 200$ —— (1)

$-800x_1 + 401x_2 = -200$ —— (2)

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(1) Multiply by 2.

$$800x_1 - 402x_2 = 400 \quad \text{---(3)}$$

Add Eq. (2) and Eq. (3)

$$\cancel{800}x_1 - 402x_2 = 400$$
$$\cancel{-800}x_1 + 401x_1 = -200$$
$$-x_2 = +200$$

$$\boxed{x_2 = -200}$$

Put the value of x_2 in

$$400x_1 - 201x_2 = 200$$
$$400x_1 = 200 + 201(-200)$$
$$x_1 = \frac{200 - 40200}{400}$$
$$x_1 = \frac{-40000}{400}$$

$$\boxed{x_1 = -100}$$

Small change in Matrix A:

$$\begin{bmatrix} 401 & -201 \\ -800 & 401 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}$$

Make augmented FORM,

$$\left[\begin{array}{cc|c} 401 & -201 & 200 \\ -800 & 401 & -200 \end{array}\right]$$

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$$\frac{160.801 - 160800}{\frac{1}{401}}$$

$$\left[\begin{array}{cc|c} 1 & -201/401 & 200/401 \\ -800 & 401 & -200 \end{array}\right] \quad R_1/401 \to R_1$$

$$\left[\begin{array}{cc|c} 1 & -201/401 & 200/401 \\ 0 & 0.002493 & \end{array}\right] \quad R_2 \to R_2 + 800R_1$$

we are Small Chang in A but large Change in Solution Matrix occur then Solution is ill Condition

$$\text{Cond}[A] = \|A\|_\infty \times \|A^{-1}\|_\infty$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 400 & -201 \\ -800 & 401 \end{vmatrix}$$

$$|A| = 160400 - 160800$$

$$\boxed{|A| = -400}$$

$$\text{Adj } A = \begin{bmatrix} 400 & 201 \\ 800 & 400 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-400}\begin{bmatrix} 401 & 201 \\ 800 & 400 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 1.0025 & 0.5025 \\ -2 & -1 \end{bmatrix}}$$

$$\|A^{-1}\|_\infty = |-2| + |-1|$$

$$\boxed{\|A^{-1}\|_\infty = 3}$$

$$\|A\|_\infty = |-800| + |401|$$

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$\boxed{\|A\|_\infty = 1201}$ Answer

Cond $[A] = \|A\|_\infty \times \|A^{-1}\|_\infty$

Cond $[A] = 1201 \times 3$

$\boxed{\text{Cond } [A] = 3603 > 1}$ Answer

ill pose Condition

$$\begin{bmatrix} 5 & 4 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$ Check That ill or well pose Condition?

Solution:

Cond $[A] = \|A\|_\infty * \|A^{-1}\|_\infty$

First we find A^{-1}

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$|A| = \begin{vmatrix} 5 & 4 \\ 3 & 3 \end{vmatrix}$$

$\boxed{|A| = 15 - 12 = 3 \neq 0}$

$$\text{Adj } A = \begin{bmatrix} 3 & -4 \\ -3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -4 \\ -3 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/3 & -4/3 \\ -3/3 & 5/3 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 1 & 1.3333 \\ -1 & 1.6667 \end{bmatrix}}$$

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$\|A\|_\infty = |5| + |4|$

$\boxed{\|A\|_\infty = 9}$

$\|A^{-1}\|_\infty = |-1| + |1.6667|$

$\boxed{\|A^{-1}\|_\infty = 2.6667}$

Now Applying FORMULA :

$Cond[A] = \|A\|_\infty * \|A^{-1}\|$

$Cond[A] = 9 * 2.6667$

$\boxed{Cond[A] = 24 > 1}$ Answer

- ill pose Condition.

$$\begin{bmatrix} 200 & -400 \\ -600 & 400 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 400 \\ -600 \end{bmatrix}$$

Solution:

$Cond[A] = \|A\|_\infty * \|A^{-1}\|_\infty$

Firstly we find A^{-1}.

$$A^{-1} = \frac{adj A}{|A|}$$

$$|A| = \begin{vmatrix} 200 & -400 \\ -600 & 400 \end{vmatrix}$$

$|A| = 80,000 - 240,000$

$\boxed{|A| = -160,000}$

$$\boxed{Adj A = \begin{bmatrix} 400 & 900 \\ 600 & 200 \end{bmatrix}}$$

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0.005
0.005

$$A^{-1} = -\frac{1}{160{,}000}\begin{bmatrix} 400 & 400 \\ 600 & 200 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{400}{160{,}000} & -\frac{400}{160000} \\ \frac{600}{160000} & \frac{200}{160000} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -0.0025 & -0.0025 \\ 0.00375 & 0.00125 \end{bmatrix}$$

$$\|A\|_\infty = |-600| + |400|$$

$$\boxed{\|A\|_\infty = 1000}$$

$$\|A^{-1}\|_\infty = |0.00375| + |0.00125|$$

$$\boxed{\|A^{-1}\|_\infty = 0.005}$$

Now Applying FORMULA:

$$\text{Cond}[A] = \|A\|_\infty * \|A^{-1}\|_\infty$$

$$\text{Cond}(A) = 1000 \times 0.005$$

$$\boxed{\text{Cond}[A] = 5 > 1}$$ ANSWER

- This is ill pose condition.

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3 Feb 2022 THURSDAY

ITTERATIVE METHOD : INDIRECT METHOD

Jacobi METHOD

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4 = 0$$

$$b_1x_1 + b_2x_2 + b_3x_3 + b_4 = 0$$

$$c_1x_1 + c_2x_2 + C_3x_3 + C_4 = 0$$

$$x_1^{(k+1)} = \frac{-1}{a_1}\left[a_2x_2^{(k)} + a_3x_3^{(k)} + a_4\right]$$

$$x_2^{(k+1)} = \frac{-1}{b_2}\left[b_1x_1^{(k)} + b_3x_3^{(k)} + b_4\right]$$

$$x_3^{(k+1)} = \frac{-1}{C_3}\left[C_1x_1^{(k)} + C_2x_2^{(k)} + C_4\right]$$

Relationship :

$$|a_1| > |a_2| + |a_3|$$

$$|b_2| > |b_1| + |b_3|$$

$$|c_3| > |c_1| + |c_2|$$

INITIAL ITTERATION :

$$x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$$

$$x_1^{(1)} = 2 \quad , x_2^{(1)} = 1 \, , \, x_3^{(1)} = 4$$

2nd itteration:

$$x_1^{(2)} = \frac{-1}{a_1}\left[x_2^{(1)} + x_3^{(1)}\right]$$

Put the value of previous itteration.

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$$\overset{\text{Greater}}{10}x_1 + 2x_2 + 4x_3 = 3$$

$$2x_1 + \overset{\text{Greater}}{10}x_2 - x_3 = 4$$

$$x_1 - 4x_2 + \overset{\text{Greater}}{10}x_3 = 1$$

Solution:

$$x_1^{(k+1)} = \frac{1}{10}\left[3 - 2x_2^{(k)} - 4x_3^{(k)}\right]$$

$$x_2^{(k+1)} = \frac{1}{10}\left[4 - 2x_1^{(k)} + x_3^{(k)}\right]$$

$$x_3^{(k+1)} = \frac{1}{10}\left[1 - x_1^{(k)} + 4x_2^{(k)}\right]$$

when $k=0$ $\left(x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0\right)$

1st itteration: put $k=0$ in (1) (2) (3)

$$x_1^{(1)} = \frac{1}{10}\left[3 - 2x_2^{0} - 4x_3^{0}\right]$$

$$x_2^{(1)} = \frac{1}{10}\left[4 - 2x_1^{0} + x_3^{0}\right]$$

$$x_3^{(1)} = \frac{1}{10}\left[1 - x_1^{0} + 4x_2^{0}\right]$$

$$x_1^{(1)} = \frac{1}{10}\left[3 - 2(0) - 4(0)\right]$$

$$x_2^{(1)} = \frac{1}{10}\left[4 - 2(0) + (0)\right]$$

$$x_3^{(1)} = \frac{1}{10}\left[1 - 0 + 4(0)\right]$$

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$$x_1^{(1)} = \frac{3}{10}$$

$$\boxed{x_1^{(1)} = 0.3}$$

$$x_2^{(1)} = \frac{4}{10}$$

$$\boxed{x_2^{(1)} = 0.4}$$

$$x_3^{(1)} = \frac{1}{10}$$

$$\boxed{x_3^{(1)} = 0.1}$$

2nd itteration: $k = 1$

When $k=1$ $x_1^{(1)} = x_2^{(1)} = x_3^{(1)} = 1$

$$\therefore x_1^{(2)} = \frac{1}{10}\left[3 - 2(0.4)^{(1)} - 4(0.1)^{(1)}\right]$$

$$x_2^{(2)} = \frac{1}{10}\left[4 - 2(0.3)^{(1)} + (0.1)^{(1)}\right]$$

$$x_3^{(2)} = \frac{1}{10}\left[1 - (0.3)^{(1)} + 4(0.4)^{(1)}\right]$$

$$\boxed{x_1^{(2)} = 0.18}$$

$$\boxed{x_2^{(2)} = 0.35}$$

$$\boxed{x_3^{(2)} = 0.23}$$

3RD ITTERATION: $(k = 2)$

$x_1^{(1)}$

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$$x_1^{(3)} = \frac{1}{10}\left[3-2(0.35)-4(0.23)\right]$$

$$x_2^{(3)} = \frac{1}{10}\left[4-2(0.18)+(0.23)\right]$$

$$x_3^{(3)} = \frac{1}{10}\left[1-(0.18)+4(0.35)\right]$$

$$x_1^{(3)} = 1.38 \Rightarrow 0.138$$
$$x_2^{(3)} = 3.87 \Rightarrow 0.387$$
$$x_3^{(3)} = 2.22 \Rightarrow 0.222$$

4^{th} ITTERATION: $k=3$

$$x_1^{(4)} = \frac{1}{10}\left[3-2(0.387)-4(0.222)\right]$$

$$x_2^{(4)} = \frac{1}{10}\left[4-2(0.222)+(0.138)\right]$$

$$x_3^{(4)} = \frac{1}{10}\left[1-(0.138)+4(0.387)\right]$$

$$x_1^{(4)} = 0.1338$$

$$x_2^{(4)} = 0.3694$$

$$x_3^{(4)} = 0.241$$

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5TH ITTERATION : $k=4$

$$x^5 = \frac{1}{10}\left[3 - 2(0.3694) - 4(0.241)\right]$$

$$y^5 = \frac{1}{10}\left[4 - 2(0.241) + 0.1338\right]$$

$$z^5 = \frac{1}{10}\left[1 - 0.1338 + 4(0.3694)\right]$$

$x^5 = 0.12972$

$y^5 = 0.36518$

$z^5 = 0.23438$

6th itteration : $k=5$

$$x^6 = \frac{1}{10}\left[3 - 2(0.36518) - 4(0.23438)\right]$$

$$y^6 = \frac{1}{10}\left[4 - 2(0.23438) + 0.12972\right]$$

$$z^6 = \frac{1}{10}\left[1 - 0.12972 + 4(0.36518)\right]$$

$x^6 = 0.133212$

$y^6 = 0.366096$

$z^6 = 0.2331$

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JABCOBI METHOD

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

Solution:

$$x^{k+1} = \frac{1}{10}\left[3 + 5y^{k} + 2z^{k}\right]$$

$$y^{k+1} = \frac{1}{10}\left[+3 + 3z^{k} + 4x^{k}\right]$$

$$z^{k+1} = \frac{1}{10}\left[-3 - x^{k} - 6y^{k}\right]$$

1st ITTERATION when $k = 0$

$$x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$$

$$x^{1} = \frac{1}{10}\left[3 + 5(y)^{0} + 2z^{0}\right]$$

$$y^{1} = \frac{1}{10}\left[3 + 3(z)^{0} + 4(x)^{0}\right]$$

$$z^{1} = \frac{1}{10}\left[-3 - x^{0} - 6y^{0}\right]$$

$$x^{1} = \frac{1}{10}\left[3\right]$$

$$y = \frac{1}{10}\left[3\right]$$

$$z = \frac{1}{10}(-3)$$

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$x^1 = 0.3$

$y^1 = 0.3$

$z^1 = -0.3$

2nd Itteration when k = 1

$$x^2 = \frac{1}{10}\left[3 + 5(0.3) + 2(-0.3)\right]$$

$$y^2 = \frac{1}{10}\left[3 + 3(-0.3) + 4(0.3)\right]$$

$$z^2 = \frac{1}{10}\left[-3 - (+0.3) - 6(0.3)\right]$$

$x^2 = 3.9 \Rightarrow 0.39$

$y^2 = 3.3 \Rightarrow 0.33$

$z^2 = -5.1 \Rightarrow -0.51$

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3RD Itteration k = 2

$$x^3 = \frac{1}{10}\left[3 + 5(0.33) + 2(-0.51)\right]$$

$$y^3 = \frac{1}{10}\left[3 + 3(-0.51) + 4(0.39)\right]$$

$$z^3 = \frac{1}{10}\left[-3 - 0.39 - 6(0.33)\right]$$

$x^3 = 0.363$

$y^3 = 0.303$

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$z^3 = -0.495$

4^{th} itteration $k = 3$

$$x^4 = \frac{1}{10}\left[3 - 5(0.303) + 2(-0.495)\right]$$

$$y^4 = \frac{1}{10}\left[3 + 3(-0.495) + 4(0.363)\right]$$

$$z^4 = \frac{1}{10}\left[-3 - 0.363 - 6(0.303)\right]$$

$x^4 = 0.3525$

$y^4 = 0.2967$

$z^4 = -0.5181$

5th itteration: $k = 4$

$$x^5 = \frac{1}{10}\left[3 - 5(0.2967) + 2(-0.5181)\right]$$

$$y^5 = \frac{1}{10}\left[3 + 3(-0.5181) + 4(0.3525)\right]$$

$$z^5 = \frac{1}{10}\left[-3 - 0.3525 - 6(0.2967)\right]$$

$x^5 = 0.34473$

$y^5 = 0.28557$

$z^5 = -0.51327$

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6th itteration k = 5

$$x^6 = \frac{1}{10}\left[3+5(0.28557)+2(-0.51327)\right]$$

$$y^6 = \frac{1}{10}\left[3+3(-0.51327)+4(0.34473)\right]$$

$$z^6 = \frac{1}{10}\left[-3-0.34473-6(0.28557)\right]$$

$x^6 = 0.340131$

$y^6 = 0.283911$

$z^6 = -0.505815$

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Gauss Siedal Method:

$$x_1^{(k+1)} = \frac{-1}{a_1}\left[a_2 x_2^{(k)} + a_3 x_3^{(k)} + a_4\right]$$

$$x_2^{(k+1)} = \frac{-1}{b_2}\left[b_1 x_1^{(k+1)} + b_3 x_3^{(k)} + b_4\right]$$

$$x_3^{(k+1)} = \frac{-1}{c_3}\left[c_1 x_1^{(k+1)} + c_2 x_2^{(k+1)} + c_4\right]$$

where $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

$|a_1| \geq |a_2| + |a_3|$

$|b_2| \geq |b_1| + |b_3|$

$|c_3| \geq |c_1| + |c_2|$

$2x_1 - x_2 + 0x_3 = 7$

$-x_1 + 2x_2 - x_3 = 1$

$-x_2 + 2x_3 = 1$

Solve by Using Gauss Siedal Method:

Solve:

$|2| \geq |-1| + |0|$

$|2| \geq 1$

$|2| \geq |-1| + |-1|$

$2 \geq 2$

$|2| > |0| + |-1|$

$2 > 1$

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$$x_1^{(k+1)} = \frac{1}{2}\left[7 + x_2^{(k)}\right]$$

$$x_2^{(k+1)} = \frac{1}{2}\left[1 + x_1^{(k+1)} + x_3^{(k)}\right]$$

$$x_3^{(k+1)} = \frac{1}{2}\left[1 + x_2^{(k+1)}\right]$$

$$x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$$

For 1st Itteration: (k=0)

$$x_1^{(1)} = \frac{1}{2}\left[7 + 0\right] \Rightarrow \boxed{x_1^{(1)} = 3.5}$$

$$x_2^{(1)} = \frac{1}{2}\left[1 + x_1^{(1)} + x_3^{(0)}\right]$$

$$= \frac{1}{2}\left[1 + 3.5 + 0\right]$$

$$= \frac{1}{2}\left[4.5\right]$$

$$\boxed{x_2^{(1)} = 2.25}$$

$$x_3^{(1)} = \frac{1}{2}\left[1 + x_2^{(1)}\right]$$

$$x_3^{(1)} = \frac{1}{2}\left[1 + 2.25\right]$$

$$x_3^{(1)} = \frac{3.25}{2}$$

$$\boxed{x_3^{(1)} = 1.625}$$

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2ND ITTERATION : (k = 1)

$$x_1^{(2)} = \frac{1}{2}\left[7 + x_2^{(1)}\right]$$

$$x_1^{(2)} = \frac{1}{2}\left[7 + 2.25\right]$$

$$x_1^{(2)} = \frac{1}{2}\left[9.25\right]$$

$$\boxed{x_1^{(2)} = 4.625}$$

$$x_2^{(2)} = \frac{1}{2}\left[1 + x_1^{(2)} + x_3^{(1)}\right]$$

$$x_2^{(2)} = \frac{1}{2}\left[1 + 4.625 + 1.625\right]$$

$$x_2^{(2)} = \frac{1}{2}[$$

$$\boxed{x_2^{(2)} = 3.625}$$

$$x_3^{(2)} = \frac{1}{2}\left[1 + x_2^{(2)}\right]$$

$$x_3^{(2)} = \frac{1}{2}\left(1 + 3.625\right)$$

$$x_3^{(2)} = \frac{4.625}{2}$$

$$\boxed{x_3^{(2)} = 2.3125}$$

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3RD Itteration: (k=2)

$$x_1^{(3)} = \frac{1}{2}\left[7 + x_2^{(2)}\right]$$

$$x_1^{(3)} = \frac{1}{2}\left[7 + 3.625\right]$$

$$x_1^{(3)} = \frac{1}{2}\left[10.625\right]$$

$$\boxed{x_1^{(3)} = 5.3125}$$

$$x_2^{(3)} = \frac{1}{2}\left[1 + x_1^{(3)} + x_3^{(2)}\right]$$

$$x_2^{(3)} = \frac{1}{2}\left[1 + 5.3125 + 2.3125\right]$$

$$\boxed{x_2^{(3)} = 4.3125}$$

$$x_3^{(3)} = \frac{1}{2}\left[1 + x_2^{(3)}\right]$$

$$x_3^{(3)} = \frac{1}{2}\left[1 + 4.3125\right]$$

$$\boxed{x_3^{(3)} = 2.65625}$$

4RTH ITTERATION: (k=3)

$$x_1^{(4)} = \frac{1}{2}\left(7 + x_2^{(3)}\right)$$

$$x_1^{(4)} = \frac{1}{2}\left(7 + 4.3125\right)$$

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$$\boxed{x_1^{(4)} = 5.65625}$$

$$x_2^{(4)} = \frac{1}{2}\left[1 + x_1^{(4)} + x_3^{(3)}\right]$$

$$x_2^{(4)} = \frac{1}{2}\left[1 + 5.65625 + 2.65625\right]$$

$$x_2^{(4)} = \frac{1}{2}(9.3125)$$

$$\boxed{x_2^{(4)} = 4.65625}$$

$$x_3^{(4)} = \frac{1}{2}\left[1 + x_2^{(4)}\right]$$

$$x_3^{(4)} = \frac{1}{2}\left[1 + 4.65625\right]$$

$$\boxed{x_3^{(4)} = 2.828125}$$

5th itteration : $(k = 4)$

$$x_1^{(5)} = \frac{1}{2}\left[7 + x_2^{4}\right]$$

$$x_1^{(5)} = \frac{1}{2}\left[7 + 4.65625\right]$$

$$\boxed{x_1^{(5)} = 5.828125}$$

$$x_2^{(5)} = \frac{1}{2}\left[1 + x_1^{(5)} + x_3^{4}\right]$$

$$x_2^{(5)} = \frac{1}{2}(1 + 5.828125 + 2.828125)$$

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$$\boxed{x_2^{(5)} = 4.828125}$$

$$x_3^{(5)} = \frac{1}{2}\left[1 + x_2^{(5)}\right]$$

$$x_3^{(5)} = \frac{1}{2}\left[1 + 4.828125\right]$$

$$\boxed{x_3^{(5)} = 2.9140625}$$

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Gauss Seidal Method:

$4x + y + 2z = 4$

$3x + 5y + z = 7$

$x + y + 3z = 3$

Solution:

$|4| > |1| + |2|$

$4 > 3$

$|5| > |3| + |1|$

$5 > 4$

$|3| > |1| + |1|$

1st itteration when $k = 0$

$x^{(0)} = y^{(0)} = z^{(0)} = 0$

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$$x = \frac{1}{4}\left[4 - y^{(k)} - 2z^{(k)}\right]$$

$$y = \frac{1}{5}\left[7 - 3x^{(k+1)} - z^{(k)}\right]$$

$$z = \frac{1}{3}\left[3 - x^{(k+1)} - y^{(k+1)}\right]$$

$$x^{(1)} = \frac{1}{4}\left[4 - y^{(0)} - 2z^{(0)}\right]$$

$$x^{(1)} = \frac{1}{4}[4]$$

$$\boxed{x^{(1)} = 1}$$

$$y^{(1)} = \frac{1}{5}\left[7 - 3x^{(1)} - z^{(0)}\right]$$

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Gauss Siedal Method:

$4x + y + 2z = 4$

$3x + 5y + z = 7$

$x + y + 3z = 3$

Solution:

$|4| > |1| + |2|$

$4 > 3$

$|5| > |3| + |1|$

$5 > 4$

$|3| > |1| + |1|$

1st itteration when $k = 0$

$x^{(0)} = y^{(0)} = z^{(0)} = 0$

$$x = \frac{1}{4}\left[4 - y^{(k)} - 2z^{(k)}\right]$$

$$y = \frac{1}{5}\left[7 - 3x^{(k+1)} - z^{(k)}\right]$$

$$z = \frac{1}{3}\left[3 - x^{(k+1)} - y^{(k+1)}\right]$$

$$x^{(1)} = \frac{1}{4}\left[4 - y^{(0)} - 2z^{(0)}\right]$$

$$x^{(1)} = \frac{1}{4}[4]$$

$$\boxed{x^{(1)} = 1}$$

$$y^{(1)} = \frac{1}{5}\left[7 - 3x^{(1)} - z^{(0)}\right]$$

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$y^{(1)} = \frac{1}{5}[7-3(1)]$

$y^{(1)} = \frac{4}{5}$

$\boxed{y^{(1)} = 0.8}$

$z^{(1)} = \frac{1}{3}[3-x^{(1)}-y^{(1)}]$

$z^{(1)} = \frac{1}{3}[3-1-0.8]$

$\boxed{z^{(1)} = 0.4}$

2nd itteration: $(k=1)$

$x^{(2)} = \frac{1}{4}[4-y^{(1)}-2z^{(1)}]$

$x^{(2)} = \frac{1}{4}[4-0.8-2(0.4)]$

$\boxed{x^{(2)} = 0.6}$

$y^{(2)} = \frac{1}{5}[7-3x^{(2)}-z^{(1)}]$

$y^{(2)} = \frac{1}{5}[7-3(0.6)-0.4]$

$\boxed{y^{(2)} = 0.96}$

$z^{(2)} = \frac{1}{3}[3-x^{(2)}-y^{(2)}]$

$z^{(2)} = \frac{1}{3}[3-0.6-0.96]$

$\boxed{z^{(2)} = 0.48}$

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3rd itteration: (k = 2)

$$x^{(3)} = \frac{1}{4}[4 - y^{(2)} - 2z^{(2)}]$$

$$x^{(3)} = \frac{1}{4}[4 - 0.96 - 2(0.48)]$$

$$x^{(3)} = \frac{1}{4}[4 - 0.96 - 0.96]$$

$$\boxed{x^{(3)} = 0.52}$$

$$y^{(3)} = \frac{1}{5}[7 - 3x^{(3)} - z^{(2)}]$$

$$y^{(3)} = \frac{1}{5}[7 - 3(0.52) - 0.48]$$

$$y^{(3)} = \frac{1}{5}[7 - 1.56 - 0.48]$$

$$\boxed{y^{(3)} = 0.992}$$

$$z^{(3)} = \frac{1}{3}[3 - x^{(3)} - y^{(3)}]$$

$$z^{(3)} = \frac{1}{3}[3 - 0.52 - 0.992]$$

$$\boxed{z^{(3)} = 0.496}$$

4th itteration: (k = 3)

$$x^{(4)} = \frac{1}{4}[4 - y^{(3)} - 2z^{(3)}]$$

$$x^{(4)} = \frac{1}{4}[4 - 0.992 - 2(0.496)]$$

$$x^{(4)} = \frac{1}{4}[4 - 0.992 - 0.992]$$

$$\boxed{x^{(4)} = 0.504}$$

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$$y^{(4)} = \frac{1}{5}\left[7 - 3x^{(4)} - z^{(3)}\right]$$

$$y^{(4)} = \frac{1}{5}\left[7 - 3(0.504) - 0.496\right]$$

$$y^{(4)} = \frac{1}{5}\left[7 - 1.512 - 0.496\right]$$

$$\boxed{y^{(4)} = 0.9984}$$

$$z^{(4)} = \frac{1}{3}\left[3 - x^{(4)} - y^{(4)}\right]$$

$$z^{(4)} = \frac{1}{3}\left[3 - 0.504 - 0.9984\right]$$

$$\boxed{z^{(4)} = 0.4992}$$

5th itteration: $k = 4$

$$x^{(5)} = \frac{1}{4}\left[4 - y^{(4)} - 2z^{(4)}\right]$$

$$x^{(5)} = \frac{1}{4}\left[4 - 0.9984 - 2(0.4992)\right]$$

$$\boxed{x^{(5)} = 0.5008}$$

$$y^{(5)} = \frac{1}{5}\left[7 - 3x^{(5)} - z^{(4)}\right]$$

$$y^{(5)} = \frac{1}{5}\left[7 - 3(0.5008) - 0.4992\right]$$

$$y^{(5)} = \frac{1}{5}\left[7 - 1.5024 - 0.4992\right]$$

$$\boxed{y^{(5)} = 0.99968}$$

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$$z^{(5)} = \frac{1}{3}\left[3 - x^{(5)} - y^{(5)}\right]$$

$$z^{(5)} = \frac{1}{3}\left[3 - 0.5008 - 0.99968\right]$$

$$z^{(5)} = \frac{1}{3}\left[1.49952\right]$$

$$\boxed{z^{(5)} = 0.49984}$$

6th itteration : (k = 5)

$$x^{(6)} = \frac{1}{4}\left[4 - y^{(5)} - 2(z)^{(5)}\right]$$

$$x^{(6)} = \frac{1}{4}\left[4 - 0.99968 - 2(0.49984)\right]$$

$$\boxed{x^{(6)} = 0.50016}$$ Answer

$$y^{(6)} = \frac{1}{5}\left[7 - 3(x)^{6} - z^{(5)}\right]$$

$$y^{(6)} = \frac{1}{5}\left[7 - 3(0.50016) - 0.49984\right]$$

$$\boxed{y^{(6)} = 0.999936}$$ Answer

$$z^{(6)} = \frac{1}{3}\left[3 - x^{(6)} - y^{(6)}\right]$$

$$z^{(6)} = \frac{1}{3}\left[3 - 0.50016 - 0.999936\right]$$

$$\boxed{z^{(6)} = 0.49982}$$ Answer

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