Monday
29/10/2018

Group:-

A non-empty set "G" is called grap if

(i) It holds **Closure law** ✓

i.e Let $a, b \in G$

if $a \dot{\times} b \in G$ then G holds Closure law.

let :: $G = \{0, 1, 2, 3, 4\}$ under modulus 5

(ii) **Associative law**

Let $a, b \in G$

if $a \dot{\times} (b \dot{\times} c) = (a \dot{\times} b) \dot{\times} c$

example $a = 2$, $b = 3$, $c = 4$

L.H.S	R.H.S
$a+(b+c)$	$(a+b)+c$
$= 2+(3+4)$	$= (2+3)+4$
$= 2+(7)$	$= (5)+4$
$= 2+2$	$= 0+4$
$= 4$	$= 4$

L.H.S = R.H.S

(iii) Identity

Let $a \in G$ if $a * e = a$ and $e \in G$ then "e" is identity.

Additive indentity is "0"
Multiplicative " " "1"

(iv) Inverse

Let $a \in G$ if $a * a^{-1} = e$ then a^{-1} is inverse of a ✓

(v) Commutative law

Let $a, b \in G$ if $a * b = b * a$ Then G is called Abelian Group. ✓

Note:- ⇒ If aset ~~sta~~ satisfies only first condition then it is monoid.

⇒ If a set satisfies first two conditions then it is called semi-Group.

⇒ If aset satisfies first Three conditions then it is called Groupoid.

⇒ If a set " " four conditions then it is called group.

⑤ mod =

$$\begin{array}{r} 1 \\ a^5\overline{)\,a^5} \\ \underline{a^5} \\ 0 \end{array}$$

Order of group:-

The number of element in group "G" is called order of group.

notation $|G| = 5$.

Order of element:-

Let a, b are elements of group G, a positive integer "n" is said to be the order of "a" if $\underline{a^n = e}$. and n is least such +ve integer.

Periodic group:-

If every element of a group "G" is of finite order then G is called periodic group

$G = \{1, a, a^2, a^3, a^4\}$ $1 = a^5$
→ Cyclic group.

Mixed group:-

If a group "G" contains elements of finite as well as infinite order, then G is called mixed group

$$G = (\mathbb{R}', \times)$$

$$\mathbb{R} = (-\infty, \infty)$$

$$\mathbb{R}' = \{(-\infty, \infty)\} - \{0\}$$

Sub-Group:-

Let H be non-empty subset of a group G, then H is sub-group of G is H itself a group under binary operation defined on G.

3×3 2×3

30/01/2018

Matrix

Arragement of digits into columns and rows is called a matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 1 & 2 & 3 \end{bmatrix}$$

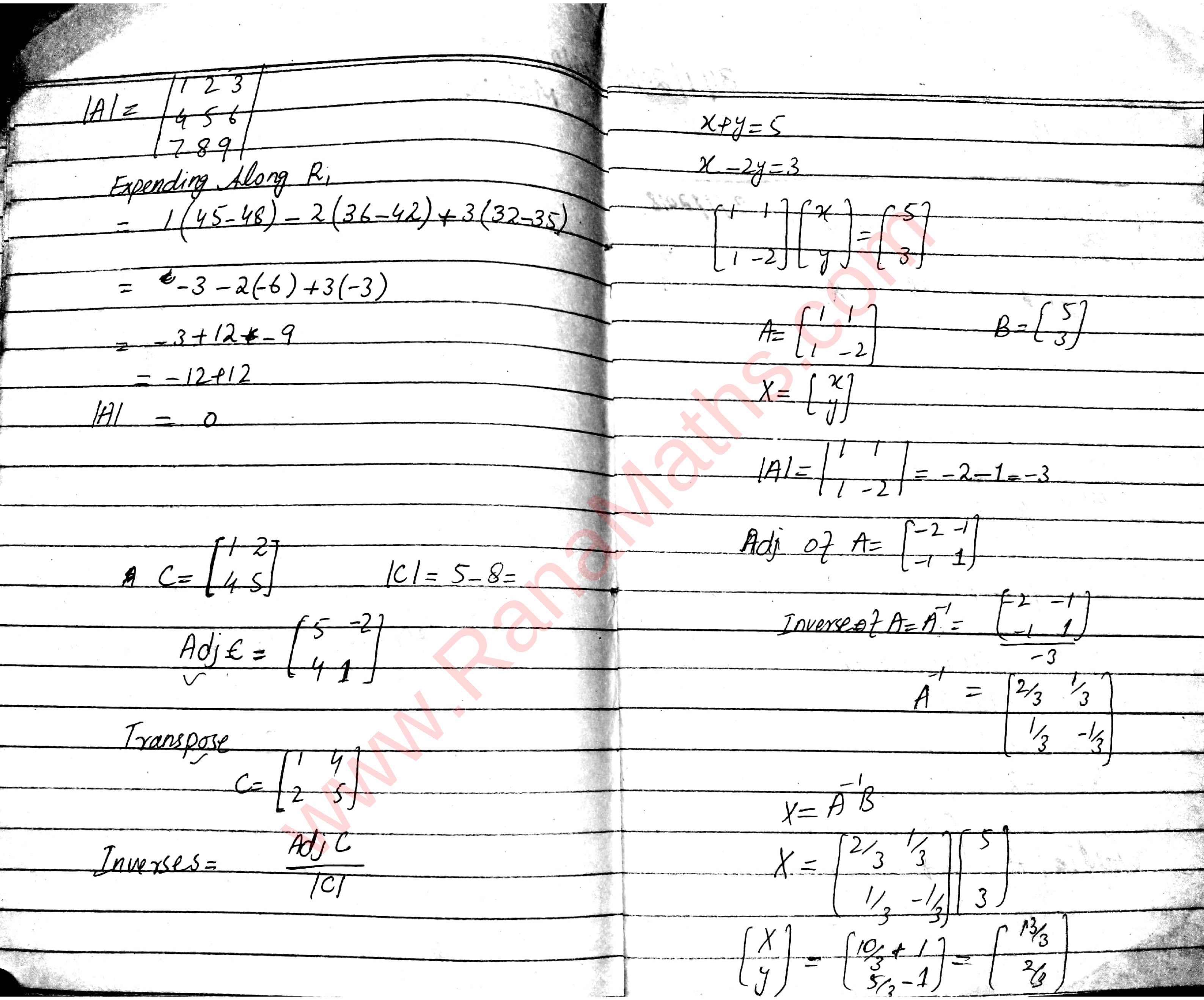
$$A+B = \begin{bmatrix} 4 & 6 & 8 \\ 10 & 12 & 14 \\ 8 & 10 & 12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3+12+3 & 4+14+6 & 5+16+9 \\ 12+30+6 & 16+35+12 & 20+40+18 \\ 21+48+9 & 28+56+18 & 35+64+27 \end{bmatrix}$$

$$AB = \begin{bmatrix} 18 & 24 & 30 \\ 48 & 63 & 78 \\ 78 & 102 & \end{bmatrix}$$

Multiplication CR

Order R×C



$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Expending Along R_1

$$= 1(45-48) - 2(36-42) + 3(32-35)$$

$$= -3 - 2(-6) + 3(-3)$$

$$= -3 + 12 \cancel{+} -9$$

$$= -12 + 12$$

$$|A| = 0$$

$$\cancel{A}\ C = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \qquad |C| = 5-8=$$

$$\text{Adj } C = \begin{bmatrix} 5 & -2 \\ 4 & 1 \end{bmatrix}$$

Transpose

$$C = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\text{Inverses} = \frac{\text{Adj } C}{|C|}$$

$$x + y = 5$$

$$x - 2y = 3$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2-1 = -3$$

$$\text{Adj of } A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{Inverse of } A = A^{-1} = \frac{\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}}{-3}$$

$$A^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ y \end{bmatrix} = \begin{bmatrix} \frac{10}{3} + 1 \\ 5/3 - 1 \end{bmatrix} = \begin{bmatrix} 13/3 \\ 2/3 \end{bmatrix}$$

31/1/2018)

$x = 13/3$, $y = 2/3$

31/1/2018

$3x_1 + 2x_2 + 5x_3 = 2$
$2x_1 + 4x_2 + 6x_3 = 2 \Rightarrow x_1 + 2x_2 + 3x_3 = 1$
$5x_1 + 3x_2 + 3x_3 = 1$

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 2 & 3 \\ 5 & 3 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$AX = B$ ✓

$X = A^{-1}B$ ✓

$$|A| = \begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 3 \\ 5 & 3 & 3 \end{vmatrix}$$

$= 3(6-9) - 2(3-15) + 5(3-10)$

$= 3(-3) - 2(-12) + 5(-7)$

$= -9 + 24 - 35$

$= 24 - 44$

$|A| = -20$

$$\text{Addjoint } A = \begin{bmatrix} -3 & -(+12) & -7 \\ -(-9) & -16 & -(-1) \\ -4 & -(4) & 4 \end{bmatrix}^T$$

$$A^{-1} = \frac{-1}{20} \begin{bmatrix} -3 & 9 & -4 \\ 12 & -16 & -4 \\ -7 & 1 & 4 \end{bmatrix}$$

$X = A^{-1}B$ ✓

$$= \frac{-1}{20} \begin{bmatrix} -3 & 9 & -4 \\ 12 & -16 & -4 \\ -7 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= -\frac{1}{20} \begin{bmatrix} -6+9-4 \\ 24-16-4 \\ -14+1+4 \end{bmatrix}$$

$$= -\frac{1}{20} \begin{bmatrix} -1 \\ 4 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/20 \\ -1/5 \\ 9/25 \end{bmatrix}$$

$x_1 = \frac{1}{20}$, $x_2 = -1/5$, $x_3 = 9/25$

Verify

$3x_1 + 2x_2 + 5x_3$

$3(\frac{1}{20}) + 2(-\frac{1}{5}) + 5(\frac{9}{25})$

$= \frac{3}{20} - \frac{2}{5} + \frac{9}{5}$

$= \frac{3}{20} + \frac{7}{5}$

$= \frac{3+28}{20}$

Invertible Matrices:- ✓

~~If~~ let A and B are square matrices of same order then If $AB=I$, then B is invertible to A.

$$A=\frac{1}{80}\begin{bmatrix}3&2&5\\1&2&3\\5&3&3\end{bmatrix}, \quad B=\frac{-1}{4}\begin{bmatrix}-3&9&-4\\12&-16&-4\\-7&1&4\end{bmatrix}$$

$$AB=\frac{-1}{20}\begin{bmatrix}-9+24-35 & 27-32+5 & -12-8+20\\ -3+24-21 & 9-32+3 & -4-8+12\\ -15+36-21 & 45-48+3 & -20-12+12\end{bmatrix}$$

$$AB=\frac{1}{-20}\begin{bmatrix}-20&0&0\\0&-20&0\\0&0&-20\end{bmatrix}$$

$$AB=\frac{-20}{-20}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$AB=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

AB=I, Invert. B to A

$A^{-1}B$

R×C

Block Matrices:-

Suppose we have a matrix

$$A=\left[\begin{array}{ccc|cc|cc}1&0&0&1&2&3&1\\0&1&0&1&1&0&1\\0&0&1&0&0&0&2\\\hline 0&1&0&1&0&0&1\\2&3&2&1&4&5&6\end{array}\right]$$

I, B, C, D, E, F

BC X

CB X

Block matrix is matrix of matrices

~~Plo~~ Polynomial matrices:-

✓

① $$P=\begin{bmatrix}0&x^2&2\\3&2x&5x\\3x-1&x^2&3x\end{bmatrix}=\underset{\text{Constt}}{\begin{bmatrix}0&0&2\\3&0&0\\-1&0&0\end{bmatrix}}+\underset{x\text{-coeffic}}{\begin{bmatrix}0&0&0\\0&2&5\\3&0&3\end{bmatrix}}x$$

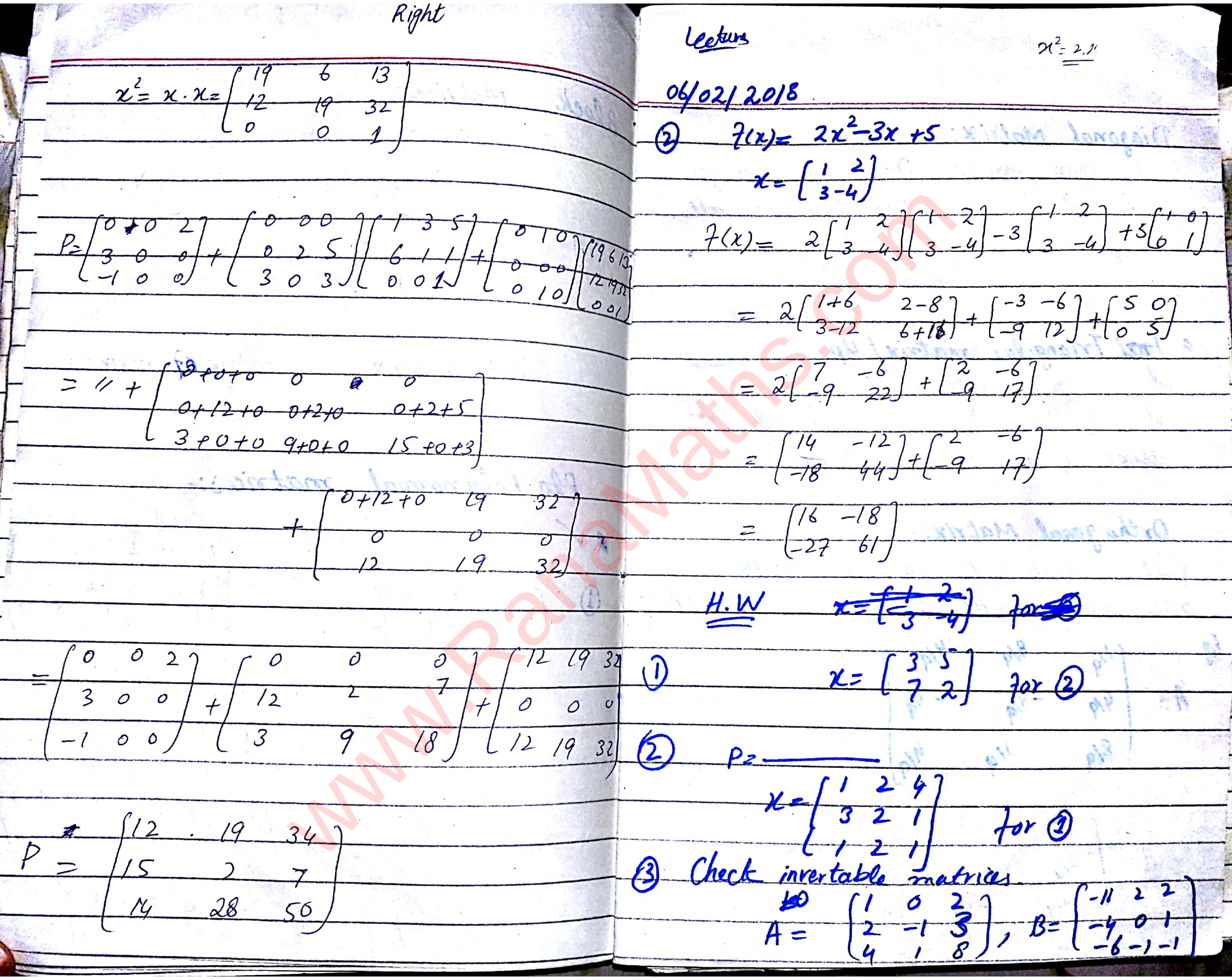
x^2 coefficients

$$+\begin{bmatrix}0&1&0\\0&0&0\\0&1&0\end{bmatrix}x^2$$

Suppose $$X=\begin{bmatrix}1&3&5\\6&1&1\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&3&5\\6&1&1\\0&0&1\end{bmatrix}$$

$$X.X=\begin{bmatrix}1+18+0 & 3+3+0 & 5+3+5\\ 6+6+0 & 18+1+0 & 30+1+1\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$



Right

$$x^2 = x \cdot x = \begin{bmatrix} 19 & 6 & 13 \\ 12 & 19 & 32 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 3 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 6 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 19 & 6 & 13 \\ 12 & 19 & 32 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \text{//} + \begin{bmatrix} 3+0+0 & 0 & 0 \\ 0+12+0 & 0+2+0 & 0+2+5 \\ 3+0+0 & 9+0+0 & 15+0+3 \end{bmatrix}$$

$$+ \begin{bmatrix} 0+12+0 & 19 & 32 \\ 0 & 0 & 0 \\ 12 & 19 & 32 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 12 & 2 & 7 \\ 3 & 9 & 18 \end{bmatrix} + \begin{bmatrix} 12 & 19 & 32 \\ 0 & 0 & 0 \\ 12 & 19 & 32 \end{bmatrix}$$

$$P = \begin{bmatrix} 12 & 19 & 34 \\ 15 & 2 & 7 \\ 14 & 28 & 50 \end{bmatrix}$$

Lecture

$x^2 = 2.x$

06/02/2018

② $f(x) = 2x^2 - 3x + 5$

$$x = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$$

$$f(x) = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1+6 & 2-8 \\ 3-12 & 6+16 \end{bmatrix} + \begin{bmatrix} -3 & -6 \\ -9 & 12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= 2\begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ -9 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -12 \\ -18 & 44 \end{bmatrix} + \begin{bmatrix} 2 & -6 \\ -9 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -18 \\ -27 & 61 \end{bmatrix}$$

H.W ~~$x = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ for ③~~

① $x = \begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix}$ for ②

② $P =$ ______

$$x = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{for } ①$$

③ Check invertable matrices

~~10~~

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ -6 & -1 & -1 \end{bmatrix}$$

Diagonal Matrix:-

A Square matrix $D=(d_{ij})$ is diagonal matrix if its non-diagonal enteries all are zeros.

Trai Triangular matrix/ Up

A square matrix $A=(a_{ij})$ is called Triangular matrix if all ~~ent~~ enteries below the diagonal ~~ent~~ are zeros.

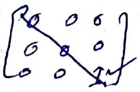
Orthagonal Matrix:-

A real matrix A is said to be orthogonal if $A(A^T)=A^TA=I$ or $A^{T}=\bar{A}'$

Eg

$$A=\begin{bmatrix} 1/9 & 8/9 & -4/9 \\ 4/9 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{bmatrix}$$

$$A^T=\begin{bmatrix} 1/9 & 4/9 & 8/9 \\ 8/9 & -4/9 & 1/9 \\ -4/9 & -7/9 & 4/9 \end{bmatrix}$$



$R_2-4R_1\rightarrow R_2$

Complex Matrix:-

A matrix containing "i"

for these matrices

$$(\bar{A})^T=(\overline{A^T})$$

e.g

$$A=\begin{bmatrix} 1 & i & 2 \\ 3i & -i & 4 \\ 6 & 7 & 8i \end{bmatrix}$$

$$\bar{A}=\begin{bmatrix} \bar{1} & \bar{i} & \bar{2} \\ \overline{3i} & \overline{-i} & \bar{4} \\ \bar{6} & \bar{7} & \overline{8i} \end{bmatrix} \Rightarrow \bar{A}=\begin{bmatrix} 1 & -i & 2 \\ -3i & i & 4 \\ 6 & 7 & -8i \end{bmatrix}$$

$$(\bar{A})^T=\begin{bmatrix} 1 & -3i & 6 \\ -i & i & 4 \\ 2 & 4 & -8i \end{bmatrix}$$

NOW

$$A^T=\begin{bmatrix} 1 & 3i & 6 \\ i & -i & 4 \\ 6 & 7 & 8i \end{bmatrix} \Rightarrow (\overline{A^T})=\begin{bmatrix} \bar{1} & \overline{3i} & \bar{6} \\ \bar{i} & \overline{-i} & \bar{4} \\ \bar{6} & \bar{7} & \overline{8i} \end{bmatrix}$$

$$(\overline{A^T})\begin{bmatrix} 1 & -3i & 6 \\ -i & i & 4 \\ 6 & 7 & -8i \end{bmatrix},$$ so $(\bar{A})^T=(\overline{A^T})$

H.W

④ $\hookrightarrow A = \begin{bmatrix} 2+8i & 5-3i & 4-7i \\ 6i & 1-4i & 3+2i \end{bmatrix}$

~~Her~~ Hermition Matrix

A Square complex matrix "A" is said to be Hermition matrix if

$$A^H = (\bar{A})^T = A$$

Skew-Hermition Matrix:-

A square complex matrix "A" is said to be skewHermition matrix if

$$A^H = (\bar{A})^T = -A$$

Unitary Matrix:-

A Square complex matrix A is Unitary Matrix if $A^H = A^{-1}$ $\because A^H = (\bar{A})^T$

$$AA^H = A^H A = I$$

Normal Matrix:-

A square complex matrix A is said to be normal If

$$\underline{A^H A = AA^H}$$

$A = \frac{1}{2}\begin{bmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{bmatrix}$

H.W

$B = \begin{bmatrix} 3 & 1-2i & 4+7i \\ 1+2i & -4 & -2i \\ 4-7i & 2i & 2 \end{bmatrix}$

$C = \begin{bmatrix} 2+3i & 1 \\ i & 1+2i \end{bmatrix}$

Check Hermition, Unitary, Normal

$\hookrightarrow A^H -$

Homework

Tuesday
06/02/2018

① $f(x) = 2x^2 - 3x + 5$

$$x = \begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix}$$

Sol:-

$$f(x) = 2\begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix} - 3\begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 9+35 & 15+10 \\ 21+14 & 35+4 \end{bmatrix} + \begin{bmatrix} -9 & -15 \\ -21 & -6 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= 2\begin{bmatrix} 44 & 25 \\ 35 & 39 \end{bmatrix} + \begin{bmatrix} -4 & -15 \\ -21 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 88 & 50 \\ 70 & 78 \end{bmatrix} + \begin{bmatrix} -4 & -15 \\ -21 & -1 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} 84 & 35 \\ 49 & 77 \end{bmatrix}$$

② $$P = \begin{bmatrix} 0 & x^2 & 2 \\ 3 & 2x & 5x \\ 3x-1 & x^2 & 3x \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 3 & 0 & 3 \end{bmatrix}x$$

$$+ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}x^2 \rightarrow Ⓐ$$

Here $$x = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Sol

$$x^2 = x.x = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+4 & 2+4+8 & 4+2+4 \\ 3+6+1 & 6+4+2 & 12+2+1 \\ 1+6+1 & 2+4+2 & 4+2+1 \end{bmatrix}$$

$$x^2 = xx = \begin{bmatrix} 11 & 14 & 10 \\ 10 & 12 & 15 \\ 8 & 8 & 7 \end{bmatrix}$$

eq Ⓐ becomes

$$\begin{bmatrix} 0 & x^2 & 2 \\ 3 & 2x & 5x \\ 3x-1 & x^2 & 3x \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 3 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 11 & 14 & 10 \\ 10 & 12 & 15 \\ 8 & 8 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0+0+0 & 0 & 0 \\ 0+6+5 & 0+4+10 & 0+2+5 \\ 3+0+3 & 6+0+6 & 12+0+3 \end{bmatrix}$$

$$+ \begin{bmatrix} 0+10+0 & 12 & 15 \\ 0 & 0 & 0 \\ 10 & 12 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 11 & 14 & 7 \\ 6 & 12 & 15 \end{bmatrix} + \begin{bmatrix} 10 & 12 & 15 \\ 0 & 0 & 0 \\ 10 & 12 & 15 \end{bmatrix}$$

$$P = \begin{bmatrix} 10 & 12 & 17 \\ 14 & 14 & 7 \\ 15 & 24 & 30 \end{bmatrix}$$ Ans

$$\longleftrightarrow$$

③ $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ -6 & -1 & -1 \end{bmatrix}$

Sol

$$AB = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ -6 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -11-8-12 \end{bmatrix}$$

③ $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ -6 & -1 & -1 \end{bmatrix}$

Sol

$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ -6 & -1 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ -6 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -11+0-12 & 2+0-2 & 2+0-2 \\ -22+4-18 & 4+0+3 & \end{bmatrix}$$

$$BA = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ +6 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -11+4+8 & 0-2+2 & -22+6+16 \\ -4+0+4 & 0+0+1 & -8+0+8 \\ +6-2-4 & 0+1-1 & +12-3-8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As $BA = I$

So A is invertable of B.

Q4 $A = \begin{bmatrix} 2+8i & 5-3i & 4-7i \\ 6i & 1-4i & 3+2i \end{bmatrix}$

Show that

$(\bar{A})^T = \overline{(A^T)}$

Sol

$\bar{A} = \begin{bmatrix} 2-8i & 5+3i & 4+7i \\ -6i & 1+4i & 3-2i \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 2-8i & -6i \\ 5+3i & 1+4i \\ 4+7i & 3-2i \end{bmatrix}$

Now

$(A^T) = \begin{bmatrix} 2+8i & 6i \\ 5-3i & 1-4i \\ 4-7i & 3+2i \end{bmatrix}$

$\overline{(A^T)} = \begin{bmatrix} 2-8i & -6i \\ 5+3i & 1+4i \\ 4+7i & 3-2i \end{bmatrix}$

Hence $(\bar{A})^T = \overline{(A^T)}$

Proved.

Q5 ~~Hermitian Matrices~~ Unitary

(i) $A = \frac{1}{2}\begin{bmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{bmatrix}$

Sol

$\bar{A} = \frac{1}{2}\begin{bmatrix} 1 & +i & -1-i \\ -i & 1 & 1-i \\ 1-i & -1-i & 0 \end{bmatrix}$

$A^H = (\bar{A})^T = \frac{1}{2}\begin{bmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{bmatrix}$

~~As $(\bar{A})^T \neq A$ so it is not Hermition~~

Now

$A^H A = \frac{1}{4}\begin{bmatrix} 1 & -i & 1-i \\ i & 1 & -1-i \\ -1-i & 1-i & 0 \end{bmatrix}\begin{bmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 1-i^2+1-i^2 & -\cancel{i}\cancel{-i}-1+\cancel{i}+\cancel{i}-i^2 & -1+\cancel{i}\cancel{-i}-i \\ \cancel{i}+\cancel{i}-1\cancel{-i}\cancel{-i}-i^2 & -i^2+1+1-\cancel{i}+\cancel{i}-i^2 & -\cancel{i}+i^2+1+\cancel{i} \\ -1-\cancel{i}+\cancel{i}-i^2+0 & +\cancel{i}+i^2+1-\cancel{i}+0 & 1-\cancel{i}+\cancel{i}-i^2+1+i^2+0 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$= \frac{4}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

As $A^H A = I$ so it is ~~H~~ Unitary

Q6 Hermition

B

$$B=\begin{bmatrix} 3 & 1-2i & 4+7i \\ 1+2i & -4 & -2i \\ 4-7i & 2i & 2 \end{bmatrix}$$

Sol

$$\bar{B}=\begin{bmatrix} 3 & 1+2i & 4-7i \\ 1-2i & -4 & 2i \\ 4+7i & -2i & 2 \end{bmatrix}$$

$$(\bar{B})^T=\begin{bmatrix} 3 & 1-2i & 4+7i \\ 1+2i & -4 & -2i \\ 4-7i & 2i & 2 \end{bmatrix}$$

As $(\bar{B})^T = B$

So it is Hermition Matrix

$$\longleftrightarrow$$

Q7 Normal Matrix:

$$C=\begin{bmatrix} 2+3i & 1 \\ i & 1+2i \end{bmatrix}$$

Sol

$$\bar{C}=\begin{bmatrix} 2-3i & 1 \\ -i & 1-2i \end{bmatrix}$$

$$C^H=(\bar{C})^T=\begin{bmatrix} 2-3i & -i \\ 1 & 1-2i \end{bmatrix}$$

Firstly

$$C^HC=\begin{bmatrix} 2-3i & -i \\ 1 & 1-2i \end{bmatrix}\begin{bmatrix} 2+3i & 1 \\ i & 1+2i \end{bmatrix}$$

$$=\begin{bmatrix} 4-9i^2-i^2 & 2-3i-i-2i^2 \\ 2+3i-2i^2+i & 1+1-4i^2 \end{bmatrix}$$

$$=\begin{bmatrix} 4-10i^2 & 2-4i+2 \\ 2+4i+2 & 2+4 \end{bmatrix}$$

$$C^HC = \cancel{4+8}\begin{bmatrix} 14 & 4-4i \\ 4+4i & 6 \end{bmatrix}$$

Now

$$CC^H=\begin{bmatrix} 2+3i & 1 \\ i & 1+2i \end{bmatrix}\begin{bmatrix} 2-3i & -i \\ 1 & 1-2i \end{bmatrix}$$

$$=\begin{bmatrix} 4-9i^2+1 & -2i-3i^2+1-2i \\ 2i-3i^2+1+2i & -i^2+1-4i^2 \end{bmatrix}$$

$$=\begin{bmatrix} 14 & 4-4i \\ 4+4i & 6 \end{bmatrix}$$

As $C^HC = CC^H$

So This is normal Matrix.

07/02/2018

Elementry ~~Matrix~~ Row Operations:

A matrix "A" is called elementry row operation.

[E_1] interchange the ith row and j-th row. $R_i \longleftrightarrow R_j$

[E_2] multiply the ith row by a non-zero scalar "k". $KR_i \longrightarrow R_i \quad (K \neq 0)$

[E_3] Replace the i-th row by K times the j-th row plus the i-th row

$$KR_j + R_i \longrightarrow R_i$$

0 6 1

let

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} R_2 \longleftrightarrow R_3$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{bmatrix} \rightarrow 6R_3$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 6 & 1 \end{bmatrix} \nearrow 6R_2 + R_3$$

Elementry Column operation:-

[E_1] interchange the i-th column and j-th column.

$$C_i \longleftrightarrow C_j$$

[E_2] multiply the i-th column by a non-zero scalar "K"

$$KC_i \longleftrightarrow C_i \quad (K \neq 0)$$

[E_3] Replace the i-th column by K time the j-th column plus th i-th column.

$$KC_j + C_i \longrightarrow C_i$$

let

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad C_2 \longleftrightarrow C_3$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{bmatrix} 6C_3, \quad E_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} 6C_2 + C_3$$

$q(x) = X^{T}A$

$q(x_1, x_2, \dots, x_n)\ \Sigma c_{ij}$

Quadratic forms:-

~~Let~~ A Quadratic form "q" in variables $x_1, x_2, \dots, x_n$ is a polynomial $q(x_1, x_2, x_3, \dots, x_n) = \Sigma c_{ij}\, x_i x_j$ where each term has degree 2.

$$q(X) = X^{T} A X$$

here A is a symmetric matrix. X is a matrix of form $X = \begin{bmatrix} x \\ \vdots \\ y \end{bmatrix}$

$$a_{ii} = c_{ii}$$

$$a_{ij} = \frac{c_{ij}}{2}$$

$\because a_{12} = a_{21}$

Example:-

$$q(x, y, z) = x^2 - 6xy + 8y^2 - 4xz + 5yz + 7z^2$$

Now $q(x_1, x_2, x_3) = x_1^2 - 6x_1x_2 + 8x_2^2 - 4x_1x_3 + 5x_2x_3 + 7x_3^2$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad X^{T} = [x\ y\ z]$$

$$q(x) = X^{T} A X$$

~~$q(X) =$~~

$$q(X) = [x\ y\ z]\begin{bmatrix} 1 & -3 & -2 \\ -3 & 8 & 5/2 \\ -2 & 5/2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Exp $q(x, y, z) = x^2 + 4xy + 5y^2 - 6xz - 8yz + 8z^2$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$X^{T} = [x\ y\ z]$$

$$q(X) = [x\ y\ z]\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \cancel{[x+2y-3z \quad 2x+5y-4z \quad -3x-4y+8z]}$$

$$= [x+2y-3z \quad 2x+5y-4z \quad -3x-4y+8z]\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{x^2+2xy}\ x^2+2xy-3xz \\ 2xy+5y^2-4yz \\ -3xz-4yz+8z^2 \end{bmatrix}$$

$$q(X) = [x^2 + 2xy - 3xz + 2xy + 5y^2 - 4yz - 3xz - 4yz + 8z^2]$$

Similar Matrix:

A matrix B is similar to A if there exist a non-zero singular matrix "P" such that

$$B = P^{-1}AP$$

$\longleftrightarrow$

Lecture

12/02/2018 Monday

Elementry Matrix:- An Elementry matrix is any matrix that can be obtained by performing elementry row/column on an identity matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} R_3 \longleftrightarrow R_2$$

$\longleftrightarrow$

LU-Factorization:-

$$L = \begin{bmatrix} 1 & 0 & 0 \\ - & 1 & 0 \\ - & - & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \qquad U = \begin{bmatrix} \square & - & - \\ 0 & \square & - \\ 0 & 0 & - \end{bmatrix}$$

$-2R_1 + R_2 \to R_2$

$-R_1 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\tfrac{1}{3}R_2 + R_3 \to R_3 \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}$$

R_1 and R_2 ke coefficient dekhny hein.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -\frac{1}{3} & 1 \end{bmatrix}$$

$$A = LU$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -\frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4-3 & 6-5 \\ 1 & 2+1+0 & 3+\frac{5}{3}-\frac{8}{3} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

Exp

$$B=\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 1\\ 3 & 1 & 1\end{bmatrix}$$

$-R_1+R_2$.

$-3R_1+R_3$ $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -2\\ 0 & -5 & -8\end{bmatrix}$

$5R_2+R_3$

$$U=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -2\\ 0 & 0 & -18\end{bmatrix}$$

We make matrix "L" from the coefficient of Row operation by changing their signs.

$$L=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 3 & -5 & 1\end{bmatrix}$$

$$A= LU$$

$$=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 3 & -5 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -2\\ 0 & 0 & -18\end{bmatrix}$$

$$=\left(\begin{matrix}1 & 2 & 3\\ 1 & 2+1 & 3-2\\ & & \end{matrix}\right.$$

Vector Space:-

let "V" be a non-empty set with two operations

(i) ~~Ad~~ Vector Addition

This assigns to any $u,v \in V$ a sum $u+v \in V$

(ii) Scalar Multiplication

This Assigns to any $u \in V$ & $k \in K$ a product $ku \in V$. (K is set of Integers)

Then "V" is called a vector space over the field "K" if the following axioms hold for any vectors $u, v, w \in V$

(Axioms)

A(I) $(u+v)+w = u+(v+w)$

A(II) There is a vector in "V" denoted by "0" and called the zero vector such that for any $u \in V$

$u+0=0+u=u$

A(III)

For Each $u \in V$ There is a vector in "V" ~~an~~ denoted by "$-u$" and called the negative of "u" such that

$u+(-u)=-u+u=0$

A(IV) for $u,v \in V$

$u+v=v+u$

M(I) For $u,v \in V$

and $k \in K$

$k(u+v)=ku+kv$

M(III) For $u \in V$ &

$k_1,k_2 \in K$

$(k_1k_2)u=k_1(k_2u)$

M(II)
~~M(III)~~ For $u \in V$ &

$k_1,k_2 \in K$

$(k_1+k_2)u=k_1u+k_2u$

Field:- Group(+) + commutative + Distributive law + Group(×)$(F-\{0\})$
e.g $\mathbb{R}-\{0\}$

M(IV) For $u \in V$

$1u=u$ where $1 \in K$

13/02/2018

{Sub space} Sub-space

Let "V" be a vector space over a field "K" and let "W" be a subset of "V" then "W" is said to be subspace of "V" if "W" is itself ~~as~~ a vector space over 'K' with respect to operation ~~at~~ of vector addition and scalar multiplication on "V"

Theorem:- (Subspace)

Suppose W be a subset of a vector space "V" Then W is a subspace of V ~~if~~ if the following two conditions hold

a):- The zero vector 0 belongs to "W"

(b):- For every $u,v \in W$ and $k \in K$

(i) $u+v \in W$

(ii) $ku \in W$

Lecture:- 13/02/2018

Linear Combination:-

Let "V" be a vector space over a field "K" a vector $v \in V$ is a linear combination of $u_1, u_2, \dots, u_n$ if there exist scalars $k_1, k_2, \dots, k_n$ such that

$$v = k_1u_1 + k_2u_2 + \dots + k_nu_n$$

Alternatively, v is a linear combination of $u_1, u_2, \dots, u_n$ if there is a solution to the vector equation

$$v = x_1u_1 + x_2u_2 + \dots + x_nu_n$$

where, $x_1, x_2, \dots x_n$ are unknown scalars

Example (Linear combination in $\mathbb{R}^n$)

Suppose we want to express ~~$v = 3, 7, -4$~~ $v = (3, 7, -4) \in R^3$ as a linear combination of vectors $u_1 = (1, 2, 3)$

$u_2 = (2, 3, 7)$

$u_3 = (3, 5, 6)$

we ~~st~~ seeks scalars x, y, z s.t

$$v = xu_1 + yu_2 + zu_3$$

$$\begin{bmatrix} 3 \\ 7 \\ -4 \end{bmatrix} = x\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} + z\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 7 \\ -4 \end{bmatrix} = \begin{bmatrix} x + 2y + 3z \\ 2x + 3y + 5z \\ 3x + 7y + 6z \end{bmatrix}$$

$3 = x + 2y + 3z$

$7 = 2x + 3y + 5z$

$-4 = 3x + 7y + 6z$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 3 \\ 2 & 3 & 5 & 7 \\ 3 & 7 & 6 & -4 \end{array}\right]$$

$R_2 - 2R_1$

$$R_3 - 3R_1 \quad \left[\begin{array}{ccc:c} 1 & 2 & 3 & 3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & -3 & -13 \end{array}\right]$$

$R_2 + R_3$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -4 & -12 \end{array}\right]$$

$x + 2y + 3z = 3 \rightarrow$ i)

$-y - z = 1 \rightarrow$ ii)

$-4z = -12$

$\Rightarrow \quad z = 3$

putting $z=3$ in (ii)

$$-y-3=1$$

$$\underline{\underline{y=-4}}$$

Putting in eq(ii)

$$x+2(-4)+3(3)=3$$

$$x\cancel{+}-8+9=3$$

$$x+1=3$$

$$\underline{\underline{x=2}}$$

$$v=2u_1-4u_2+3u_3$$

Linear Combination in P(t):-

Linear Combination P(t) denots ~~all~~ the set of all real polynomials of form $p(t)\in P(t)$

$$p(t)=k_0+k_1t+k_2t^2+\dots+k_mt^m$$

where $k_0,k_1,k_2,\dots,k_n \in K$.

will be found on 14/02/2018 lecture Wed

14/02/2018 Wednesday

~~Exp~~. Example:-

Suppose we want to express polynomial

$$v = 3t^2 + 5t - 5$$

as a linear combination of polymials

$$P_1 = t^2 + 2t + 1 \quad , P_2 = 2t^2 + 5t + 4$$

$$P_3 = t^2 + 3t + 6$$

So

$$v = xP_1 + yP_2 + zP_3 \longrightarrow (A)$$

putting values

$$3t^2 + 5t - 5 = x(t^2+2t+1) + y(2t^2+5t+4)$$
$$+ z(t^2+3t+6)$$

$$(3t^2+5t-5) = (x+2y+z)t^2 + (2x+5y+3z)t$$
$$+ (x+4y+6z)$$

Comparing coefficients of t^2, t and const.

t^2 $\Rightarrow$ $3 = x + 2y + z$

$$5 = 2x + 5y + 3z$$

$$-5 = x + 4y + 6z$$

in matrix form

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 5 & 3 & 5 \\ 1 & 4 & 6 & 5 \end{array}\right]$$

Tor make it uper triangular matrix

$$\begin{array}{l} -2R_1 + R_2 \\ -R_1 + R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 5 & -8 \end{array}\right]$$

$$-2R_2 + R_3 \rightarrow R_3 \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3 & -6 \end{array}\right]$$

$$x + 2y + z = 3$$
$$y + z = -1$$
$$3z = -6$$

$$\boxed{z = -2}$$

$$\boxed{y = 1}$$

$$\boxed{x = 3}$$

Putting in (A)

$$v = 3P_1 + P_2 \cancel{+} -2P_3 \text{ Ans}$$

Spanning Set:-

Let "V" be a vector space the vectors $u_1, u_2, u_3, \ldots, u_m \in V$ are said to span V or to form spanning set of V if every v in V is a linear combination of vectors $u_1, u_2, \ldots, u_m$ if there exist scalars, $k_1, k_2, \ldots, k_m \in K$ such that

$$v = k_1u_1 + k_2u_2 + \ldots + k_mu_m$$

Examples:-

Consider the vector space $V = \mathbb{R}^3$

Exp 1

We claim that the following vectors form a spanning set of $\mathbb{R}^3$

$$e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1)$$

Now linear combinat

$$v = k_1e_1 + k_2e_2 + k_3e_3$$

and let $v = (x, y, z)$

$$(x,y,z) = k_1(1,0,0) + k_2(0,1,0) + k_3(0,0,1)$$

$$\Rightarrow x = k_1$$
$$y = k_2$$

$$v = xe_1 + ye_2 + ze_3$$

Ex (iii)

$$w_1 = (1,1,1)$$
$$w_2 = (1,1,0)$$
$$w_3 = (1,0,0)$$

Now

$$v = k_1w_1 + k_2w_2 + k_3w_3 \rightarrow Ⓐ$$

$$v = k_1(1,1,1) + k_2(1,1,0) + k_3(1,0,0)$$

$$(x,y,z) = (k_1, k_1, k_2) + (k_2, k_2, 0) + (k_3 + 0 + 0)$$

$$(x,y,z) = (k_1 + k_2 + k_3,\ k_1 + k_2,\ k_1)$$ ✓

$$\boxed{k_1 = z}$$

$$k_1 + k_2 = y \Rightarrow k_2 = y - k_1 \Rightarrow \boxed{k_2 = y - z}$$

$$\boxed{k_1 + k_2} + k_3 = x$$

$$y + k_3 = x$$

$$k_3 = x - y$$

$$v = zw_1 + (y-z)w_2 + (x-y)w_3$$

4/02/2018

Exp we claim that following vectors form a spanning set of $\mathbb{R}^3$.

$u_1 = (1,2,3) \quad u_2 = (1,3,5)$

$u_3 = (1,5,9)$

$V = (2,7,8)$

Now linear combination is given

$V = k_1u_1 + k_2u_2 + k_3u_3 \rightarrow ①$

$(2,7,8) = k_1(1,2,3) + k_2(1,3,5) + k_3(1,5,9)$

$\Rightarrow \quad 2 = k_1 + k_2 + k_3 \;,\; 7 = 2k_1 + 3k_2 + 5k_3$

$8 = 3k_1 + 5k_2 + 9k_3$

make matrix from equation

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 3 & 5 & 7 \\ 3 & 5 & 9 & 8 \end{array}\right]$$

$$\begin{array}{l} R_2 - 2R_1 \to R_2 \\ R_3 - 3R_1 \to R_2 \end{array} \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & 6 & 2 \end{array}\right]$$

$$R_3 - 2R_2 \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

$k_1 + k_2 + k_3 = 2$

$k_2 + 3k_3 = 3$

u_1, u_2, u_3 donot span~~ning~~ $\mathbb{R}^3$ bcz $\boxed{K_3 \text{ does not exist}}$.

Propositions:-

Let $V(F)$ be a vector space. If O_V and O_F be the additive identities of the group V and field F respectively Then

1) $aO_V = O_V \;,\; \forall a \in F$

2) $O_F v = O_V \;,\; \forall v \in V$

3) If $av = O_V \Rightarrow$ Either $a = O_F$ or $v = O_V$

4) $(-a)v = a(-v) = -(av) \;,\; \forall a \in F$ and $v \in V$

5) $a(v_1 - v_2) = av_1 - av_2 \;,\; \forall a \in F$ and $\forall v_1, v_2 \in V$

6):- $av = v \Leftrightarrow a = 1_F \in F$

(1) $aO_V = O_V \quad , \forall a \in F$

Proof ~~(i) For $a \in F$ and $v \in V$ then $av \in V$~~

(i) For $a \in F$ and $v \in V$ then $av \in V$

$a\underline{V} = a(O_V + V)$ $\quad \because V = O_V + V$

$aV + O_V = aO_V + aV$ $\quad \because aV \in V \Rightarrow av = av + O_V$

$O_V + av = a O_V + av$

$O_V + av - av = aO_V + av - av$ $\quad \because a, O_V, V \in V$

So $a(O_V + V) = aO_V + av$

$\Rightarrow O_V = aO_V$ $\quad \because av \in V$ Then $-av \in V$

2) $VO_F = O_V \quad , \forall v \in V$

Proof (i) For $O_F \in F$ and $v \in V$, $a \in F$, $av \in V$

$av = (O_F + a)V$ $\quad \because a = O_F + a$

$av + O_V = O_F V + aV$ $\quad \because av + O_V = av$

$O_V + av = O_F V + av$ $\quad O_F, a, v \in V \Rightarrow (a+b)c = ac + bc$

$O_V + av - av = O_F V + av - av$

$\Rightarrow O_V = VO_F$

Lect2

20/02/2018

(iii) $av = O_V \Rightarrow$ Either $a = O_F$ or $V = O_V$

Suppose that $a \neq O_F$ and $V \neq O_V$ then $a^{-1} \in F$

$aV = O_V$

$a^{-1}(av) = a^{-1}O_V$ $\quad \because a \neq O_F \Rightarrow a^{-1} = F$

$(a^{-1}a)V = O_V$ $\quad \because$ Associative law holds w.r.t multiply

By part (1) i.e $aV = O_V$ $\quad a^{-1}O_V \in V \Rightarrow O_V \in V$

$1_F V = O_V$ $\quad \because$ Mutiplicative Identity

$V = O_V$

which is ~~cond~~ contradiction to our supposition that $V \neq O_V$

this prove can be completed by the argument supposing that $V \neq O_V$

(iv) $(-a)V = a(-V) = -(av)$

Sol:- By Part (2)

$0_F V = 0_V$

$[a+(-a)]V = 0_V$ ∵ $0_F = a+(-a)$

$av + (-a)v = 0_V$ Using right dist. law

$-(av) + av + (-a)V = -av + 0_V$ Taking additi inverse of av on both sides.

$0_V + (-a)v = av$

$(-a)V = -av$ ∵ additive identity

Similarly By part (1)

$a\, 0_V = 0_V$ ∵ $0_V \in V$

$a(v+(-v)) = 0_V$ $\Rightarrow v+(-v) = 0_V$

$av + a(-v) = 0_V$ ∵ Left dist. law

$-(av) + av + a(-v) = 0_V - \{av\}$ ∵ $av \in V \Rightarrow -av \in V$

$0_V + a(-v) = -av + 0_V$ $-(av) + av = 0_V$

$a(-v) = -av$ ∵ 0_V is additive Identity.

(5) $a(v_1 - v_2) = av_1 - av_2$

Sol:-

$L.H.S = a(v_1 - v_2)$

$= a[v_1 + (-v_2)]$

$= av_1 + a(-v_2)$ ∵ Using Dist. law

$= av_1 - av_2$ ∵ $a(-v_2) = -av_2$

(6) $av = v \Leftrightarrow a = 1_F \in F$

(Sol) Suppose that $0_F \neq a$ and $av = v$ for $v \neq 0_v$

$av = v$

$av - v = v - v$ $\quad$ $\because v \in V$

$av + (-v) = 0_v$ $\quad$ $\Rightarrow -v - \in V$

$\because v + (-v) = 0_v$

$(a + (-1_F))v - 0_v$

$\Rightarrow a - 1_F = 0_F$ or $v = 0_v$

$a - 1_F + 1_F - 0_F + 1_F$ $\quad$ $A \because 1_F \in F$

$\Rightarrow -1_F \in F$

$a = 1_F$ $\quad$ $-1_F + 1_F = 0_F$

Now $\quad a = 1_F$

As $v \in V$ Multiplying by v

$av = 1_F v$ $\quad$ $1_F \in F$ and $v \in V$

$av = v$ $\quad$ $\Rightarrow 1_F v = v \in V$

Theorem

Let $V(F)$ be vector space then the following cancellation laws hold within ~~vector~~ $V(F)$

(1) $\alpha v = \beta v$

$\Rightarrow \alpha = \beta$

$\forall \alpha, \beta \in F$

$\forall v \in V$

$v \neq 0_v$

(2) $\alpha v_1 = \alpha v_2$

$\Rightarrow v_1 = v_2$ $\quad$ $\alpha \in F, v_1, v_2 \in V$

$v_1 \neq 0_v$

$v_2 \neq 0_v$

$\alpha \neq 0_F$

Proof

(1) $\alpha \cancel{\beta} v - \beta v$ $\quad$ $\because \beta v \in V$

$\Rightarrow -\beta v \in V$

$\alpha v - \beta v = \beta v - \beta v$

$(\alpha - \beta)v = 0_v$ $\quad$ $\because \beta v + (-\beta v) = 0_v$

$\Rightarrow \alpha - \beta = 0_F \;, v = 0_v$ $\quad$ $\because av = 0_v$

$\Rightarrow$ Either $\alpha = 0$

$v = 0_v$

But $v \neq 0_v$ (Given)

So $\alpha - \beta = 0_F$

$\alpha + -\beta + \beta = 0_F + \beta$ ∵ $-\beta \in F$

$\alpha - \beta$ $\Rightarrow \beta \in F$

∵ $(-\beta) + \beta = 0_F$

Proof

(2) $\alpha v_1 = \alpha v_2$

$v_1 \neq 0_V$

$v_2 \neq 0_V$

$\cancel{\alpha}$ $\alpha \neq 0_F$

⇒ Sol

$\alpha v_1 - \alpha v_2 = \alpha v_2 - \alpha v_2$ ∵ $\alpha v_2 \in V$

$\Rightarrow -\alpha v_2 \in V$

$\cancel{\alpha v_1 + (-\alpha v_2) = 0_F}$ ∵ $-aV = (a)V$

$= a(-V)$

$\alpha v_1 + \alpha(-v_2) = 0_V$

$\alpha(v_1 + (-v_2)) = 0_V$

$\Rightarrow \alpha = 0_F$ or $v_1 + (-v_2) = 0_V$

But $\alpha \neq 0_F$ (Given)

So, $v_1 + (-v_2) = 0_V$ $-v_2 \in V$

$\Rightarrow v_2 \in V$

$v_1 + (-v_2) + v_2 = 0_V + v_2$ ∵ $0_V + v_2 = v_2$

∵ $v_1 + 0_V = v_1$

$v_1 + 0_V = v_2$

$v_1 = v_2$

Linearly Dependent:- (L.D)

Let $S = \{v_1, v_2, \dots, v_n\}$ be a subset of a vector Space $V(F)$. "S" is said to be L.D if there exist scalars $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \in F$ not all zero, such that

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots + \alpha_n v_n = 0_V \rightarrow ①$$

it may be understood that there exist a non-zero solution $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n = 0$ in F^n of the Eq① that is $\alpha_i \neq 0$ for some i, then

$$\alpha_i v_i = \cancel{\alpha_i} - \alpha_1 v_1 - \alpha_2 v_2 - \dots - \alpha_{i-1} v_{i-1} - \alpha_{i+1} v_{i+1}$$
$$- \alpha_{i+2} v_{i+2} - \dots - \alpha_n v_n \quad \text{for } i \leq n$$

$$v_i = \left(-\frac{\alpha_1}{\alpha_i}\right) v_1 + \left(-\frac{\alpha_2}{\alpha_i}\right) v_2 + \dots + \left(-\frac{\alpha_{i-1}}{\alpha_i}\right) v_{i-1}$$
$$+ \left(-\frac{\alpha_{i+1}}{\alpha_i}\right) v_{i+1} + \dots + \left(-\frac{\alpha_n}{\alpha_i}\right) v_n$$

Linearly Independent:-

In a Vector Space $V(F)$ a finite subset $S = \{v_1, v_2, v_3, \dots, v_n\}$ of $V(F)$ is called linearly Independent if there exist scalars $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n \in F$ such that the equation

$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = 0_v$

has only its zero solution and ~~non~~ non-zero solution exist at all.

$\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$

⟷

21/02/2018

Exp $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$

$e_3 = (0, 0, 1)$

$\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = 0_v = (0, 0, 0)$

$\alpha_1(1, 0, 0) + \alpha_2(0, 1, 0) + \alpha_3(0, 0, 1) = (0, 0, 0)$

$\Rightarrow (\alpha_1, 0, 0) + (0, \alpha_2, 0) + (0, 0, \alpha_3) = (0, 0, 0)$

$(\alpha_1, \alpha_2, \alpha_3) = (0, 0, 0)$

$\alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 0$

Linearly Independent.

Exp $V_1 (1, 1, 0)$, $V_2 = (1, 0, 1)$, $V_3 = (0, 1, 1)$

Here

$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0_v = (0, 0, 0)$

$\alpha_1(1, 1, 0) + \alpha_2(1, 0, 1) + \alpha_3(0, 1, 1) = (0, 0, 0)$

$(\alpha_1, \alpha_1, 0) + (\alpha_2, 0, \alpha_2) + (0, \alpha_3, \alpha_3) = (0, 0, 0)$

$(\alpha_1 + \alpha_2, \alpha_1 + \alpha_3, \alpha_2 + \alpha_3) = (0, 0, 0)$

$\alpha_1 + \alpha_2 = 0 \rightarrow (i)$, $\alpha_1 + \alpha_3 = 0 \rightarrow (ii)$, $\alpha_2 + \alpha_3 = 0 \rightarrow (iii)$

~~Eq (i) – (ii)~~ ~~$\alpha_1 = \alpha_2$~~ ~~Eq = α~~

~~$\alpha_1 + \alpha_2 = 0$~~

~~$-\alpha_1 \pm \alpha_3 = 0$~~

Eq (i) – (ii)

$\alpha_1 + \alpha_2 = 0$

$\underline{\pm\alpha_1 \pm \alpha_3 = 0}$

$\alpha_2 - \alpha_3 = 0$

$\alpha_2 = \alpha_3 \rightarrow (iv)$

Putting in (iii)

$\alpha_2 + \alpha_2 = 0$

$2\alpha_2 = 0$

$\boxed{\alpha_2 = 0}$

from (iv) $\boxed{\alpha_3 = 0}$

from (i) $\boxed{\alpha_1 = 0}$

Matrix from equations

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \dots + \alpha_n V_n = 0$

has only its zero ~~solution~~ ~~and so~~

non-zero solution exist at all.

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$|A| \neq 0$

$= 1(-1) - (1)$ Linear Independent

$= -2 \neq 0$

Exp $V_1 = (2, 3, -1, -2)$

$V_2 = (-6, -9, -3, 6)$

Sol:-

$\alpha_1 V_1 + \alpha_2 V_2 = (0, 0, 0, 0)$

$\alpha_1(2, 3, -1, -2) + \alpha_2(-6, -9, -3, 6) = (0,0,0,0)$

$(2\alpha_1, 3\alpha_1, -\alpha_1, -2\alpha_1) + (-6\alpha_2, -9\alpha_2, -3\alpha_2, 6\alpha_2) = (0,0,0,0)$

$(2\alpha_1 - 6\alpha_2, 3\alpha_1 - 9\alpha_2, -\alpha_1 - 3\alpha_2, -2\alpha_1 + 6\alpha_2) = (0,0,0,0)$

$\Rightarrow 2\alpha_1 - 6\alpha_2 = 0 \to (i), \quad 3\alpha_1 - 9\alpha_2 = 0 \to (ii), \quad -\alpha_1 - 3\alpha_2 = 0 \to (iii)$

$-2\alpha_1 + 6\alpha_2 = 6 \to (iv)$

taking "2" common from Eq (i)

$\alpha_1 - 3\alpha_2 = 0$

adding in (iii) $+\alpha_1 - 3\alpha_2 = 0$

$-6\alpha_2 = 0$

$\alpha_2 = 0$

from (i)

$2\alpha_1 - 0 = 0$

$\boxed{\alpha_1 = 0}$

As all α_i's are zero so this is linear Independent combination.

Q1 $V_1 = (1, 2, 3)$ → Check **H.W**

$V_2 = (2, -1, 0)$ Independent or dependent

Q2 $V_1 (1, 1, 0)$, $V_2 = (0, 1, 1)$, $V_3 = (1, 0, 1)$

Check standard basis.

Basis of a Vector Space:-

Let $V(F)$ be a vector space over a field F, a subset B of $V(F)$ is said to form a basis of the V.S $V(F)$ if

(1):- $L(B) = V(F)$

i.e Linear span of B is the whole of the vector space $V(F)$

(2): The set B is a L.I subset of $V(F)$.

$e_1 = (1,0,0)$, $e_2 = (0,1,0)$ and $e_3 = (0,0,1)$ are standard basis.

For Exp $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

$$V = (b_1, b_2, b_3)$$

$$V = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 \Rightarrow Ⓐ$$

$$(b_1, b_2, b_3) = \alpha_1(1,0,0) + \alpha_2(0,1,0) + \alpha_3(0,0,1)$$

$$(b_1, b_2, b_3) = (\alpha_1, \alpha_2, \alpha_3)$$

$$\alpha_1 = b_1 \; ; \; \alpha_2 = b_2 \, , \; \alpha_3 = b_3$$

Eq Ⓐ becomes

$$(b_1, b_2, b_3) = b_1 e_1 + b_2 e_2 + b_3 e_3$$

~~let $(b_1, b_2, b_3) = (1, 2, 3)$~~

~~$(1, 2, 3) = 1(1,0,0) + 2(0,1,0) + 3(0,0,1)$~~

~~$(1, 2, 3) = (1, 0, 0) +$~~

Check for $e_1(1,0,0) = (b_1, b_2, b_3)$

$$S = \{e_1, e_2, e_3\}$$

No. of basis in a vector space is called dimensions of basis.

Exp $V_1 = (1, 2, 1)$

$V_2 = (2, 9, 0)$

$V_3 = (3, 3, 4)$ ~~Show that~~

~~Sol:~~ Show That $S = \{V_1, V_2, V_3\}$ is a basis ~~of~~ in $\mathbb{R}^3$

Sol:-

$$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0_V$$

$$\alpha_1(1,2,1) + \alpha_2(2,9,0) + \alpha_3(3,3,4) = (0,0,0)$$

$$(\alpha_1, 2\alpha_1, \alpha_1) + (2\alpha_2, 9\alpha_2, 0) + (3\alpha_3, 3\alpha_3, 4\alpha_3) = (0,0,0)$$

$$\alpha_1 + 2\alpha_2 + 3\alpha_3 = 0 \rightarrow (i) \quad 2\alpha_1 + 9\alpha_2 + 3\alpha_3 = 0 \rightarrow (ii),$$

$$\alpha_1 + 4\alpha_3 = 0 \rightarrow (iii)$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

$= 1(36) - 2(8-3) + 3(-9)$

$= 36 - 10 - 27$

$= -1 \neq 0$ Independenent

xing (i) with ~~α~~ $\underset{\approx}{2}$ and subtracting ~~from (ii)~~

$\cancel{2\alpha_1} + 4\alpha_2 + 6\alpha_3 = 0$

$\pm \cancel{2\alpha_1} \pm 9\alpha_2 \pm 3\alpha_3 = 0$

$-5\alpha_2 + 3\alpha_3 = 0 \rightarrow (iv)$

~~multiply (iii) 5α~~ with 5 $\quad 5\alpha_2 = 3\alpha_3$

$\alpha_2 = \frac{3}{5}\alpha_3 \rightarrow Ⓐ$

putting in eq (i)

$\alpha_1 + \frac{6}{5}\alpha_3 + 3\alpha_3 = 0$

$\frac{5\alpha_1 + 6\alpha_3 + 15\alpha_3}{5} = 0$

$5\alpha_1 + 21\alpha_3 = 0 \rightarrow (v)$

a~~dding in (iii)~~

Multiplying (iii) with 5 and -ing from (v)

$\cancel{\mp 5\alpha_1} \pm 20\alpha_3 = 0$

$+\alpha_3 = 0$

from Ⓐ

$\boxed{\alpha_2 = 0}$

from (iii)

$\boxed{\alpha_1 = 0}$

So these are linear ~~ly~~ Indeapendent combination/set.

Now $\quad V = (b_1, b_2, b_3)$

$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = V = (b_1, b_2, b_3) \rightarrow Ⓐ$

~~$\alpha_1(1,2,3) + \alpha_2(2,9,3) + \alpha_3(1,0,4) = (b_1, b_2, b_3)$~~

~~$(\alpha_1 + 2\alpha_2 + \alpha_3, 2\alpha_1 + 9\alpha_2, 3\alpha_1 + 3\alpha_2 + 3\alpha_3) = (b_1, b_2, b_3)$~~

~~$\alpha_1 + 2\alpha_2 + \alpha_3 = b_1, 2\alpha_1 + 9\alpha_2$~~

$\alpha_1(1,2,1) + \alpha_2(2,9,0) + \alpha_3(3,3,4) = (b_1, b_2, b_3)$

$(\alpha_1 + 2\alpha_2 + 3\alpha_3, 2\alpha_1 + 9\alpha_2 + 3\alpha_3, \alpha_1 + 4\alpha_3) = (b_1, b_2, b_3)$

$\alpha_1 + 2\alpha_2 + 3\alpha_3 = b_1 \rightarrow (i)$

$2\alpha_1 + 9\alpha_2 + 3\alpha_3 = b_2 \rightarrow (ii)$

$4\alpha_1 + 4\alpha_3 = b_3 \rightarrow (iii)$

xing eq (i) with "2" and substracting from (ii)

$\cancel{2\alpha_1} + 9\alpha_2 + 3\alpha_3 = b_2$

$\pm \cancel{2\alpha_1} \pm 4\alpha_2 \pm 6\alpha_3 = 2b_1$

$5\alpha_2 - 3\alpha_3 = b_2 - 2b_1$

$$5\alpha_2 = b_2 - 2b_1 + 3\alpha_3$$

$$\alpha_2 = \frac{1}{5}(b_2 - 2b_1 + 3\alpha_3) \rightarrow (iv)$$

* putting in eq (ii)

$$4\alpha_1 + 9/5(b_2 - 2b_1 + 3\alpha_3) + 3\alpha_3 = b_2$$

$$4\alpha_1 + \frac{9}{5}b_2 - \frac{18}{5}b_1 + \frac{27\alpha_3}{5} + 3\alpha_3 = b_2$$

$$4\alpha_1 + \frac{42}{5}\alpha_3 = b_2 - \frac{9}{5}b_2 + \frac{18}{5}b_1$$

$$\cancel{4\alpha_1} + \frac{42}{5}\alpha_3 = \frac{-4}{5}b_2 + \frac{18}{5}b_1$$

Subtracting Eq(iii) from above

$$\pm 4\alpha_1 \pm 4\alpha_3 = \pm b_3$$

$$\frac{22}{5}\alpha_3 = -\frac{4}{5}b_2 + \frac{18}{5}b_1 - b_3$$

xing "5"

$$22\alpha_3 = -4b_2 + 18b_1 - 5b_3$$

$$\boxed{\alpha_3 = \frac{1}{22}(-4b_2 + 18b_1 - 5b_3)}$$

Putting in (iv)

$$\alpha_2 = \frac{1}{5}\left[b_2 - 2b_1 + \frac{3}{22}(-4b_2 + 18b_1 - 5b_3)\right]$$

$$\alpha_2 = \frac{1}{5}\left[b_2 - 2b_1 - \frac{6}{11}b_2 + \frac{27}{11}b_1 - \frac{15}{22}b_3\right]$$

$$\alpha_2 = \frac{1}{5}\left[\frac{5}{11}b_2 + \frac{5}{11}b_1 - \frac{15}{22}b_3\right]$$

$$\boxed{\alpha_2 = \frac{1}{11}\left(b_2 + b_1 - \frac{3}{2}b_3\right)}$$

Eq(i) becomes

$$\alpha_1 + 2/11\left(b_2 + b_1 - \frac{3}{2}b_3\right) + 3/22(-4b_2 + 18b_1 - 5b_3) = b_1$$

$$\alpha_1 + \frac{2}{11}b_2 + \frac{2}{11}b_1 - \frac{3}{11}b_3 - \frac{6}{11}b_2 + \frac{27}{11}b_1 - \frac{15}{22}b_3 = b_1$$

$$\alpha_1 + \frac{29}{11}b_1 - \frac{4}{11}b_2 - \frac{21}{22}b_3 = b_1$$

$$\alpha_1 = b_1 - \frac{29}{11}b_1 + \frac{4}{11}b_2 - \frac{21}{22}b_3$$

$$\alpha_1 = \frac{-18}{11}b_1 + \frac{4}{11}b_2 - \frac{21}{22}b_3$$

$$\alpha_1 = \frac{1}{11}\left(-18b_1 + 4b_2 - \frac{21}{2}b_3\right)$$

So Eq ①

$$\frac{1}{11}\left(-18b_1 + 4b_2 - \frac{21}{2}b_3\right)V_1 + \frac{1}{11}\left(b_2 + b_1 - \frac{3}{2}b_3\right) + \frac{1}{22}(4b_2 + 18b_1 - 5b_3)$$

$$= V = (b_1, b_2, b_3)$$

Home work

Sunday
25/02/2018

Q1 $V_1 = (1,2,3)$ $V_2 = (2,-1,0)$

Check dependent or Independent

Sol:- we can write linear combination for this as

$\alpha_1 V_1 + \alpha_2 V_2 = 0_V = (0,0,0)$

$\alpha_1(1,2,3) + \alpha_2(2,-1,0) = (0,0,0)$

$(\alpha_1, 2\alpha_1, 3\alpha_1) + (2\alpha_2, -\alpha_2, 0) = (0,0,0)$

$(\alpha_1 + 2\alpha_2, 2\alpha_1 - \alpha_2, 3\alpha_1) = (0,0,0)$

$\alpha_1 + 2\alpha_2 = 0 \to (i)$ $2\alpha_1 - \alpha_2 = 0 \to (ii)$ $3\alpha_1 = 0 \to (iii)$

$\boxed{\alpha_1 = 0}$

from (i) or (ii)

$0 + 2\alpha_2 = 0$ $2(0) - \alpha_2 = 0$

$\alpha_2 = 0$ $\alpha_2 = 0$

$\boxed{\alpha_2 = 0}$

As all a_i's are zero so this combination is linearly independent.

Q2 check standard basis

$V_1 = (1,1,0)$, $V_2 = (0,1,1)$, $V_3 = (1,0,1)$

Sol first of all we check linear dependency of this

So

$\alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0_V = (0,0,0)$

$\alpha_1(1,1,0) + \alpha_2(0,1,1) + \alpha_3(1,0,1) = (0,0,0)$

$(\alpha_1, \alpha_1, 0) + (0, \alpha_2, \alpha_2) + (\alpha_3, 0, \alpha_3) = (0,0,0)$

$(\alpha_1 + \alpha_3, \alpha_1 + \alpha_2, \alpha_2 + \alpha_3) = (0,0,0)$

$\alpha_1 + \alpha_3 = 0 \to (i)$ $\alpha_1 + \alpha_2 = 0 \to (ii)$ $\alpha_2 + \alpha_3 = 0 \to$ (iii

by Eq(i) − Eq(ii)

$\cancel{\alpha_1} + \alpha_3 = 0$

$\pm\cancel{\alpha_1} \pm \alpha_2 = 0$

$-\alpha_2 + \alpha_3 = 0 \to (iv)$

by Eq(iii) + Eq(iv)

$\cancel{\alpha_2} + \alpha_3 = 0$

$-\cancel{\alpha_2} + \alpha_3 = 0$

$2\alpha_3 = 0$

$\boxed{\alpha_3 = 0}$

putting $\alpha_3=0$ in Eq (iv)

$\boxed{\alpha_1=0}$

putting in Eq (ii)

$\boxed{\alpha_2=0}$

As all ai's are zero so this combination is linearly independent.

Now

$$\alpha_1 v_1+\alpha_2 v_2+\alpha_3 v_3=V=(b_1,b_2,b_3) \rightarrow Ⓐ$$

$$\alpha_1(1,1,0)+\alpha_2(0,1,1)+\alpha_3(1,0,1)=(b_1,b_2,b_3)$$

$$(\alpha_1,\alpha_1,0)+(0,\alpha_2,\alpha_2)+(\alpha_3,0,\alpha_3)=(b_1,b_2,b_3)$$

$$(\alpha_1+\alpha_3,\alpha_1+\alpha_2,\alpha_2+\alpha_3)=(b_1,b_2,b_3)$$

$$\alpha_1+\alpha_3=b_1 \rightarrow (i),\quad \alpha_1+\alpha_2=b_2 \rightarrow (ii),\quad \alpha_2+\alpha_3=b_3 \rightarrow (iii)$$

by Eq (i) − Eq (ii)

$$\cancel{\alpha_1}+\alpha_3=b_1$$

$$\underline{\pm\cancel{\alpha_1}\pm\alpha_2=\pm b_2}$$

$$-\alpha_2+\alpha_3=b_1-b_2 \rightarrow (iv)$$

by Eq (iv) + Eq (iii)

$$-\cancel{\alpha_2}+\alpha_3=b_1-b_2$$

$$\underline{\cancel{\alpha_2}+\alpha_3=b_3}$$

$$2\alpha_3=b_1-b_2+b_3$$

$$\boxed{\alpha_3=\frac{1}{2}b_1-\frac{1}{2}b_2+\frac{1}{2}b_3} \Rightarrow \boxed{\alpha_3=\frac{1}{2}(b_1-b_2+b_3)}$$

putting in Eq (iii)

$$\alpha_2+\frac{1}{2}b_1-\frac{1}{2}b_2+\frac{1}{2}b_3=b_3$$

$$\boxed{\alpha_2=\frac{1}{2}b_2-\frac{1}{2}b_1+\frac{1}{2}b_3} \Rightarrow \boxed{\alpha_2=\frac{1}{2}(b_2-b_1+b_3)}$$

$$=\frac{2-1}{2}b_3=\frac{1}{2}b_3$$

Putting in eq (ii)

$$\alpha_1+\frac{1}{2}b_2-\frac{1}{2}b_1+\frac{1}{2}b_3=b_2$$

$$\boxed{\alpha_1=\frac{1}{2}b_1-\frac{1}{2}b_3+\frac{1}{2}b_2} \Rightarrow \boxed{\alpha_1=\frac{1}{2}(b_1-b_3+b_2)}$$

Putting values of α_1,α_2 and α_3 in eq Ⓐ

$$\frac{1}{2}(b_1-b_3+b_2)v_1+\frac{1}{2}(b_2-b_1+b_3)v_2+\frac{1}{2}(b_1-b_2+b_3)v_3=(b_1,b_2,b_3) \rightarrow Ⓑ$$

Check Ⓑ for $v_1=(1,1,0)=(b_1,b_2,b_3)$

$$\frac{1}{2}(1-0+1)(1,1,0)+\frac{1}{2}(1-1+0)v_2+\frac{1}{2}(1-1+0)v_3=(1,1,0)$$

$$\frac{1}{\cancel{2}}(\cancel{2})(1,1,0)+0+0=(1,1,0)$$

$(1,1,0)=(1,1,0)$ True for v_1

Now for $v_2=(0,1,1)=(b_1,b_2,b_3)$

Eq Ⓑ $\Rightarrow$

$$\frac{1}{2}(0-1+1)v_1+\frac{1}{2}(1-0+1)(0,1,1)+\frac{1}{2}(0-1+1)v_3=(0,1,1)$$

$$0+(0,1,1)+0=(0,1,1)$$

$(0,1,1)=(0,1,1)$ True for v_2

Now for $V_3 = (1,0,1) = (b_1, b_2, b_3)$

Eq Ⓑ ⇒

$$\frac{1}{2}(1-1+0)V_1 + \frac{1}{2}(0-1+1)V_2 + \frac{1}{2}(1-0+1)(1,0,1) = (1,0,1)$$

$$0 + 0 + \frac{1}{2}(2)(1,0,1) = (1,0,1)$$

$$(1,0,1) = (1,0,1) \text{ True}$$

So the subset $S = \{V_1, V_2, V_3\}$ forms a linear span of V.S.

Hence V_1, V_2 and V_3 are basis of Vector Space:.

— × — × —

Monday

26/02/2018

Dimensions of Basis:-

A vector Space $V(F)$ with basis B ($B \subseteq V(F)$) is called m-dimensional vector space if no. of vectors in the basis "B" are finite in no. equal to "m" otherwise there exists no ∃ such finite "m" $V(F)$ is understood infinite dimensional

Mid till here

Subspace of a Vector Space V(F):-

1st defn.

⇒ Let $V(F)$ be a vector space over a Field "F". A subset U of $V(F)$ is called a subspace of $V(F)$

If

(i) U is subgroup of $V(F)$

(ii) U is closed under the field multiplication

$\alpha \in F$, $u_1 \in U$

$\Rightarrow \alpha u_1 \in U$

2nd defination:-

⇒ A subset U of a V.S $V(F)$ is said to form a subspace of $V(F)$ if

(i) $u_1 - u_2 \in U$

$u_1, u_2 \in U$

(ii) $\alpha u \in U$

where $u \in U$, $\alpha \in F$

3rd defination:-

A subset U of a vector space $V(F)$ is called a subspace of $V(F)$ if

(i) $\forall\ \alpha, \beta \in F$ and $\forall\ u_1, u_2 \in U$

$\alpha u_1 + \beta u_2 \in U$

05/03/2018

Immediate Deduction from the definition

1):- If V(F) is a vector space then each subgroup U of abelian group (V, +) is a sub-space of V(F), which is invariant under the scalar multiplication by the elements of F.

2):- Subspace must be a vector space, over the same field as that of super space V.

3):- The Vector Space V(F) itself a subspace of its own vector space.

4):- The zero vector of V(F) is contained in each subspace of V(F).

5):- The subset ~~ze~~ $\{0_V\}$ of V(F) form ~~of~~ a subspace of V(F) it is the only subspace containing one vector of V(F).

It is called zero space or Null space.

6):- The subspace $\{0_V\}$ and the space V(F) are called trivial ~~sub~~/improper subspaces of V(F). and other subspaces are called non-trivial/proper subspaces.

7):- If U_1 and U_2 are two subspaces of vector space V(F) then $U_1 \cap U_2$ is also a subspace V(F).

—×—×—

Tuesday:- 06/03/2018

Theorem:-

Let U_1 and U_2 be two subspaces of a vector space V(F) then the subset $U_1+U_2=\{u_1+u_2 : u_1 \in U_1, u_2 \in U_2\}$ forms a subspace of V(F)

Proof:-

Let $x = u_1+u_2$ and $y = u_1'+u_2$

$u_1 \neq u_1'$

$u_2 \neq u_2'$

where $u_1, u_1' \in U_1$ and $u_2, u_2' \in U_2$

$\alpha x + \beta y$

$= \alpha(u_1 + u_2) + \beta(u_1' + u_2')$ where $\alpha, \beta \in F$

$= \alpha u_1 + \alpha u_2 + \beta u_1' + \beta u_2'$ By Dist. law.

$= \alpha u_1 + \beta u_1' + \alpha u_2 + \beta u_2'$ By commutative law.

$= (\alpha u_1 + \beta u_1') + (\alpha u_2 + \beta u_2')$

where $\alpha u_1 + \beta u_1' \in U_1$ bcz U_1 is a subspace

Similarly $\alpha u_2 + \beta u_2' \in U_2$ bcz U_2 is a subspace

So,

$\alpha x + \beta y = (\alpha u_1 + \beta u_1') + (\alpha u_2 + \beta u_2') \in U_1 + U_2$

then $U_1 + U_2$ is a subspace.

13/03/2017

Change of Basis / Replacement of Basis

Let $\underbrace{S = \{x_1, x_2, x_2, \ldots, x_n\}}$ is a basis in V.S $V(F)$.

let $y = \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \underline{\alpha_i x_i} + \ldots + \alpha_n x_n$ be a non-zero vector and $\alpha_j \neq 0$. Then we can write

$$x_j = \left(-\frac{\alpha_i}{\alpha_j}\right)x_1 + \left(-\frac{\alpha_2}{\alpha_j}\right) + \ldots + \left(-\frac{\alpha_{j-1}}{\alpha_j}\right)x_{j-1} + \left(\frac{1}{\alpha_j}\right)y + \left(\frac{-\alpha_{j+1}}{\alpha_j}\right)x_{j+1} + \ldots + \left(\frac{-\alpha_n}{\alpha_j}\right)x_n$$

So, $\{x_1, x_2, x_3, \ldots, \underbrace{x_{j-1}, y, x_{j+1}}, \ldots, x_n\}$ be a basis.

Extra lecture with self

In class 02/04/2018

$V_1 = (1, 0, 2)$ $V_2 = (2, 1, 0)$

Linear Combination $V_1 = k_1 u_1 + k_2 u_2 + k_3 u_3$

$(1, 0, 2) = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3$

$= \alpha_1(1, 0, 0) + \alpha_2(0, 1, 0) + \alpha_3(0, 0, 1)$

$(1, 0, 2) = (\alpha_1, \alpha_2, \alpha_3)$

$\alpha_1 = 1$

$\alpha_2 = 0$

$\alpha_3 = 3$

$(1, 0, 2) = e_1 + 2e_3$

Now Replace $\{e_1, e_2, e_3\}$ to $\{V_1, e_2, e_3\}$

$\{(1,0,2), (0,1,0), (0,0,1)\}$

Now

$$(1,0,2) = \alpha_1(1,0,2) + \alpha_2(0,1,0) + \alpha_3(0,0,1)$$

$$(1,0,2) = (\alpha_1, 0, 2\alpha_1) + (0, \alpha_2, 0) + (0, 0, \alpha_3)$$

$$(1,0,2) = (\alpha_1, \alpha_2, 2\alpha_1 + \alpha_3)$$

$= \alpha_1 = 1$

$\alpha_2 = 0$

$2\alpha_1 + \alpha_3 = 2$

$\alpha_3 = 0$

$(1, 0, 2) = V_1$

Now let

$\{(1,0,0), (1,0,2), (0,0,1)\}$

Now

$$(1,0,2) = \alpha_1(1,0,0) + \alpha_2(1,0,2) + \alpha_3(0,0,1)$$

[illegible]

$\alpha_1 + \alpha_2 = 1$

$2\alpha_2 + \alpha_3 = 2$

let $\alpha_2 = 0$

$\alpha_3 = 2$

$V_1 = V$

Now, $\{V_2, e_2, e_3\}$

$$V_2 = \alpha_1 V_2 + \alpha_2 e_2 + \alpha_3 e_3$$

$$(2, 1, 0) = \alpha_1(2,1,0) + \alpha_2(0,1,0) + \alpha_3(0,0,$$

$2\alpha_1 = 2 \Rightarrow \alpha_1 = 1$

$\alpha_1 + \alpha_2 = 1$

$\alpha_2 = 0$

$\alpha_3 = 0$

$(2,1,0) = V_2$

$\{V_1, V_2, e_3\}$

$V_1 = \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 e_3$

$(1,0,2) = \alpha_1(1,0,2) + \alpha_2(2,1,0) + \alpha_3(0,0,1)$

$\alpha_1 + 2\alpha_2 = 1 \Rightarrow \alpha_1 = 1$

$\alpha_2 = 0$

$2\alpha_1 + \alpha_3 = 2$

$\alpha_3 = 0$

$V_2 = \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 e_3$

$(2,1,0) = \alpha_1(1,0,2) + \alpha_2(2,1,0) + \alpha_3(0,0,1)$

$\alpha_1 + 2\alpha_2 = 2 \Rightarrow \alpha_1 = 0$

$\alpha_2 = 1$

$2\alpha_1 + \alpha_3 = 0$

$\alpha_3 = 0$

02/04/2014

$V_2 = (2,1,0)$

(e_1, e_2, V_2)

$V_2 = K_1 e_1 + K_2 e_2 + K_3 V_2$

$(2,1,0) = K_1(1,0,0) + K_2(0,1,0) + K_3(2,1,0)$

$K_1 + 2K_3 = 2$

$K_2 + K_3 = 1$

let $K_3 = 1$

$K_2 = 0$

$K_1 = 0$

~~$(2,1,0) = (1,0,0) + (0,1,0)$~~

$=$

Inner product

Let "V" be a real vector space suppose to each pair of vector $u, v \in V$ there is assigned a real number denoted by $\langle u, v \rangle$ this function is called a real inner product on "V" if it satisfies the following axioms.

I_1 I_1):- (Linear property)

$\langle au_1 + bu_2, v \rangle = a\langle u_1, v \rangle + b\langle u_2, v \rangle$

$\sqrt{-1}$ $\sqrt{1} =$ (-1) $-|x-1=1$

I_2):- Symmetric Property

$\langle u, v\rangle = \langle v, u\rangle$

I_3):- Positive definite property

$\langle u, u\rangle \geq 0$

$\langle u, u\rangle = 0$ if and if $u = 0$

The Vector space "V" with an inner product is called **Inner product space**

Tuesday **characteristics** **03/04/2018**

Axiom (I_1) ~~is~~ is equilant to following two condition

(a) :- $\langle u_1 + u_2, v\rangle = \langle u_1, v\rangle + \langle u_2, v\rangle$

(b) :- $\langle au, v\rangle - a\langle u, v\rangle$

Using I_1 and I_2 (Symmetric Property) ~~by~~ we obtain

$\langle u, cv_1 + dv_2\rangle = \langle cv_1 + dv_2, u\rangle$ By I_2

$= c\langle v_1, u\rangle + d\langle v_2, u\rangle$ by I_1

$= c\langle u, v_1\rangle + d\langle u, v_2\rangle$ by I_2

which is equilant to

(a) $\langle u, v_1 + v_2\rangle = \langle u, v_1\rangle + \langle u, v_2\rangle$

$\|u\|$ "u" norm

(b) $\langle u, cv\rangle = c\langle u, v\rangle$

That is the inner product function $\langle u, v\rangle$ is also a linear in its 2nd position. By Induction we obtain

$\langle a_1u_1 + a_2u_2 + \dots + a_nu_n, v\rangle = a_1\langle u_1, v\rangle + a_2\langle u_2, v\rangle + \dots + a_n\langle u_n, v\rangle$

and

$\langle u, b_1v_1 + b_2v_2 + b_3v_3 + \dots + b_nv_n\rangle =$ ~~ue~~ $b_1\langle u, v_1\rangle + b_2\langle u, v_2\rangle + \dots + b_n\langle u, v_n\rangle$

By I_3 $\langle u, u\rangle \geq 0$ and, hence its +ve real square root exists and we use the notation

$\|u\| = \sqrt{\langle u, u\rangle}$

this non-negative real number $\|u\|$ is called "norm" or "length of "u""

Consider the vector space $\mathbb{R}^n$ the dot product (scalar product) in $\mathbb{R}^n$ is defined by

$u.v = a_1b_1 + a_2b_2 + \dots + a_nb_n$

$u(a_1, a_2, a_3, \dots, a_n)$

$v = (b_1, b_2, b_3, \dots, b_n)$

the function defines an inner product on

$\mathbb{R}^n$. The norm $\|u\|$ of the vector u is

$$\|u\| = (a_i)$$

in this space follows

$$\|u\| = \sqrt{u.u} = \sqrt{a_1^2 + a_2^2 + a_3^2 + \dots + a_n^2}$$

Cuachy-Schwarz Inequality

For any vectors $u, v \in V$

$$\langle u,v\rangle^2 \leq \langle u,u\rangle\langle v,v\rangle$$

OR

$$|\langle u,v\rangle| \leq \|u\| . \|v\|$$

$$\langle u,v\rangle^2 \leq \langle u,u\rangle\langle v,v\rangle$$

Properties of Norm:-

Let "V" be inner product space then the $\|u\|$ in V satisfies the following properties

N_1):-

$$\|v\| \geq 0$$

$$\|v\| = 0 \text{ iff } v = 0$$

N_2):-

$$\|kv\| = k\|v\|$$

N_3):- $\|u+v\| \leq \|u\| + \|v\|$

N_3 is called Triangle inequality. In other words v is orthagonal to "u" "v" satisfies a homogeneous equation whose coefficient are the element of u

$$v_1 = (1,3,5) , v_2 = (0,1,4) \text{ in } \mathbb{R}^3$$

Find orthogonal vector.

Let $w = (x, y, z)$

$\langle v_1, w\rangle = 0$, $\langle v_2, w\rangle = 0$

$x + 3y + 5z = 0 \rightarrow ①$ $y + 4z = 0 \rightarrow ②$

$y = -4z$

let $z = 1$

$① \Rightarrow x - 12 + 5 = 0$ $\boxed{y = -4}$

$\boxed{x = 7}$

$w = (7, -4, 1)$

check also

Orthogonal compliment:-

Let "S" be a subset of an inner product space V the orthogonal compliment of "S" denoted by $S^{\perp}$ consist of those vectors in V ~~with~~ which are orthogonal to every vector $u \in S$

$$S^{\perp} = \{v \in V : \langle v, u\rangle 0 \text{ for every } u \in S\}$$

Orthogonal and orthonormal Sets:-

A set "S" of vectors in V is called orthogonal if each pair of vectors in "S" are orthogonal. and S is called orthonormal if "S" is orthogonal and each vectors in "S" has unit length.

Orthogonal $\Rightarrow \langle u_i, u_j\rangle = 0$ if $i \neq j$

Orthonormal $\Rightarrow \langle u_i, u_j\rangle = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

$e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$, $e_3 = (0, 0, 1)$ in $\mathbb{R}^3$

$\langle e_1, e_2\rangle =$

$= 0$

$\langle e_2, e_3\rangle = 0$

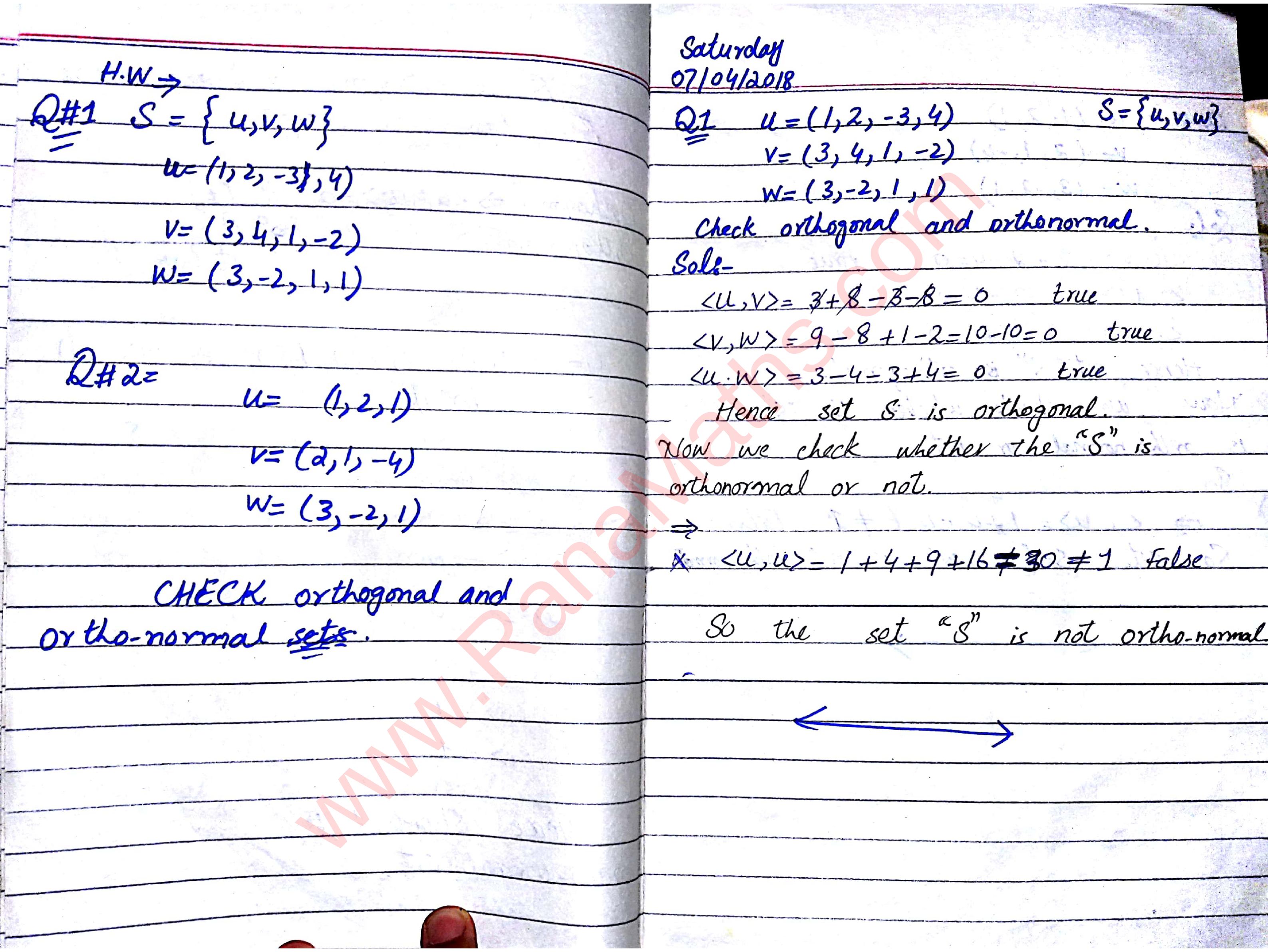
$\langle e_3, e_2\rangle = 0$

$\langle e_1, e_1\rangle = 1$

$\langle e_2, e_2\rangle = 1$

$\langle e_3, e_3\rangle = 1$

Here Standard Basis are ortho-Normal set.



H.W →

Q#1 $S = \{u, v, w\}$

$u = (1, 2, -3, 4)$

$v = (3, 4, 1, -2)$

$w = (3, -2, 1, 1)$

Q#2 =

$u = (1, 2, 1)$

$v = (2, 1, -4)$

$w = (3, -2, 1)$

CHECK orthogonal and ortho-normal sets.

Saturday
07/04/2018

Q1 $u = (1, 2, -3, 4)$ $S = \{u, v, w\}$

$v = (3, 4, 1, -2)$

$w = (3, -2, 1, 1)$

Check orthogonal and orthonormal.

Sol:-

$\langle u, v\rangle = \cancel{3} + \cancel{8} - \cancel{3} - \cancel{8} = 0$ true

$\langle v, w\rangle = 9 - 8 + 1 - 2 = 10 - 10 = 0$ true

$\langle u . w\rangle = 3 - 4 - 3 + 4 = 0$ true

Hence set S is orthogonal.

Now we check whether the "S" is orthonormal or not.

⇒

✗ $\langle u, u\rangle = 1 + 4 + 9 + 16 \neq 30 \neq 1$ False

So the set "S" is not ortho-normal.

⟷

Q#2 $u = (1, 2, 1)$

$v = (2, 1, -4)$

$w = (3, -2, 1)$

Sol:

$\langle u, v \rangle = 2 + 2 - 4 = 0$ true

$\langle v, w \rangle = 6 - 2 - 4 = 0$ true

$\langle u, w \rangle = 3 - 4 + 1 = 0$ true

Hence set "S" is orthogonal.

Now. we check whether the "S" is ortho-normal or not

So

$\Rightarrow \langle u, u \rangle = 1 + 4 + 1 = 6 \neq 1$ False

So set $S = \{u, v, w\}$ is not ortho-normal

09/04/2018

Q#1 continued.

$u = (1, 2, -3, 4)$

$\|u\|^2 = 30$

$\|u\| = \sqrt{30}$

$\hat{u} = \left(\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{-3}{\sqrt{30}}, \frac{4}{\sqrt{30}}\right)$

$V = (3, 4, 1, -2)$

$\|V\| = \sqrt{30}$

$\hat{V} = \left(\frac{30}{\sqrt{30}}, \frac{4}{\sqrt{30}}, \frac{1}{\sqrt{80}}, \frac{-2}{\sqrt{30}}\right)$

$\langle \hat{u} \cdot \hat{V} \rangle = \frac{3}{30} + \frac{8}{30} - \frac{3}{30} - \frac{8}{30}$

$= 0$

$\langle \hat{u} \cdot \hat{u} \rangle = \frac{1}{30} + \frac{4}{30} + \frac{9}{30} + \frac{16}{30}$

$= \frac{30}{30} - 1$

Projections:-

Consider a non-zero vector w in an inner product space $\mathbf{V}$. for any $v \in V$ the projection of $\vec{v}$ along w is given by

$$\text{proj}(v,w) = \frac{\langle v,w \rangle}{\langle w,w \rangle} w = \frac{\langle v,w \rangle}{\|w\|^2} w$$

Example:-

$$V = (1,2,3,4)$$

$$w = (1,-3,4,-2) \quad \text{in } \mathbb{R}^4$$

$\text{proj}(v,w) - ?$

Sol:-

$$\langle v,w \rangle = 1 \cancel{+} -6 + 12 - 8 = -1$$

$$\langle w,w \rangle = 1 + 9 + 16 + 4 = 30$$

So

$$\text{Proj}(V,w) = -\frac{1}{30}(1,-3,4,-2)$$

Now

$$= \left(\frac{-1}{30}, \frac{1}{10}, \frac{-2}{15}, \frac{1}{15}\right)$$

$$\langle v,v \rangle = 1 + 4 + 9 + 16 = 30$$

So

$$\text{Proj}(w,v) = \frac{-1}{30} V$$

$$\text{Proj}(w,v) - \frac{-1}{30}(1,2,3,4)$$

$$= \left(-\frac{1}{30}, \frac{-1}{15}, \frac{1}{10}, \frac{-2}{15}\right)$$

Positive ~~denfit~~ definite matrix:-

Let "A" be Real, symmetric matrix there exist a non-sigular matrix ~~B~~ P such that

$B = P^T A P$ is diagonal and that the number of +ve entries in B is an in-varient of A. The matrix A is said to be +ve definite ~~matrix~~ if all The diagonal enteries of B is +ve. In other words, A is +ve definite if

$$X^T A X > 0$$

for every non-zero vector X in $\mathbb{R}^n$.

for example:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ -1 & -2 & 8 \end{bmatrix}$$

$$R_1 + R_3 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 2 & 7 \end{bmatrix}$$

~~$2R_2+$~~

$$2R_2+R_3 \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

Now

$$R_1+\frac{1}{3}R_3 \to R_1 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\frac{2}{3}R_3+R_2 \to R_2 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Tuesday
10/04/2018

Inner Product space (complex):-

Let "V" be a vector space over "C" suppose ~~two~~ to each pair of vectors $u, v \in V$ There is assigned a complex number denoted by ~~inne~~ $\langle u, v \rangle$ this fn is called (complex) inner product on V if it satisfies the fallowing axioms:

~~$[I_1]$~~ **(I_1^*) (Linear Property)**

$$\langle au_1+bu_2, v\rangle = a\langle u_1, v\rangle + b\langle u_2, v\rangle$$

(I_2^*) (Conjugate Symmetric property)

$$\langle u, v\rangle = \overline{\langle v, u\rangle}$$

(I_3^*) (Positive definite Property)

$$\langle u, u\rangle \geq 0 \text{ and}$$
$$\langle u, u\rangle = 0 \text{ iff } u=0$$

The vectorspace V over C with an inner product is called a (complex) inner product space.

Normed Vector Space

Let 'V' be Real or Complex vector space suppose to each v belongs to V there is assigned a real number denoted by $||V||$ This fn $||\cdot||$ is called a norm on V if it satisfies the following axiom

(N₁) $||V|| \geq 0$ and
$||V|| = 0$ iff $v=0$

(N₂) $||KV|| = |K| \, ||V||$

(N₃) $||u+V|| \leq ||u|| + ||V||$

The Vector Space V with a norm is called a normed vector Space.
Distance b/w u and V is given by

$$||u-V|| = d(u,v)$$

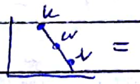
Metric Space:-

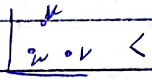
Let V be a normed vector space then the fn $d(u,u) = ||u-V||$ satisfies the following three axioms of a metric space

(M₁) $d(u,v) \geq 0$ and
$d(u,v) = 0$ iff $u=v$

(M₂) $d(u,v) = d(v,u)$

(M₃) $d(u,v) \leq d(u,w) + d(w,v)$





Example

$u = (1,-5,3)$
$v = (4,2,-3)$ in $\mathbb{R}^3$

(Norms)

$l_\infty \Rightarrow \max(|a_i|)$

$l_1 \Rightarrow |a_1| + |a_2| + \cdots + |a_n|$

$l_2 \Rightarrow \sqrt{(a_1)^2 + (a_2)^2 + \cdots + (a_n)^2}$

So $\|u\|_\infty = 5$, $\|v\|_\infty = 4$

$\|u\|_1 = 9$ $\|v\|_1 = 9$

$\|u\|_2 = \sqrt{1+25+9} = \sqrt{35}$, $\|v\|_2 = \sqrt{16+4+9}$

$= \sqrt{29}$

Now

$u - v = (-3, -7, 6)$

$d_\infty(u, v) = 7$, $d_1(u, v) = 16$

$d_2(u, v) = \sqrt{9+49+36} = \sqrt{94}$